Mathematical Reasoning and Proving for Prospective Secondary Teachers

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The design-based research approach was used to develop and study a novel capstone course: Mathematical Reasoning and Proving for Secondary Teachers. The course aimed to enhance prospective secondary teachers' (PSTs) content and pedagogical knowledge by emphasizing reasoning and proving as an overarching approach for teaching mathematics at all levels. The course focused on four proof-themes: quantified statements, conditional statements, direct proof and indirect reasoning. The PSTs strengthened their own knowledge of these themes, and then developed and taught in local schools a lesson incorporating the proof-theme within an ongoing mathematical topic. Analysis of the first-year data shows enhancements of PSTs' content and pedagogical knowledge specific to proving.

Keywords: Reasoning and Proving, Preservice Secondary Teachers, Design-Based Research

Our NSF-funded 3-year project addresses the limited practical and theoretical knowledge base on how to prepare PSTs to teach in ways that emphasize mathematical reasoning and proving (Ko, 2010; Stylianides & Stylianides, 2015). We designed, implemented and studied a novel capstone course: *Mathematical Reasoning and Proving for Secondary Teachers*. The focus of reasoning and proving was motivated by the persistent discrepancy between the value of proof as advocated by researchers (e.g., Hanna & deVillers, 2012) and policy documents (NCTM, 2009; CCSS, 2010) and the marginal place of proof in school mathematics, which is often viewed by students and teachers alike as redundant confirmation of known results, rather than a means for deepening understanding (Knuth, 2002; Kotelawala, 2016).

The course consists of modules corresponding to four proof-themes: quantified statements, conditional statements, direct proof and indirect reasoning, which were identified in the literature as challenging for students and PSTs (Antonini & Mariotti, 2008; Weber, 2010). Each module has activities to enhance PSTs' knowledge of a certain proof theme, followed by developing and teaching lessons at a local school integrating that proof-theme with current mathematical topics.

We used multiple sources of data to evaluate how PSTs' knowledge of content and pedagogy, and their dispositions towards proof evolved throughout the course. These included pre- and post- measures of mathematical knowledge for teaching proof and dispositions towards proving. We collected PSTs' lesson plans and 360° video-recordings of their lessons, which captured simultaneously the PSTs teaching performance and the school students' engagement with proof-oriented lessons. The PSTs also submitted self-reflections after each lesson, and cumulative teaching portfolios at the conclusion of the course.

Preliminary data analysis of the first-year course implementation shows improvement in PSTs' content knowledge of the four proof-themes. The repeated cycles of lesson development, implementation and video-supported reflection contributed to PSTs' pedagogical knowledge for proving. However, analysis reveals that integrating the proof-themes with pedagogical practices can be challenging for PSTs. To better support PSTs in this aspect, we plan to further conceptualize and enhance instructional scaffolding of the course in the subsequent iterations of the study. Through this process we seek to generate an evidence-based instructional model, and four proof-modules that can be adopted by other courses or institutions to improve preparation of secondary mathematics teachers.

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