Student reasoning with complex numbers in upper-division physics

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Abstract: *Students encounter complex numbers in many physics courses. In particular physics uses complex exponentials to describe oscillatory phenomena and requires that students use multiple representations (algebraic, x vs t graphs, complex plane). In this poster we will examine student responses suggesting difficulties with the connection between complex numbers and oscillation, drawn from students in upper-division physics courses in math methods.*

Description

This work is part of a collaboration to investigate student learning and application of mathematics in the context of upper-division physics courses. In particular the project focuses on a course required by most physics departments focusing on developing mathematical methods for upper-division physics. Throughout, we seek to go beyond procedures and to probe conceptual understanding and the development of quantitative reasoning skills.

Results suggest that procedural understanding of complex algebra is often not enough for students to connect mathematics with relevant physics contexts. Students had difficulty in relating complex numbers to oscillatory phenomena. It was not immediately clear whether incorrect responses reflected difficulties with procedures or conceptual understanding. For example, students were asked on a course exam to show, using expressions with complex exponentials, how two waves would destructively interfere given a π phase difference. Of ten students answering after instruction, only three gave correct answers, none using polar form.

To probe student reasoning, we have used a variety of tasks including both procedural symbolic manipulations and more conceptual questions. While students were largely successful on procedural tasks, their responses suggested a key disconnect with the use of complex numbers to describe oscillations. For example, students were asked to sketch the real part of the function Ae^{iwt} (8 sections, N = 107). Student written responses were examined and coded based on correctness and the explanation; the relevant codes after several iterations included the overall graph template (oscillatory / exponential / linear), the value of the function at t = 0, and, in the case of oscillatory sketches, whether the amplitude was constant or changing. Figure 1 shows a correct response and one showing exponential growth.

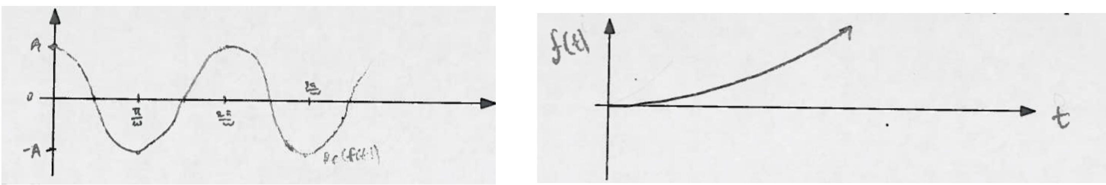

Figure 1 Scans of student sketches of the real part of the function Ae^{iwt}. About a third of students sketched responses like the second example, showing exponential growth.

About 28% of responses were categorized as correct and another 15% showed an oscillatory function with incorrect features (e.g., phase shift or decreasing amplitude). Many responses, however, did not show oscillation; 33% of responses were categorized as showing exponential *growth*. Sadaghiani (2005) reported similar confusion between e^{kx} and e^{ikx} in quantum mechanics examples.

In this context and others, our data suggest that experience with mathematical procedures is not sufficient for students to make sense of the meaning of the procedures and apply them to physics contexts. The course text includes only procedural exercises with a handful of applications, including oscillations, separated into a later section at the end of the relevant chapter. We believe students need dedicated curricular materials focusing on these ideas.

References

[1] M.E. Loverude and B.S. Ambrose, “Focused Collection on Upper Division PER,” *Physical Review Special Topics in Physics Education Research* **11**, 020002 (2015).

[2[R. E. Pepper, S. V. Chasteen, S. J. Pollock, and K. K. Perkins, “Observations on student difficulties with mathematics in upper-division electricity and magnetism,” *Physical Review Special Topics Physics Education Research* **8**, 010111 (2012).

[3] B.S. Ambrose, “Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction,” *American Journal of Physics*, *72*(4), 453-459 (2004).

[4] T. I. Smith, J. R. Thompson and D. B. Mountcastle, “Student Understanding of Taylor Series Expansions in Statistical Mechanics,” *Physical Review Special Topics - Physics Education Research* 9, 020110 (2013).

[5] H. R. Sadaghiani, “Conceptual and mathematical barriers to students learning quantum mechanics,” Ph.D. Dissertation, the Ohio State University (2005).

[6] E. F. Redish, “Problem solving and the use of math in physics courses.” In Conference World View on Physics Education in 2005: Focusing on Change, Delhi, August 21–26, 2005.

[7] M.E. Loverude, “Assessment to complement research-based instruction in upper-level physics courses, 2011 PERC Proceedings [Omaha, NE, August 3-4, 2011], edited by N. S. Rebello, P. V. Engelhardt, and C. Singh [AIP Conf. Proc. 1413, 51-54 (2012);

[8] M. E. Loverude, “Surprisingly, there is an actual physical application,” Student understanding in Math Methods, 2013 PERC Proceedings [Portland, OR, July 17-18, 2013], edited by P. V. Engelhardt, A. D. Churukian, and D. L. Jones.

[9] M. Boas, *Mathematical Methods in the Physical Sciences,* Wiley (2006).

[8] B Wilcox et al, “Analytic framework for students’ use of mathematics in upper division physics,” *Phys. Rev. ST-PER* **9**, 0201191 (2013).