Children's Topological Thinking

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Abstract: *This poster presents an ongoing investigation into children's topological thinking. Prior research identified and advanced children's informal ideas about topological equivalence – and equivalence and invariance more broadly. This investigation extends that research into children's thinking about related ideas such as order, separation, and coverings. Newly identified forms of geometric thinking have implications for the teaching and learning of geometry and for research into students' mathematical thinking.*

Keywords: Geometric reasoning, Topology, Equivalence, Teaching experiment

Children's experiences in geometry throughout elementary school are entirely Euclidean. However, research finds that they also possess intuitive topological ideas (Greenstein, 2014; Laurendeau & Pinard, 1970; Piaget & Inhelder, 1956). These findings lay the foundation for the claim that there are forms of topological reasoning available to young learners that can be identified as mathematical, are significant, and can be seen to develop in ways that would have implications both for research into students' mathematical reasoning and as a focus for further curriculum development and design.

The poster we are proposing will illustrate our current investigation into children's thinking about topological equivalence, as well as the foundations for this investigation in prior research (Greenstein, 2014). That research found that a microworld for topological equivalence (Greenstein, 2017) supported two children's constructions of ways of thinking about topological equivalence. They used those schemes (von Glasersfeld, 1995) to build equivalence classes of shapes and identify the properties of shapes within equivalence classes. Broadly speaking, it was evident from this investigation that engagement with topology provides learners with powerful forms of mathematical engagement that are not available to them in Euclidean geometry.

Our current investigation seeks to develop a superseding model (Steffe & Thompson, 2000) of children's thinking about topological equivalence, and extend the focus of our prior research into additional aspects of equivalence relations that arise in the context of topology, including notions of order, covering, and separation. For example, through their investigations of topological equivalence and invariance, children are also engaging with the ideas of order through examinations of points along a curve; of coverings through a task that provides the child with a square and calls for a collection of shapes that can adequately cover that square; and of separation through a task that provides a collection of distinct points and calls for shapes that can be used to confirm their separation.

Findings from this study are beneficial to students whose topological ideas have yet to be engaged in schools and also to the community of mathematics educators whose research has only nominally investigated them.

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