Constant Rate of Change: The Reasoning of a Former Teacher and Current Doctoral Student

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In this work, I provide brief illustrations of multiple ways of reasoning about constant rate of change that I observed in a mathematics education doctoral student's activity when tasked to draw graphs relating two varying quantities. These ways of reasoning suggest that textbook authors and instructors critically examine those illustrations and experiences provided to students in order for students to come away from mathematics courses with consistent and productive reasonings about rate of change.

Keywords: Covariational Reasoning, Rate of Change, Calculus

Researchers have reported that a productive meaning for the idea of *rate of change* involves one to conceptualize relationships between two covarying quantities (Thompson, 1994). Covariational reasoning describes the mental actions involved in one coordinating such varying quantities (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). In recent decades, researchers have observed and characterized students' and teachers' mental actions while engaged in tasks to model covarying quantities (Carlson et al., 2002; Castillo-Garsow, 2012; Coe, 2007). Researchers have also identified students and teachers having difficulty with covariational reasoning (Carlson et al., 2002; Johnson, 2015). In particular, Musgrave and Carlson investigated mathematics graduate students' meanings of average rate of change (Musgrave & Carlson, 2016). They found that these students often held non-conceptual meanings for average rate of change that were primarily focused on computations or geometric interpretations involving secant lines on graphs. With the goal of developing students' and teachers' covariational reasoning, it is productive to construct models of individuals' thinking about rate of change as a means for creating rich experiences in which students and teachers can develop sophisticated ways of reasoning. The work of this study contributes to expanding and broadening models of individuals' covariational reasonings by providing insights into those reasoning processes that continued mathematics users (i.e., mathematics teachers turned graduate students) engage in.

In this poster, I present multiple and inconsistent ways of reasoning about constant rate of change that I observed in the activity of one mathematics education doctoral student with high school mathematics teaching experience. The study involved clinical interviews in which I asked the participant to draw graphs relating two varying quantities in an animated situation. The participant's reasonings resulted in inconsistent conclusions and suggest that he did not interpret or describe rate of change covariationally by imagining changes. I characterize these ways of reasoning as tangent line reasoning and constant ratio reasoning. *Tangent lines* involved the participant constructing and reasoning geometrically with tangent lines he constructed on his graph. *Constant ratio* involved the participant identifying that the two accumulated quantities he was graphing could be related computationally by a scale factor of some fixed unit magnitude of each quantity (which he identified as the "constant" in a constant rate of change relationship). These ways of reasoning did not seem productive for the participant and yet seem to be suggestive of certain non-quantitative curricular treatment of rate of change. These illustrations suggest that mathematics educators and textbook authors critically examine those reasonings of their students and the experiences they provide students in order for students to develop more consistent and productive reasoning abilities about rate of change.

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