Using Everyday Examples to Understand the Concept of Basis

Jessica Knapp Michelle Zandieh Aditya P. Adiredja
Arizona State University University of Arizona

The purpose of this paper is to explore everyday examples given by students to explain the notion of basis. By exploring key aspects of the examples generated by the students we can see what roles and characteristics of basis the students attend to.

Keywords: Examples, Linear Algebra, Basis, Generating Examples

In natural language, it is common to use an everyday example as a conceptual metaphor for abstract ideas. Research suggests that digesting formal definitions can be a stumbling block for students (Edwards & Ward, 2004; Knapp, 2009). Several researchers have studied how students come to understand the concepts of basis, span, and linear (in)dependence (e.g. Aydin, 2014; Stewart & Thomas, 2010; Trigueros & Possani, 2013; Plaxco &Wawro, 2015). Adiredja & Zandieh (2017) developed a framework for exploring students' generation of everyday examples for basis. We use a modified version of this framework to explore student everyday examples.

Data Collection and Analysis

Data was collected by our second author in Germany with nine Applied Math upper division and masters university students. Interviews were held in English and videotaped by a speaker fluent in English and German. Adiredja & Zandieh (2018) studied ways that students understand the notion of basis in linear algebra. Two aspects of that understanding were how the basis vectors related to the space (*roles*) and the nature of the set of basis vectors (*characteristics*). The roles codes are Generating, Covering, Structuring, Traveling and Supporting. Characteristic codes are Minimal, Essential, Representative, Non-redundant, and Different. We will use the data to illustrate some of these roles and characteristics.

Results and Discussion

After engaging in mathematical activities on basis, students were asked how they might describe the idea of basis to someone who had not yet learned the concept. The students developed everyday examples as part of their explanation. One student, Andreas, began with a description of a sailboat; however, he decided to think of a different example because the sailboat example had a flaw "[if] you can't assess the point already with one of the other vectors or a combination then it isn't actually a new vector and it can be crossed out or x'ed out from the basis." The student here was attending to the characteristic of non-redundancy. The sailboat example didn't have a mechanism to keep there from being redundant vectors in the basis.

Andreas then chose a star fish to describe basis. "You have sea stars; … they just walk straight in one of those directions." He thought of the star fish legs as the vectors in the basis. "So they just declare one of their legs as front. And then they march on. And so they can access any point on the sea floor which is our plane again." In using the verbs "march, access, move, and walk," Andreas's star fish example illustrates the Traveling code. He reflected on the characteristics of the star fish example, "by moving in those five directions, the question is, is this even necessary to have five directions." Before Andreas was focused on "x-ing out" redundant vectors, now his focus has shifted to consider the minimal amount of legs needed to access the entire sea floor. This illustrates the characteristic of minimal.

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