

## Content Analysis of Introductory Textbooks in Point-Set Topology

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*This study compared twelve point-set topology textbooks at the introductory level. The goal was to differentiate each textbook according to its overall conceptual approach to the field, as well as its mathematical approach to four fundamental topological ideas. The analysis indicated significant differences in the conceptual and mathematical presentation of those topics among the twelve textbooks. These findings highlight the need for researchers to distinguish between the conceptual and mathematical approaches found in textbooks for proof-intensive courses in undergraduate mathematics education.*

**Keywords:** Concept, Proof, Topology, Textbook analysis

Textbooks play an important role in outlining the learning progressions that students are expected to follow as they first construct their conceptual understanding for new ideas. The purpose of this study was to delineate some of the major conceptual approaches to point-set topology that are introduced by well-known textbooks on the subject, and therefore highlight the ways that students are expected to conceptualize ideas in their undergraduate topology courses.

### **Methodology**

Twelve introductory textbooks in point-set topology were selected for analysis (see References). The textbook choices were primarily based on the Mathematical Association of America's list of recommendations for undergraduate libraries (MAA, 2017), supplemented by a survey of recommendations published online by authors in the field. The textbooks were chosen to include a variety of conceptual approaches to the topic, but excluded specialized content, outline-style treatments, and out-of-print texts. One popular online textbook was also selected for comparison. The textbooks were examined and compared according to their overall presentation of point-set topology, as well as their approaches to four key analytical and topological concepts. These topics were: open sets, closed sets, sequence/limits, and continuous functions.

### **Findings**

There were significant differences found among the twelve textbooks, both in terms of the theoretical perspective taken on the field of topology, and in the conceptual progressions that were used to present individual topics. Differences in the textbooks' broad approaches to topology were often reflected in each author's choice of the definitions, theorems, and order of presentation of the concepts screened in the analysis. The textbooks' conceptual approaches to the introduction of topology were grouped into three categories, labelled the *metric-analytic*, *geometric-intuitive*, and *abstract-axiomatic* approaches. Textbooks that followed the *metric-analytic* approach tended to generalize concepts from the study of real analysis, often relying on sequential and metric-based techniques. The *geometric-intuitive* approach tended to focus on low-dimensional surfaces and spatial imagery to establish concepts based on physical intuition. Textbooks that followed the *abstract-axiomatic* approach established definitions and theorems from the topological axioms, employing abstract examples or counterexamples to illustrate concepts. Concept-specific comparisons will be presented in table format on the poster.

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