Student Intuition Behind the Chain Rule and How Function Notation Interferes

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Recently, Speer and Kung (2016) informed the RUME community on what was missing from our research. In an effort to begin to fill these gaps in the literature, we explored students conceptual understanding of the chain rule in Calculus I classrooms taught by the first author. In this teaching experiment (Steffe & Thompson, 2000), our preliminary results indicate that if students are afforded opportunities to engage in experientially real tasks (Freudenthal, 1991; Rasmussen & King, 2000) on the chain rule, they understand the purpose it serves and can extend that understanding to varied contexts. However, the largest interference to this understanding was function notation, particularly nested function notation. Implications indicate that the instruction of chain rule could be enhanced by preempting a chain rule unit with nested function notation, while still maintaining tasks centered around a conceptual understanding of the chain rule.

Keywords: chain rule, function notation, student mathematical thinking

While little is known about student understanding of the chain rule, much is known about student understanding of function and function notation. From this extensive body of research, we know that challenging activities, particularly constructive ones, aid in the development of the function concept (Carlson, 1998). Additionally, research highlights the importance of linking representations of functions and how that connects to learning function concepts (Even, 1998; Ronda, 2015). Research has also shed light on common misconceptions students have about functions. For example, students have been shown to not fully understand the use and meaning of parentheses in function notation (Carlson, 1998). In this study, we aim to answer the following research question: How do Calculus I students interpret various forms of notation when related to their understanding of the chain rule?

Methods and Preliminary Results

In this teaching experiment, we filmed a full unit of chain rule from two Calculus I sections taught at a public university in the eastern United States. Focus of video data was always on small group work while students were solving tasks designed by the first author. The context for the tasks was as follows: A student, Mary, is taking a hike between two nearby towns. Students were given a graph of Mary's elevation height in terms of time and a table of the temperature of her location given an elevation height. Ultimately students were prompted to develop a need for the chain rule when ascertaining the change in Mary's temperature based on time.

Analysis is ongoing; yet, our preliminary results indicate that students were able to understand the need for the chain rule. That is, if a function r depends on the function s, which itself depends on the variable t, the rate of change of $r(t)$ is the rate of change of r in terms of s times the rate of change of s in terms of t, $\frac{dr}{dt} = \frac{dr}{ds} \cdot \frac{ds}{dt}$. However, when confronted with the parenthetical notation of nested functions, $r(s(t))$, students' ability to generalize the chain rule was impeded. Oftentimes students considered this parenthetical notation to be an indication of multiplication which lead them to misconceptions. Future work will consider ways to redesign instruction to preempt this pitfall in student thinking.

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