Research on Concept-based Instruction of Calculus

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Abstract: This study, involving 254 college-level calculus students and 3 teachers, investigated the misunderstanding of concepts in calculus and designed concept-based instruction to help students understand concepts. Multiple achievement measures were used to determine the degree to which students from different instructional environments had mastered the concepts and the procedures. The midterm examination and the final examination results showed that the students enrolled in the concept-based learning environment scored higher than the students enrolled in the traditional learning environment and the investigation at the end of the semester showed that most of students like the concept-based learning environment.

Key words: Concept-based Instruction, Misunderstandings, Teaching design

In the context of mass higher education, the ability of college freshmen is generally in a lower level than before. Many college students can do simple works on calculus, but they cannot understand the idea behind the concept, and as a result, usually have fuzzy understanding of the relationship between concepts. Therefore, to find the cognitive difficulties of the students on the concepts of calculus and to design the concept instruction are the keys to the reform of the teaching on Calculus.

This research presented a study on calculus course in three freshmen classes by carrying out the teaching design and teaching experiment. Research methods such as design research, questionnaires, interviews and classroom observation were adopted. There were 3 teachers and 254 students participated in the practice. Based on the findings of this study, the following conclusions could be drawn:

Firstly, college students' concept image of the fundamental concepts of calculus was one-sided, and some even wrong. Some students couldn't define the limit by correct words. Most of the students usually thought of the slope of the tangent when seeing the derivative, rather than the rate of change. There was confusion in the understanding of the geometrical meaning of differential and linear approximation. Some students know that the definite integral can express the area, but they can't make sure the area of what region; some students did not know which amount was sliced when they calculated the integral.

Secondly, we constructed principles on concept instruction in calculus as follows: (1) Concepts were introduced and demonstrated in a genetic way. (2) Help students understand the concepts by means of geometric or intuitive examples. (3) Paying attention to the elaboration of the relations of the concept between them. The results of teaching experiment showed that the students enrolled in the concept-based learning environment scored higher (M=34.42) than the students enrolled in the traditional learning environment (M=30.27) on the 40 point Conceptual Understanding Subscale and the students enrolled in the concept-based learning environment scored significantly higher (M=48.68) than the students enrolled in the traditional learning environment (M=42.65) on the 60 point Procedural Skill Subscale in the examination.

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