Adaption of Sherin's Symbolic Forms for the Analysis of Students' Graphical Understanding

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We describe a methodological presentation of Sherin's (2001) symbolic forms, discussing adaptions made to the framework to analyze graphical reasoning. Symbolic forms characterize the ideas students associate with patterns in an expression. To expand symbolic forms beyond equations, we supplement it with another framework that considers modeling as discussing mathematical narratives. This affords the language to describe how students think about the process or "story" that could have given rise to a graph. By considering registrations in general terms as structural features students attend to (parts of the "story"), when students assign ideas to registrations (parts of an equation or regions of a graph), they are using symbolic forms.

Keywords: mathematical reasoning, symbolic forms, rates, chemistry

Sherin (2001) developed symbolic forms as a means to characterize how students used mathematical ideas to reason about equations when solving problems in physics. This framework has its roots in the constructivist idea of "phenomenological primitives" (p-prims), which describe intuitive ideas developed based on experience (Bodner, 1986; diSessa, 1993). Symbolic forms can be seen as mathematical p-prims, involving students associating ideas (conceptual schema) with a pattern of symbols (symbol template); for example, students associating the idea of "balancing" with the symbolic form "$\square = \square$", where the boxes are generic placeholders for algebraic terms (Sherin, 2001). This is important because without explicit instruction students associate ideas with patterns that are productive when learning concepts (e.g., opposing forces in physics). This framework has been utilized across different discipline-based education research (DBER) fields to explore student understanding of integration, the differential (*dx)*, area and volume, and mathematical expressions in physics and chemistry (Becker & Towns, 2012; Jones 2013, 2015a, 2015b; Dorko & Speer, 2015; Marredith & Marrongelle, 2008; Von Korff & Rubello, 2014). We assert students have similar ideas about graphs and seek to expand symbolic forms to move beyond equations, which has broad applicability across DBER fields.

A central tenet of our adaption of symbolic forms to graphical reasoning is Nemirovsky's (1996) conceptualization of "mathematical narratives" as the integration of events with symbolic notations (i.e., modeling). Nemirovsky (1996) used mathematical narratives to focus on student descriptions of "stories" that could give rise to a particular graph in the context of graphical representations of velocity, distance, and time. Viewing modeling as "story-telling" is particularly useful when considering students' graphical reasoning because it provides the language to describe students' discussion of the series of events represented by a graph. In the literature "registrations" have been used to describe features students focus on in computer simulations; we adopt this terminology to describe structural features students attend to in representations, and when students "register" or associate specific ideas with these features, they are reasoning using symbolic forms (Roschelle, 1991; Sengupta and Willensky, 2009).

Although it has been suggested that symbolic forms can be adapted to graphical reasoning, in practice it has not yet been taken up in the literature (Izak, 2000; Lee & Sherin, 2006; Sherin, 2001). Our presentation will provide examples of how we functionalize this adapted framework, using chemistry as a rich context to study students' reasoning associated with graphs that describe the rate of change of chemical compounds over time, since research has shown that students have difficulty with ideas related to the derivative and rate (Orton, 1983; Rasmussen, Marrongelle, & Borba, 2014; White & Mitchelmore, 1996).

References

Becker, N., & Towns, M. (2012). Research and Practice Students' understanding of mathematical expressions in physical chemistry contexts : An analysis using Sherin's symbolic forms. *Chemistry Education Research and Practice*, *13*, 209–220.

Bodner, G. M. (1986). Constructivism: A Theory of Knowledge. *Journal of Chemical Education, 63*(10), 873-878.

diSessa, A. A. (1993). Toward an Epistemology of Physics. *Cognition and Instruction,* 10(2-3), 105-225.

Dorko, A., & Speer, N. (2015). Calculus students' understanding of area and volume units, *Investigations in Mathematics Learning, 8*(1), 23-46.

Izak, A. (2000). Inscribing the Winch: Mechanisms by Which Students Develop Knowledge Structures for Representing the Physical World With Algebra, *Journal of the Learning Sciences, 9*(1), 31-74.

Jones, S. R. (2013). Understanding the Integral: Students' Symbolic Forms. *Journal of Mathematical Behavior, 32*(2), 122-141.

Jones, S. R. (2015). Areas, anti-derivatives, and adding up pieces: Definite integrals in pure mathematics and applies science contexts, *Journal of Mathematical Behavior*, *38*, 9-28.

Jones , S. R. (2015). The prevalence of area-under-a-curve and anti-derivative conceptions over Riemann sum-based conceptions students' explanations of definite integrals, *International Journal of Mathematical Education in Science and Technology, 46*(5), 721-736.

Lee, V. R., & Sherin, B. (2006). Beyond transparency: How students make representations meaningful. In S. Barab, E. Hay, & D. Hickey (Eds.), *Proceedings of the 7th International Conference on the Learning Sciences* (pp. 397-403). Mahway, NJ: Erlbaum.

Meredith, D. C., & Marrongelle, K. A. (2008). How Students Use Mathematical Resources in an Electrostatics Context. *American Journal of Physics, 6, 570-578.*

Nemirovsky, R. (1996). Mathematical narratives, modeling, and algebra. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra: Perspectives for research and teaching* (pp. 197–220). Dordrecht, The Netherlands: Kluwer Academic.

Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics*, *14*(3), 235-250.

Rasmussen, C., Marrongelle, K., & Borba, M. C. (2014). Research on calculus: what do we know and where do we need to go? *ZDM Mathematics Education*, *46*, 507–515.

Roschelle, J. M. (1992). Students' construction of qualitative physics knowledge: Learning about velocity and acceleration in a computer microworld. University of California, Berkley.

Sengupta, P., & Wilensky, U. (2011). Lowering the learning threshold: Multi-agent-based models and learning electricity. In *Models and Modeling* (pp. 141-171). Springer Netherlands.

Sherin, B. L. (2001). How Students Understand Physics Equations. *Cognition and Instruction, 19*(4), 479-541.

Von Korff, J., & Rubello, N. S. (2014) Distinguishing between "change" and "amount" infinitesimals in first-semester in calculus-based physics, *American Journal of Physics, 82,* 695-705.

White, P., & Mitchelmore, M. (1996). Conceptual Knowledge in Introductory Calculus. *Journal for Research in Mathematics Education*, *27*(1), 79–95.