An APOS Perspective of Meaning in Mathematics Teaching

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The 'meaning of' mathematics can be thought of as mathematical understandings whereas the 'meaning for' mathematics can be understood as understanding the significance of math for non-mathematical purposes. Studies have suggested instructors have difficulty addressing both senses of meaning simultaneously while other studies have indicated factors that affect graduate teaching assistants' (GTA) instruction. Using APOS theory as a theoretical lense, this study examines how these factors affect GTA instruction of the derivative and in turn, how GTAs navigate differing senses of meaning. Through interviews, the researcher found many parallels between GTA instruction and proposed decompositions of the derivative. Regarding meaning, the researcher found when GTAs experience tension between the two senses of meaning, instructional decisions may be taken that anticipate GTA instructional concerns.

Keywords: APOS Theory, Mathematical Meaning, Derivative, Graduate Teaching Assistants

Brownell (1947) defined the "meaning of" mathematics as mathematical understandings and the "meaning for" mathematics as understanding its significance. Studies on instruction in certain contexts, like service learning, have shown tension between these senses of meaning for instructors (Carducci, 2014; Connor, 2008; Donnay, 2014; Hadlock, 2013; Rousseau, 2004; Schulteis, 2013; Zack & Crow, 2013). Whether this occurs in a 'typical' math class needs further study. With respect to graduate teaching assistants' (GTAs') instruction, studies have identified factors such as content knowledge, responsibilities, and control (Addy & Blanchard, 2010; Bond-Robinson & Rodriques, 2006; Hill, Rowan & Ball, 2005). By adapting APOS theory as done Martin, Loch, Cooley, Dexter, and Vidakovic (2010), a decomposition of the derivative was used to categorize instruction while the framework of meaning categorized the 'why' behind those decisions. By doing so, this study aimed to see how affective factors effect instruction.

Participants were mathematics GTAs (Ann and Inigo). Data included interviews and emails which explored instruction of the derivative, beliefs, concerns, and instructional goals. Using a research-based genetic decomposition (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997; Cottrill, Dubinsky, Nichols, Schwingendorf, Thomas, & Vidakovic, 1996; Hähkiöniemi, 2006), responses on instruction were coded as action, process, or object depending on level of elicited understanding while the reasons for choices were coded as "meaning of" or "meaning for"

Results showed the GTAs covered much of the decomposition, eliciting action up through object level understandings. On derivative rules, Inigo (lacking content control, but content with the set syllabi) would have students go through derivations while Ann (who took issue with the syllabi and mentioned competing responsibilities) only would in some cases to be able to stay on schedule. Cutting engagement with derivations subsequently cuts engaging with the limiting process and perhaps results in a pre-object understanding of derivatives as noted by Zandieh (2000). Interestingly, Ann was concerned students do not connect limits and calculus. If pressure to cover material is a case of focusing on the 'meaning for' and the strictly conceptual aims are cases of 'meaning of', attending to the 'meaning for' seems to have anticipated a teaching concern for Ann. While preliminary, perhaps understanding meaning for instructors may serve as an organizing framework of how affective factors reciprocally influence instruction.

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