Multivariable Calculus Textbook Analysis Highlights a Lack of Representation for Non-Cartesian Coordinate Systems

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Upper-division undergraduate physics coursework necessitates a grasp of mathematical knowledge, including an understanding of non-Cartesian coordinate systems. To fully grasp what upper-division physics' students understanding of non-Cartesian coordinates is, it is worthwhile to study the mathematics course where non-Cartesian coordinate systems are taught most extensively, Multivariable Calculus. Seven Multivariable Calculus textbooks were examined using content analysis techniques. Additionally, textbook items in four textbooks were qualitatively coded by coordinate system. Results indicate that there were few instances where non-Cartesian coordinate systems were present. These findings suggest that before upper-division physics coursework, students' instruction on non-Cartesian coordinate systems is minimal and that it might be difficult for students to employ mathematical techniques that involve non-Cartesian coordinates in their upper-division courses.

Keywords: Content Analysis, Non-Cartesian Coordinates, Multivariable Calculus

Understanding non-Cartesian coordinate systems is essential for upper-division physics courses. Published literature suggests that student understanding of non-Cartesian coordinate systems is weak; studies by Moore, Paoletti, and Musgrave (2014) observed mathematics students having continued difficulty with polar coordinates after taking mathematics through Calculus III (Multivariable), and studies by Sayre and Wittman (2007) of junior-level physics students also suggested that students' understanding of the polar coordinate system was still under formation when compared to their understanding of Cartesian coordinate systems. Multivariable Calculus textbooks typically introduce three-dimensional non-Cartesian coordinate systems and study polar coordinate systems at a greater depth. This study examines seven textbooks as sources that can potentially enable or obstruct student understanding of non-Cartesian coordinate systems. To capture a comprehensive examination of these textbooks, qualitative content analysis and quantitative content analysis were performed. Qualitative analysis techniques were used, for example, to examine the coordinate systems new topics were introduced in. Quantitative content analysis categorized examples, definitions, and problems/exercises according to their coordinate system(s). Results demonstrated that non-Cartesian coordinate system representation was minimal. New Multivariable Calculus topics were always introduced in Cartesian coordinates and sometimes did not utilize non-Cartesian coordinates at all. Further, only 21% of textbook chapters included any instance of non-Cartesian coordinates. Of those chapters, 73% of items qualitatively coded according to their coordinate systems were Cartesian. When present, these instances of non-Cartesian coordinate systems often involve simply converting from one coordinate system to another rather than posing questions that elicit a higher level of understanding of when to apply particular coordinate systems. This work implies that Multivariable Calculus textbooks do not require a high level of understanding of non-Cartesian coordinate systems, suggesting that textbooks, which serve as a resource for professors and students, could be part of what limits student understanding and application of non-Cartesian coordinate systems at higher levels of mathematics and physics.

References

Moore, K. C., Paoletti, T., & Musgrave, S. (2014). Complexities in students' construction of the polar coordinate system. *Journal of Mathematical Behavior, 36,* 135-149.

Sayre, E.C., & Wittmann, M. C. (2008). Plasticity of intermediate mechanics students' coordinate system choice. *Physical Review: Special Topics- Physical Education Research, 4,* 020105.