A Hypothetical Learning Trajectory (HLT) for Preservice Secondary Teachers' Construction of Congruence Proofs

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With the advent of the Common Core State Standards, there has been renewed interest in teaching geometry from a transformation perspective; however, most geometry teachers are unfamiliar with this approach as they learned geometry from a perspective based on Euclid's Elements. Consequently, there is little knowledge of how teachers who come from this traditional perspective learn geometry from a transformation approach. One major difference that teachers must reconcile is in the construction of congruence and similarity proofs. As such, there is a need to understand how teachers learn these proofs from a transformation perspective. We propose to present a poster reporting a hypothetical learning trajectory (HLT) for preservice teachers' construction of such congruence proofs, based on the coursework of 15 preservice secondary teachers and cognitive interview responses to geometry tasks.

Keywords: hypothetical learning trajectory, transformational geometry, proofs

Simon (1995) defined a *hypothetical learning trajectory* (HLT) as "the learning goal, the learning activities, and the thinking and learning in which the students might engage" (p. 133). HLTs can be influential in improving curriculum and instruction (e.g., Sztajn et al., 2012) and have been examined extensively for K-8 levels (e.g,, Daro et al., 2011).

We contend that HLTs could be similarly influential in improving the instruction of geometry from a transformation approach, including for pre-service secondary teachers who themselves learned geometry from a more traditional perspective and who may have to teach from a transformation perspective in the future. The central difference distinguishing these two perspectives on geometry is in the construction of congruence and similarity proofs. Thus we address: *What thinking and learning do pre-service teachers progress through while learning to construct congruence proofs?* We focus on congruence as it is typically viewed as a prerequisite for learning similarity.

Based on the mechanism of reflection on activity-effect relationships for generating an HLT (Simon & Tzur, 2004), we identified components of a HLT as specified by Simon (1995). The data for this study are 12 written assignments, selected using Simon and Tzur's (2004) framework, from 15 pre-service teachers (PSTs) (7 white females, 8 white males) enrolled in a course on geometry from a transformation perspective; and 4 students' responses to cognitive interviews. The PSTs are undergraduates at a research-intensive, doctoral granting institution.

The main result of our analysis is an HLT for pre-service secondary teachers' learning to construct congruence proofs. The HLT spans initial learning of rigid motions, to constructing proofs for triangle congruence criterion, to constructing proofs incorporating more complex and/or multi-component geometric objects. Results from this study can be applied to improving undergraduate education of pre-service secondary teachers and potentially informs task design to support concepts of congruence and similarity.

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