Quantum Physics Students' Reasoning about Eigenvectors and Eigenvalues

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Eigentheory is an important mathematical tool for modeling quantum mechanical systems, but little is known about how physics students reason about eigenvectors and eigenvalues as they transition from linear algebra courses into quantum mechanics. In this poster, we share examples of the resources (elements of students' knowledge) we have identified in physics students' reasoning about the eigenvectors and eigenvalues of real 2x2 matrices, as well as connections among these resources within and across students.

Keywords: Linear Algebra, Eigentheory, Quantum Physics, Student Reasoning, Resources

This work is part of a larger research project that is examining the various ways students reason about and symbolize concepts related to eigentheory in quantum physics, as well as how students' language and symbols for concepts related to eigentheory compare and contrast across mathematics and quantum physics contexts. Expanding on the work done by Henderson, Rasmussen, Zandieh, Wawro, and Sweeney (2010), for this poster we focus on the following research question: What ways of reasoning about eigenvectors and eigenvalues of real 2x2 matrices exist for physics majors at the beginning of a quantum mechanics course?

To operationalize the research question, we use a Resources Framework, a type of fine-grained constructivism (Redish, 2004) initially proposed by Hammer (2000): "A resource is a basic cognitive network that represents an element of student knowledge or a set of knowledge elements that the student tends to consistently activate together" (Sabella & Redish 2007, p. 1018). Resources are activated depending on how individuals frame a given situation; resources can be linked to other resources, in which activation of one resource can promote or demote activation of others; and resources may internally consist of finer-grained resources linked in a particular structure (Sayre & Wittmann, 2008). We seek to identify resources that characterize how our participants reasoned about eigenvectors and eigenvalues of real 2x2 matrices.

Data come from semi-structured (Bernard, 1988) individual interviews conducted in the first week of the semester with eight students enrolled in a senior-level quantum mechanics course at a public research university in the Northeast United States. The linear algebra prerequisite at this university was either a linear algebra or combined differential equations and linear algebra course, both sophomore-level and taught in the mathematics department. The interview question analyzed asked students to reason about the equals sign and solutions to $A\begin{bmatrix}x\\y\end{bmatrix} = 2\begin{bmatrix}x\\y\end{bmatrix}$, where A is a 2x2 matrix, and to find the eigenvectors and eigenvalues of a specific 2x2 matrix.

Each interview was transcribed and then watched independently by the three authors (one physics and two mathematics education researchers) in an attempt to identify students' conceptual and procedural resources (Wittmann & Black, 2015). For example, conceptual resources include eigenvectors being stretched by eigenvalues, scalar multiples of eigenvectors also are eigenvectors, or that the result of the product $A\begin{bmatrix}x\\y\end{bmatrix}$ is the same object as the result of the product $2\begin{bmatrix}x\\y\end{bmatrix}$. Procedural resources include, for example, performing the mathematical steps to rewrite $Ax = \lambda x$ as a system of equations or finding the roots of a characteristic polynomial to find a matrix's eigenvalues. This presentation will feature several examples of these identified resources, along with connections between these resources within and across students.

References

Bernard, R. H. (1988). *Research methods in cultural anthropology*. London: Sage.

Hammer, D. (2000). Student resources for learning introductory physics. *American Journal of Physics*, *68*(7), S52–S59.

Henderson, F., Rasmussen, C., Sweeney, G., Wawro, M, & Zandieh, M. (2010). Symbol sense in linear algebra. *Proceedings of the 13th Annual Conference on Research in Undergraduate Mathematics Education, Raleigh, NC*. Retrieved from: http://sigmaa.maa.org/rume/crume2010

Redish, E. F. (2004). A theoretical framework for physics education research: Modeling student thinking. In E. F. Redish & M. Vicentini (Eds.), *Proceedings of the International School of Physics "Enrico Fermi" Course CLVI: Research on Physics Education* (pp. 1-63). Burke, VA: IOS Press.

Sabella, M. S., & Redish, E. F. (2004). Knowledge organization and activation in physics problem solving. *American Journal of Physics*, *75*(11), 1017–1029.

Sayre, E. C., & Wittmann, M. C. (2008). Plasticity of intermediate mechanics students' coordinate system choice. *Physical Review Special Topics – Physics Education Research*, *4*, 2–14.

Wittmann, M. C., & Black, K. E. (2015). Mathematical actions as procedural resources: An example from the separation of variables. *Physical Review Special Topics – Physics Education Research*, *11*, 1–13.