

Development of Students' Mathematical Discourse through Individual and Group Work with Nonstandard Problems on Existence and Uniqueness Theorems

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Research shows that students' learning is affected by the types of tasks. We explore how the use of nonstandard problems influences understanding of the Existence and Uniqueness Theorems (EUTs) by a group of engineering students. The focus is on the development of students' mathematical discourse during the individual and group work with nonstandard problems. We present the evidence indicating that students developed new mathematical routines gaining a deeper understanding of EUTs and appreciated the experience.

Key words: ordinary differential equations, existence and uniqueness theorems, design research, mathematical discourse, individual work, group work, nonstandard problems.

Description of the Study

Ordinary differential equations (ODEs) is one of important post-calculus courses in university STEM (Science, Technology, Engineering and Mathematics) education. Nevertheless, the recent review of the literature related to research on ODEs in undergraduate education during the last decade surveys only about twenty papers dealing with the understanding of the concepts of solution of an ODE, a system of ODEs and bifurcation (Rasmussen and Wawro, 2017). EUTs are among very few theoretical results included nowadays in standard ODE courses for engineering students. Understanding and the correct use of the EUTs present serious challenges for students (Raychaudhuri, 2007), as even the concept of a solution of an ODE itself (Arslan, 2010). In our study, students consecutively produced three scripts of solutions to the set of six nonstandard problems designed by the lecturer to challenge students' conceptual understanding of the EUTs: individual solutions obtained in the first tutorial, individual solutions submitted as a homework, and solutions submitted after the discussion with peers in small groups and group presentations of solutions during the second tutorial. We analyzed three scripts, pre- and post-activity surveys, and audio recordings of the peer discussions and of the presentations.

Research Questions

- 1) How can nonstandard problems challenge students and help to develop analytical skills and further conceptual understanding of mathematical routines in an ODE course?
- 2) To what extent have individual work and group discussions contributed to the development of students' mathematical discourse?

Conclusions

Working on the problems, students made use of theorems and definitions, generalized and designed examples, verified validity of statements and analyzed reasoning. All these practices promoted students' conceptual understanding and contributed to the development of a new mathematical discourse because, using a commognitive lens, "learning mathematics means modifying one's present discourse so that it acquires the properties of the discourse practiced by mathematical community." (Sfard, 2009). We believe that lecturers should ask more nonstandard questions that they know their students will find difficult and may not be able to answer, and do it more often. Our research has shown that students valued the experience and gained a deeper understanding of the EUTs.

References

- Arslan, S. (2010). Do students really understand what an ordinary differential equation is? *International Journal of Mathematical Education in Science and Technology*, 41, 873-888.
- Rasmussen, C., & Wawro, M. (2017). Post-calculus research in undergraduate mathematics education In J.Cai (Ed.), *Compendium for Research in Mathematics Education* (pp. 551-581). Reston, VA: National Council of Teachers of Mathematics.
- Raychaudhuri, D. (2007). A layer framework to investigate student understanding and application of the existence and uniqueness theorems of differential equations. *International Journal of Mathematical Education in Science and Technology*, 38, 367-381.
- Sfard, A. (2009). Moving between discourses: from learning-as-acquisition to learning-as-participation, AIP Conference Proceedings 1179, 55 (2009); <https://doi.org/10.1063/1.3266753>.