Student's Attention to the Conclusion During Proofs

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This study investigates students' use of conclusions to structure their proofs for a standard statement in introductory Group Theory. We surveyed 65 students across three classes asking them to evaluate the truth of a statement and provide a proof. We found students tend to use hypothesis-driven second level proof framework (rather than conclusion-driven). These students were then less likely to produce a deductive argument that aligned with the original statement. We conclude with implications for the treatment of proof analysis and proof frameworks to support students' proving activity.

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In courses, such as group theory, students frequently prove statements about structure-preserving properties such as the following statement: Let *f* be an isomorphism from (G, o) to (H, *). If *G* is an abelian group, then *H* is an abelian group. In order to approach such statements, students must structure their proofs around the conclusion to argue about arbitrary elements of *H* rather than arguing about the image of elements in *G.* We designed a study to test the conjecture that students do not necessarily attend to the conclusion when proving. We surveyed 65 students across three group theory classes using either the isomorphism prompt (n=32) or an alternate false version with 1-1 homomorphism missing the necessary requirement of onto (n=33).

To analyze students' proof approaches, we use two framings: proof frameworks (Selden & Selden, 1995) and proof analysis (c.f., Marchi, 1980; Lakatos, 1976). The proof framework is the "representation of the 'top-level' logical structure of a proof" (p. 129) which is tied directly to the statement to be proven. In order to approach the isomorphism prompt above, one option is to employ the appropriate second-level proof framework (Selden & Selden, 2015): using the conclusion to structure proof (i.e. starting with elements in *H*). An alternate approach would be the selection of a second level proof framework beginning with elements in *G*, arriving at a statement about the images of these elements then using proof analysis (c.f., Marchi, 1980) to recognize that the deductive argument does not align with the statement. We coded surveys based on (1) second-level proof frameworks, (2) validity, and (3) proof corrections.

We found that students used a *G*-first proof framework (n=39) compared to *H*-first (n=17) at a rate significantly higher than chance (p=0.0016). This approach was consistent across the true and false prompt where students produced deductive arguments about the image of *G* rather than *H*. For the true statement, we further analyzed the likelihood of arriving at a valid deductive argument finding that only 2 of 16 *G*-first students arrived at a valid proof with 7 of 9 *H*-first students arriving at a valid proof, a statistically significant difference. Our results reflect that many students are not attending to the conclusion of statements when proving. Instructors may need to work with students to help the students understand the importance of using the conclusion to structure the proof. Further, proof analysis techniques (comparing the statement and deductive proofs, searching for counterexamples) could also support students in producing arguments that better align with original statements.

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