If $f(2)=8$, then $f'(2)=0$, A Common Misconception, Part 2

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This study reports calculus students' failure to differentiate the cubing function when represented piecewise as $f(x) = x^3$ if $x \neq 2$, $f(x) = 8$ if $x = 2$. The data reported here suggest that students did not fail simply due to inattention to the function definition; when reminded that 2 cubed is 8 and prompted to compare the graph of f to that of the cubing function, student performance increased, but was still poor, indicating the presence of deep-seated misunderstandings.

Keywords: function, derivative, calculus, representation

Harel and Kaput (1991) observed a troubling phenomenon among calculus students. Students claimed that for $g(x) = sin(x)$ if $x \neq 0, g(x) = 1$ if $x = 0$, then $g'(0) = 0$ (due to the constant rule). A study I presented at RUME 2017 addresses how common this sort of error is (Mirin, 2017) by discussing student performance on the task of evaluating $f'(2)$ when $f(x) = x^3$ *if* $x \neq 2, f(x) = 8$ *if* $x = 2$. This poster describes a follow-up investigation that begins to address why students perform so poorly at that task, henceforth called "The Task".

Instead of focusing on a particular topic in math education (e.g. "functions"), this study centers students' performance and understanding on a specific task. There is precedent for this sort of investigation: various studies have centered on students' failure on the "student-professor problem" (Clement, 1982; Clement, Lochhead, & Monk, 1981; Wollman, 1983). Just as studying the student-professor problem provided insight into students' understandings of various mathematical constructs such as variable and equation, studying the problem I discuss here can provide insight into students' understandings of fundamental mathematical ideas such as function, graph, derivative, and multiple representations that will be of interest to the RUME community. This study should not be considered in a vacuum, but instead as contributing to the larger body of math education research on students' understanding of the ideas embodied in the problem (function, graph, etc).

A participant at RUME 2017 observed that the piecewise definition of *f* was rather contrived and therefore students might simply assume that *f* is a discontinuous function without realizing that *f* and the cubing function agree on $x = 2$. In the original study, several students provided a graph with a single point discontinuity and answered in a way consistent with presuming that 2 cubed is not 8. Hence, it seems that inattention, rather than a major conceptual misunderstanding, was at fault in *some* students' responses. Utilizing the data from the 2017 open-ended version, I readministered The Task in multiple choice form, first prompting students to calculate 2^3 and to compare the graph of *f* to that of $y = x^3$. With this prompting, students improved significantly ($\chi^2 = 5.365$, p=.021), but still performed poorly, with a 19.6% success rate.

As in the original study, many students produced incorrect graphs of *f*, e.g. graphing *y*=8. As expected, students who produced correct graphs of *f* did do significantly better on The Task than the students who produced incorrect graphs (χ^2=6.182 p = .010). However, only 31.7% of the students who produced correct graphs succeeded at The Task, suggesting that for the remaining 68.3%, the graph of a function does not determine its derivative.

References

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