Development of Reasoning about Rate of Change, Based on Quantitative and Qualitative Analysis

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Pre-calculus and Calculus are two big compartments as we consider their developmental and complemental attribute. I analyzed data quantitatively from a series of pre-calculus assessments conducted in a large public university 2017 fall, then investigated the result and its impacts qualitatively in calculus context focused on rate of change. The two-part analysis consists of discerning intrinsic factors in the assessment items that have a large effect on overall performance followed by clinical interviews about meaning of the Fundamental Theorem of Calculus and its applications. The results support my claim that the ability to conceptualize constant rate of change has a considerably positive effect on students' reasoning about rate of change and the Fundamental Theorem of Calculus as well.

Keywords: Quantitative Analysis, Constant Rate of Change, Rate of Change, Fundamental Theorem of Calculus

Quantitative analysis on pre-calculus assessment with reform curricula (Carlson, Oehrtman et al. 2013) shows that 34 items in the assessment have 11 principal components by factor analysis and each item assesses students' understanding independently by regression analysis. I focused on items in the first component since they have a huge impact on students' overall performance, coding them with R_i(reasoning abilities), F_i(understandings of various function types), U_i(understandings of various concepts), and A_i(other abilities) suggested from CCR (The Calculus Concept Readiness) taxonomy (Carlson, Madison et al. 2015). It turns out that the combination of R3 (Quantitative and covariational Reasoning), U3(Constant rate of change) and A4(Understand and use function notation to express one quantity in terms of another) becomes a critical factor to students' conceptualization in terms of their further learning.

The result mentioned above and my experiences of teaching pre-calculus and working with DIRACC (Developing and Investigating A Rigorous Approach to Conceptual Calculus) project led me to ponder on what would be the most problematic part for students as connecting the two compartments of pre-calculus and calculus. I hypothesize that student struggles stem from fragmented interpretations on constant rate of change and the fragmented knowledge hinders comprehensive understanding of rate of change and the Fundamental Theorem of Calculus. Accordingly, I elaborately devised open-ended interview items so that each item could reflect developmental aspects by inquiring on constant rate of change, rate of change, net change, and meaning of fundamental theorem of calculus.

Qualitative analysis on the interview shows that the way of students' reasoning on constant rate of change has different layers in various contexts of graphical, verbal, physical, and symbolic representations based on the framework for the concept of derivative (Zandieth, M. 2000). Also, it supports that each layer becomes distinct pivots of their interpretation on rate of change, having a large effect on the way of reasoning the Fundamental Theorem of Calculus as a continuum of rate of change. I believe that the results from my two-part study will be of interest to the mathematic education community because students' conceptualization on rate of change will be a foundation to the next step of learning.

References

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