University Teachers' Meanings for Average Rate of Change: Impacts on Student Feedback

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Previous research that has used Thompson's mathematical "meanings" framework has focused on secondary teachers' meanings for mathematics. We examine the meanings that graduate students and professors hold for average rate of change. Further, we attempt to connect meanings to the facets of student work that graduate students and professors notice. This work lets us start to extend the meanings framework and has implications for graduate student professional development.

Keywords: Meanings, Mathematical Knowledge for Teaching, Average Rate of Change

Using two average rate of change items from the Mathematical Meanings for Teaching Secondary Mathematics (MMTsm) (Thompson, 2016; Yoon et al., 2015), we designed a task-based interview protocol and gathered written and interview data from graduate students and professors to answer these questions 1) what mathematical meanings do graduate students and professors hold for average rate of change (AROC)? 2) Based on these meanings, what do graduate students and professors notice about students' meanings of AROC displayed in written work? Goals for this poster are to present the interview design as well as the results from at least three pilot interviews, and for RUME attendees to discuss how the mathematical meanings framework (Thompson, 2016) connects to Ball and colleagues' framework for mathematical knowledge for teaching (Ball et al., 2008). These discussions will help shape connections we draw between the frameworks after further analysis and data collection.

Average rate of change is a concept commonly included in pre-calculus and calculus curricula in universities across the US. It has been identified as a concept that students *must* grasp to be prepared for calculus (Carlson et al., 2003). Recent work has identified meanings that secondary teachers hold for AROC (Yoon et al., 2015) and for underlying concepts that support AROC such as quotient, measure, covariation, and rate of change (Byerley & Thompson, 2017). In this vein of research, the focus is the teacher-centric personal meaning that each teacher holds for the mathematics they teach. By examining this construct, we can draw conclusions about the ways that student meanings may be supported or limited by teachers' meanings. By utilizing the mathematical meanings framework to examine university professor meanings, we aim to build on existing work by drawing connections between professors' and graduate teaching assistants' meanings and what they notice in student work on AROC tasks.

Results from pilot interviews suggest that graduate students may not have fully coherent meanings for AROC. Interviews were analyzed by coding for meanings identified in Yoon and colleagues' (2015) study. On the MMTsm items, one student conveyed a weak meaning for unit, and another student conveyed a formulaic meaning as the arithmetic mean. Both students were limited by a meaning for rate of change as the slope of a secant line, which made it difficult to convey their meaning in any way besides a graphical representation. Interestingly, in contrast to the professor, both graduate students' noticing of the mathematical meaning in student written work was framed by how similar their meaning was to that of the student. These findings can be used to further refine the meanings framework and inform efforts to design professional development for graduate students.

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