

Students' Understanding of Quadratic Equations

Jonathan López Torres
North Carolina State
University

Izraim L. Robles Hernández
University of Puerto Rico at
Mayagüez

Rafael Martínez-Planell
University of Puerto Rico at
Mayagüez

Action–Process–Object–Schema theory (APOS) was applied to study student understanding of quadratic equations with one variable. This requires proposing a detailed conjecture (called a genetic decomposition) of mental constructions students may use to understand quadratic equations. The genetic decomposition, which was proposed, can contribute to help students achieve an understanding of quadratic equations with improved interrelation of ideas and more flexible application of solution methods. Semi-structured interviews with eight beginning undergraduate students explored which of the mental constructions conjectured in the genetic decomposition students could do, and which they had difficulty doing. Two of the mental constructions that form part of the genetic decomposition are highlighted and corresponding further data was obtained from the written work of 121 undergraduate science and engineering students taking a multivariable calculus course. The results suggest the importance of explicitly considering these two highlighted mental constructions.

Keywords: Quadratic Equations; APOS Theory; Genetic Decomposition; Calculus

Many secondary school students, and undergraduate students, do not truly understand quadratic equations or the rules they use to solve them (Didis, Bas, & Erbas, 2011). Some studies suggest that problems like these may stem from the lack of details in books, which, consequently, teachers tend to not emphasize (Sönnerhead, 2009). Some investigations on student understanding of quadratic equations refer to specific misconceptions (Bossé & Nandakumar, 2005; Ochoviet & Oktac, 2009, 2011; Vaiyavutjamai, Ellerton, & Clements, 2005). In Puerto Rico, the Department of Education established it is not until tenth grade that students learn how to solve quadratic equations. These quadratic equations are simpler by design, and can be solved using the following techniques: factoring, using square roots, completing the square, the quadratic formula, and by using technology. Given the described context held by incoming first year students attending a Puerto Rican university, this article investigated students' understanding of quadratic equations by: (1) establishing a conjecture of their mental constructions (stated in terms of the constructs of Action–Process–Object–Schema (APOS) theory) that beginning university students may do in order to understand how to solve quadratic equations; (2) using semi-structured interviews in order to investigate which of the conjectured mental constructions students can do and which they have difficulty doing; and (3) using written work from more advanced undergraduate students to investigate their use and understanding of two specific mental constructions conjectured in the genetic decomposition.

Results and Conclusions

The genetic decomposition and results from this study highlight two specific mental constructions that play a key role in students' understanding of quadratic equations, that students have difficulty doing and that they seem to be overlooked in traditional instruction. Another result of the study underscores the importance of numerical and graphical explorations into the nature of the possible solutions of a quadratic equation.

References

- Bossé, M. J., & Nandakumar, N. R. (2005). The factorability of quadratics: motivation for more techniques. *Teaching Mathematics and its Applications*, 24(4), 143-153.
- Didiş, M., Baş, S., & Erbaş, A. (2011). Students' reasoning in quadratic equations with one unknown. In *Proceedings of the 7th Congress of the European Society for Research in Mathematics Education*. Rzeszów, Poland.
- Ochoviet, C., & Oktaç, A. (2009). If $AB = 0$ then $A = 0$ or $B = 0$?. *The Mathematics Enthusiast*, 6(1), 113-136.
- Ochoviet, C., & Oktaç, A. (2011). Algunos aspectos del desarrollo del pensamiento algebraico: el concepto de raíz y de variable en ecuaciones polinómicas de segundo grado (Some aspects of the development of algebraic thought: the concepts of root and variable in polynomial equations of the second degree). *Educación Matemática*, 23(3), 91-121.
- Sönnnerhed, W. W. (2009). Alternative approaches of solving quadratic equations in mathematics teaching: An empirical study of mathematics textbooks and teaching material or Swedish Upper-secondary school.
- Vaiyavutjamai, P., Ellerton, N. F., & Clements, M. A. (2005). Students' attempts to solve two elementary quadratic equations: A study in three nations. *Building connections: Research, theory and practice*. Sydney, Australia: Mathematics Education Research Group of Australia.