How Diagrams are Leveraged in Introduction to Proof Textbooks

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According to research, diagrams can play a vital role in the constructing and understanding of proofs (Samkoff, Lai, & Weber, 2012). Introduction to Proof (ITP) courses are usually a student's first exposure to proofs. Therefore, the ITP curriculum reflects important opportunities for students to develop proof construction and proof understanding skills. We analyzed how diagrams were presented in the top four market share ITP textbooks across a set of standard topics. Through this process, we categorized the role and nature of diagrams in the curricula. We found that a majority of diagrams were used to illustrate statements and definitions. Other important roles such as supporting proof construction, building conjectures, or finding counterexamples were infrequent.

Keywords: Introduction to Proof, diagrams, textbook analysis

Mathematicians find using diagrams beneficial when constructing and understanding proofs (Samkoff et al., 2012). ITP courses are usually a student's first exposure to proofs. During ITP courses, students develop their proof skills. Textbooks reflect an important component of the intended curriculum and opportunities for students to learn (e.g., Thompson, Senk, & Johnson, 2012). We conducted a textbook analysis to explore how diagrams are leveraged in the curricula. In particular, we investigated how diagrams are used in the proving process in ITP textbooks.

We selected four ITP textbooks to analyze: Chartrand, Polimeni, and Zhang (2012), Hammack (2013), Smith, Eggen, and St. Andre (2010), and Velleman (2006). David and Zazkis (2017) identified these four ITP textbooks as having the top market share use in ITP courses, as well as, covering standard topics for ITP courses: sets, logic, proof techniques, relations, functions, and cardinality. We identified the textbook sections corresponding to these topics then used a thematic analysis (Braun & Clarke, 2006) approach to open-code and develop themes related to the nature and role of diagrams. We coded 173 diagrams across the textbooks.

We identified the following roles of diagrams in ITP textbooks: conjecturing (CJ), identifying counterexamples (CE), instantiating definitions (DEF), instantiating statements (ST), illustrating a procedure (PRO), illustrating the key idea to a proof (KIP), organizing/synthesizing information (ORG), being part of the proof (PP), and other. Most diagrams were used to instantiate definitions and theorems, which only represents a small subset of how mathematicians use diagrams (Samkoff et al., 2012). The poster contains a detailed comparison of same topics across different textbooks.

Table 1. *Frequency of the role of diagrams in the standard ITP topics across four textbooks.*

Topic	CJ	CE	DEF	ST	PRO	KIP	ORG	PP	OTHER
Sets	0	1	38	11	2	3	0	1	5
Cardinality	3	2	13	5	2	13	0	0	0
Relations	0	0	34	1	0	2	3	1	0
Functions	2	1	17	5	1	0	2	4	1
Total	5	4	102	22	5	18	5	6	6
Percentage	2.89%	2.31%	58.96%	12.72%	2.89%	10.4%	2.89%	3.47%	3.47%

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