Relationships Between Calculus Students' Ways of Coordinating Units and their Ways of Understanding Integration

Jeffrey Grabhorn
Portland State University

Steven Boyce
Portland State University

Cameron Byerley
Colorado State University

This poster describes results from a paired-student teaching experiment focused on college calculus students' understandings of integration. Our aim was to model relationships between students' covariational reasoning, quantitative reasoning, and numerical reasoning as they were developing meanings for integration, via teaching sessions that were concurrent but independent from the students' "traditionally-taught" second-term calculus course. We will discuss commonalities between students' ways of reasoning multiplicatively, ways of reasoning about linear rates of change, and ways of understanding integration.

Keywords: Covariational Reasoning, Integration, Numerical Reasoning, Quantitative Reasoning

A key aspect for conceptualizing the fundamental theorem of calculus, the *accumulation function*, $F(x) = \int_a^x f(t)dt$, requires coordinating three varying values: that of an independent variable, *t,* as it varies from *a* to *x,* that of a dependent variable, $f(t)$, as *t* varies, and that of the accumulation of values of $f(t)$ as $f(t)$ and *t* co-vary (Swiden & Yerushalmy, 2016; Thompson, 1994; Thompson & Silverman, 2008). Research with K-12 students points to the necessity of students' construction of a structure for coordinating three *levels of units* for (a) reasoning flexibly with (im)proper fractions, e.g., for thinking of '9/7' as "containing" potential multiplicative relations with '1', '1/7', '1/9' and '7/9', and (b) reasoning flexibly with algebraic equations in the middle grades (Hackenberg & Lee, 2015). Students sometimes experience success in school mathematics if they learn to reason with three levels of units *in activity*, which means they "build" an ephemeral third level of units as part of their way of reasoning rather than assimilating situations with a units (of units (of units)) structure (Ulrich, 2015). Indeed, some students assimilating with two levels of units pursue STEM majors in college: Boyce and Wyld (2017) described constraints in two such differential calculus students' reasoning about function inverses and function composition, and Byerley (2016) described how students' reasoning with fractions was (and was not) associated with their success in different aspects of introductory calculus.

We report on an 8-session constructivist teaching experiment (Steffe & Thompson, 2000) exploring connections between students' units coordination and understandings of integral calculus. Our poster focuses on contrasting the reasoning of a pair of students, one who assimilated with two levels of units and one who could assimilate with three levels of units. Our poster will exemplify contrasts (and commonalities) in (a) their units coordination (b) their ways of reasoning about linear rates of change (c) their meanings for the quantities represented in the statement $F(x)= \int_0^x sin(t)dt$, and (d) their associated justifications for why $\int_0^\pi sin(t)dt = 2$. The results provide conjectures of how differences in the constraints students face in conceptualizing the accumulation function (and fundamental theorem of calculus) may be attributed to differences in their ways of coordinating units.

References

Boyce, S., & Wyld, K. (2017). Supporting students' understanding of calculus concepts: Insights from middle-grades mathematics education research. In A. Weinberg, C. Rasmussen, J. Rabin, M. Wawro, S. Brown (Eds.), *Proceedings of the 20th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 1152-1157). San Diego, CA: SIGMAA on RUME.

Byerley, C. (2016). *Secondary teachers and calculus students' meanings for fraction, measure, and rate of change.* (Unpublished doctoral dissertation). Arizona State University, Tempe, AZ.

Hackenberg, A., & Lee, M. Y. (2015). Relationships between students' fractional knowledge and equation writing. *Journal for Research in Mathematics Education, 46*(2), 196-243.

Swiden, O., & Yerushalmy, M. (2016). Conceptual structure of the accumulation function in an interactive and multiple-linked representation environment. *International Journal of Research in Undergraduate Mathematics Education, 2,* 30-58.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education* (pp. 267-. 307). Hillsdale, NJ: Erlbaum

Thompson, P. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. Educational Studies in Mathematics, 26(2–3), 229–274.

Thompson, P. W., & Silverman, J. (2008). The concept of accumulation in calculus. In M. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics*, 117-131. Washington, DC: Mathematical Association of America.

Ulrich, C. (2015). Stages in constructing and coordinating units additively and multiplicatively (part 1). *For the Learning of Mathematics, 35*(3), 2-7.