# Teachers' Reasoning with Frames of Reference in US and Korea

Surani Joshua, Pat Thompson
Arizona State University

*We gave approximately 180 US and 380 Korean teachers frame of reference tasks, and coded the open responses with rubrics intended to rank responses by the extent to which their responses demonstrated conceptualized and coordinated frames of reference. In both countries less than half of teachers scored at the highest level on almost every task, showing that teachers frequently struggle to keep track of quantities within a frame of reference in a meaningful way. Our US-Korean comparison also shows that US teachers struggle on most of these tasks significantly more than Korean teachers.*

*Keywords:* Frames of Reference, Secondary Teachers, International Comparison

## Theoretical Perspective

When we speak of frame of reference, we mean that an individual can think of a measure as merely reflecting the size of an object relative to a unit or he can think of a measure within a system of potential measures and comparisons of measures. An individual conceives of measures as existing within a *frame of reference* if the act of measuring entails: 1) committing to a unit so that all measures are multiplicative comparisons to it, 2) committing to a reference point that gives meaning to a zero measure and all non-zero measures, and 3) committing to a directionality of measure comparison additively, multiplicatively, or both. […] An individual is coordinating two frames of reference if she conceives each frame as a valid frame, stays aware of the need to coordinate quantities' measures within them, and carries out the mental process of finding a relation between the frames while keeping all relative quantities and information in mind (Joshua, Musgrave, Hatfield, & Thompson, 2015).

## Methodology

From 2012 to 2015, the Project Aspire team created the 48-item assessment Mathematical Meanings for Teaching – secondary math (MMTsm). Two items were categorized as frame of reference items: "Willie Chases Robin" and "Nicole Chases Ivonne". We gave them to 177 US and 359 Korean teachers and coded their free-response answers; our poster has both the aggregate data and sample responses.

## Results & Discussion

It is our hope that the data collected will orient professional development leaders to consider their teacher's meanings for the mathematics that they teach, and guide their PD to focus on helping teachers build more productive meanings. While professional development projects continue to administer the MMTsm, the data discussed in our poster is telling. In both countries less than half of teachers scored at the highest level on every task except for Korean teachers on Part A of Willie Chases Robin. There was also a statistically significant difference between US and Korean teachers on every task. Additionally, the Korean data shows us that the US data cannot be ignored simply by arguing that these tasks are inappropriate to give to high school teachers; there are a lot of gains with US teachers that could be made simply up to the current Korean teacher levels. Our data show that the U.S. teachers in our sample are not prepared to help their students reason through tasks involving multiple frames of reference.

## References

Joshua, S., Musgrave, S., Hatfield, N., & Thompson, P. W. (2015). *Conceptualizing and reasoning with frames of reference.* Paper presented at the Proceedings of the 18th Meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education, Pittsburgh, PA.