Historical Analysis on Predictive Practices: The Case of Chaotic Dynamics

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This poster focuses on the historical analysis of three main characters of the history of chaos: the Poincare's error in his memoire about the three-body problem, the ideas of Edward Lorenz about the deterministic non-periodic flow, and the work of Robert May about the logistic map. This into the Variational Thinking and Language research program from the Socioepistemological Theory. The results show a predictive practice characterized for four main actions: to search periodicities, to recognize the uncertain, to compare temporal states, and to classify kinds of behaviors. We assume that the promotion of landscapes and activities out of school are a way for construct specialized mathematical knowledge, and the incorporation of the ability to wait for the unexpected is necessary for teachers and students living this century.

Keywords: Variation, Chaotic Dynamics, Historical Analysis, Socioepistemology,

Introduction and motivation

Faced with the lack of meaning in the central ideas of the mathematics of change, mainly with the notion of variation (Carlson, Jacobs, Coe, & Hsu, 2003; Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2012; Thompson, Byerley, & Hatfield, 2013; Tall, 2013; Cuevas, 2014; Moreno, 2014), this research looks at the dynamical systems, focus on chaotic ones, as source of mathematical objects that provides different kind of examples, behaviors and *practices* that, currently, are not part of the scholar context. This work assume that Education is not synonymous of schooling but only an aspect (UNESCO, 2012; Rosas Colin, 2014; Valero, 2015), and the promotion of landscapes and activities out of school are a way to construct specialized mathematical knowledge. In another hand we agree with the Morin's idea about the humans living in an uncertain world and the incorporation of the ability to wait for the unexpected (Morin, 1999), and Ghys' discourse concerning to the necessity of include (not only) in mathematics career specific training to teach topics like chaos theory in order communicate these ideas to other scientist or non-scientist (Ghys, 2015).

The fundamental objective of this study is the *search and characterization of the actions* in the transition from the stable periodicities to unstable ones (transition from predictable to unpredictable), first doing a historical analysis looking for *actions over mathematical objects* and the *description of the phenomena* where this kind of behavior appears, and second recollecting data that supports the actions subtracted from the precedent analysis. This poster focuses on the analysis of three main characters of the history of chaos: the Poincare's error in his memoire about the three-body problem (Poincaré, 1898; Barrow-Green, 1997), the ideas of Edward Lorenz about the deterministic non-periodic flow (Lorenz, 1963; 1993) and the methods and tools used by Robert May in the analysis of the logistic map traying to understand the complicated dynamic from a "simple" mathematical model (May, 1974; 1976).

From the Socioepistemological methodology (Cantoral, 2016) over the historical analysis some results show the evolution of a *predictive practice* that is characterized for four main emergent actions over mathematical and physical objects: to search periodicities, to recognize the uncertain, to compare temporal states, and to classify kinds of behaviors. These actions will lead the construction of an experimental instrument to get data about how students and teachers in the last semester of high school and the first of university in scientific and engineer careers face the chaotic behavior.

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