How Experts Conceptualize Differentials: The Results of Two Studies

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The mathematical symbol "dx" is a symbol for which there can exist different views about its characteristics, purposes, and roles. We conducted two studies to see how experts viewed the dx in a variety of contexts. For our first study, we interviewed four mathematicians in order to understand their various concept images of the dx, and for our second study, we interviewed two mathematicians and one physicist about both their own concept images and the concept images they would like for their students to have. Overall, we found little agreement among all of the experts' responses, and we believe that that further study of experts' concept images of the differential is warranted.

Keywords: Calculus, Differentials, Concept Image

The differential is a symbol that is found in various, commonly-used mathematical notations, such as the derivative $\left(\frac{dy}{dx}\right)$ and integrals $\left(\int f(x)dx, \int_a^b g(t)\,dt\right)$. However, while these notations are standard, the meaning behind the differentials in these notations is not necessarily so: do those differentials represent small amounts of a quantity (Hu & Rebello, 2013; Von Korff & Rebello, 2012), do they only exist to indicate important variables (Artigue, 1991; Jones, 2015), are they merely notation with no intrinsic meaning (Artigue, 1991; Hu & Rebello, 2013), or can they possess some combination of all three of these meanings (Tall, 1993)? We wished to interview experts about differentials not only in an attempt to understand their concept images (Tall & Viner, 1981) of differentials, but also to see how much agreement existed among all interview subjects.

Our first study (McCarty & Sealey, 2017), conducted during the summer of 2016, involved interviewing four mathematics professors about how they perceived differentials in various contexts. In no context did all four subjects view the differential similarly, and while every context had agreement between some subjects, no two successive contexts had agreement between the same subjects. Three of the four subjects exhibited strong, personal images throughout all contexts, but these individual images were dissimilar, suggesting that no formal, unifying concept image can be found. Our second study, conducted during the summer of 2017, involved interviewing two mathematicians and one physicist about differentials in various contexts, as well as giving them potential concept images of differentials and asking if they would accept these potential images from their students. Again, there were no contexts in which all subjects agreed. Moreover, the responses from the physicist differed markedly from those of the mathematicians, and we found instances where the subjects might hold a concept image that they would not want their students to possess and vice versa.

In both studies, there were many and varied rich concept images suggested by some interview subjects; nevertheless, we conclude that there is no formal concept definition for the differential. We feel that these studies can be used to stimulate additional research, including, but not limited to, deeper study of differential concept images, further explorations into either how mathematician and physicist concept images might differ, or the differences between expert concept images and acceptable concept images from their students. We wish to present our results to encourage feedback and suggestions, as we move forward with this work.

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