Preservice Secondary Mathematics Teachers' Conceptions of the Nature of Theorems in Geometry

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Proof plays an important role in school mathematics curriculum across grade levels and content areas. Being able to understand and apply the axiomatic system, such as with theorems, is considered as a high level of proof and reasoning ability in geometry. By adopting a collective case study design, I investigated preservice secondary mathematics teachers' (PSMTs) conceptions of theorems in geometry, in order to develop knowledge about PSMTs' current conceptions and provide mathematics educators and researchers with a possible means to unpack PSMTs' conceptions. This proposal focuses on one dimension of PSMTs' conceptions, the nature of theorems (NoT) in geometry. The Findings include interpretations of PSMTs' conceptions of the NoT, in terms of the ways they claimed the truth of mathematical statements, examined the validity of given proofs, and disproved given statements, as well as the role of task-based interviews in understanding their conceptions.

Keywords: theorems, conceptions, proof and reasoning, geometry, teacher education

Proof and theorems form part of the core content of secondary geometry curriculum, and should be well grasped by secondary math teachers and their students (NCTM, 2000, 2003, 2012). Studies show that both secondary teachers and students have encountered challenges in teaching and learning proofs (Cirillo, 2009; Knuth, 2002; McCrone & Martin, 2004; NCES, 1998; Senk, 1985). In this study, I examined the essential elements of three PSMTs' conceptions of the NoT through research-informed task-based interviews, in order to answer the research questions: What conceptions do PSMTs hold regarding the NoT in geometry? And how do research-informed task-based interviews help unpack PSMTs' conceptions of the NoT in geometry?

I created a set of principles of the NoT that served as the conceptual framework for the development of the task-based interviews, including the elements *theorem has to be proved (NoT 1), theorem is true for all instances (NoT 2),* and *one counterexample is sufficient to disprove (NoT 3)* (Cirillo, 2014; Dreyfus & Hadas, 1987; Duval, 2007; McCrone & Martin, 2004). Each of the PSMTs participated in an individual task-based interview that addressed the above principles. The data analysis process started by "dividing the overall data set into categories or groups based on predetermined typologies" (Hatch, 2002, p. 152). An analytical framework was developed to identify the typologies of the data, including the definitions of PSMTs' *goals of the task*, *goal-directed activities (GAs)*, *sequence of actions* within the GAs, and *effects* of their GAs (Simon, Tzur, Heinz, & Kinzel, 2004; Tzur, 2007; Tzur & Simon, 2004).

The findings included interpretations about PSMTs' clarity of understanding about NoT 1, confusion about NoT 2 that the validity of the proving result and the validity of the proving process could be evaluated separately, and varied understandings about NoT 3 in terms of the definition of a counterexample and its role in disproving. In addition, the study discussed the role of the task-based interviews, in terms of providing an accessible problem-solving environment, encouraging free problem-solving, encouraging PSMTs' reflection, and letting the researcher be open to unforeseen activities during the interview (Goldin, 2000; Lin, Yang, Lee, Tabach, & Stylianides, 2012). The implications of the use of prompts in the interviews were also discussed.

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