Transformers! More than Meets the Eye!

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In this study, we characterize a conceptual model some students draw upon in their problem-solving activity when engaged in definite integral tasks. We call this model an Integral as a Transformer conception as it is invoked by students as a means to transform a quantitative relationship suitable for constant values into a structure appropriate for co-varying quantities.

Keywords: Definite Integral, Adding Up Pieces, Quantification, Problem-Solving

This poster explores a conceptual model underlying two common misconceptions demonstrated by students attempting to apply definite integrals to problems in context. The literature often distinguishes between these two errors by the symbolic forms (Sherin, 2001) which cued the need for integration (Jones, 2013; Meredith & Marrongelle, 2008; Nguyen & Rebello, 2011). The first error, observed in students cued by the dependence symbolic form, is characterized by a student placing a given, or derived, quantity in for the integrand without consideration for its physical contextual relationship to the differential. The student might omit the differential entirely or only justify its presence as signifying the changing variable within the integrand. Meredith and Marrongelle described this reasoning as a dead end regarding successful student integration when the integrand is not a rate of change or density. When students were instead cued by the parts of a whole symbolic form some were observed to give quantitative meaning to the differential but only viewed the accumulation process as applying to the integrand; Jones described this as Adding up the Integrand. Through classroom observations, we noticed the dependence misunderstanding emerged in some students' reasoning even when they were cued to integrate by the parts of a whole symbolic form. It also appeared this error did not necessarily prevent students from making progress through definite integral tasks. In light of this, we hypothesized there might be an underlying tool (Dewey, 1938; Hickman, 1990) students utilize in their problem-solving process which motivates these misconceptions.

This tool, which we called an Integral as a Transformer conception, entails a student invoking a definite integral to convert a mathematical model that is appropriate for constant values of its constituents (e.g., distance = velocity · time) into a model applicable for contexts in which the constituent quantities co-vary. It should be noted that for a simple rate of change and density problems this conception often provides students with a heuristic for composing correct integral structure despite an incorrect quantitative interpretation.

Planning to challenge this heuristic, we developed our study using Dewey's theory of Inquiry which characterizes knowledge as a byproduct of the dialectic interplay between a student's selection, application, testing, and refinement of a conceptual tool when faced with a problematic situation. We videotaped interviews with nine students, eight in pairs and one alone, as they worked through a series of increasingly difficult contextual definite integral tasks. Our analysis found that every group in the study used the Integral as a Transformer conception at least once in their problem-solving process, despite many pairs also justifying the need for integration in terms of the parts of a whole symbolic form. In our poster presentation, we will discuss the numerous forms in which the Integral as a Transformer conception appeared throughout the interviews, when and how it proved problematic, and more importantly how it interacted with other conceptual tools in students' mathematical modeling activity.

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