An Instructional Resource for Improving Students' Conceptual Understanding of Functions through Reflective Abstraction

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It has been widely documented that undergraduate-level students' understanding of functions is rigid and indicative of an action view which constrains conceptual understanding (Carlson, Jacobs, Coe, & Hsu, 2002). Duval affirms that, "to understand the difficulties that many students have with comprehension of mathematics, we must determine the cognitive functioning underlying the diversity of mathematical processes" (2006, p.103). What are the underlying cognitive skills students need to gain a better conceptual understanding of functions? How should instruction of function content and training in these cognitive skills be combined? We propose the theoretical model "Structural-Schema Development for a Function", to address these questions. This model defines developmental stages students pass through to form a global view for the function concept, identifies underlying cognitive mechanisms involved in each stage, and develops instructional exercises that combine content with cognitive skills training for these cognitive mechanisms.

Keywords: Schema, Functions, Understanding, Reflective Abstraction, Structuralism

The proposed model in this study is influenced by Duval's (2006) framework for *Treatments and Conversion* and Piaget's theory *Reflective Abstraction* (Arnon et. al., 2014; Dubinsky & Lewin, 1986). In Dubinsky and Lewin's view, "reflective abstraction includes the act of reflecting on one's cognitive action and coming to perceive a collection of thoughts as a structured whole" (1986, p.63). It can also be thought of as coordinating multiple lower-level structures and reflecting on these structures to combine them into a new higher-level structure (Dubinsky & Lewin). This is the glue that binds the four developmental stages of this model: Identification, Informal Classification, Formal Classification and Ordering. Students who reach the Identification stage can identify whether or not some object satisfies the formal definition of a function. Once students have identified enough objects that are elements of a function space they can begin to construct informal properties of these elements and classify them based on informal properties. Students who can describe a collection of functions as sharing the same informal properties (i.e. these functions are all smooth, these functions have jumps, etc.) have reached the Informal Classification Stage. These informal classifications can then become updated content on which formal classifications are either constructed or presented. These classifications are then coordinated for Ordering. The Ordering stage is reached when students can make substructures within the structure by ordering or nesting classifications using set relations and intuitions guided by informal classifications. Formation of a student's structural-schema for a function can occur in any of the developmental stages. The stage that a student reaches determines a threshold for how rich their structural schema has the potential to be. The student's structural-schema continually undergoes Maintenance through *assimilation* and *accommodation*. The proposed model anticipates to offer instructional resources designed to promote the skills students need to reach higher level developmental stages and ultimately gain a better conceptual understanding of functions.

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