Curricular Presentation of Static and Process-Oriented Views of Proof to Pre-service Elementary Teachers

Taren Going
Michigan State University

Engaging students in proof-related reasoning is an important but often challenging task for pre-service elementary teachers. Given that limited mathematics content courses and their associated textbooks offer some of the only opportunities for preservice elementary teachers to engage with proof, it is vital to understand what opportunities they offer to understand proof. I conducted an analysis of two textbooks used for elementary mathematics content courses to investigate the view(s) of proof promoted within and the opportunities to learn about proof-related reasoning. My findings suggest a mixed emphasis on static and dynamic views of proof and proving, but also many opportunities for instructors of mathematics content courses to promote an explanatory, process-oriented view of proof.

Keywords: Preservice Elementary Teachers, Proof, Curriculum Analysis

Introduction

Increasingly, calls for mathematical reform in the U.S. specify that mathematics instruction should enable students to "recognize reasoning and proof as fundamental aspects of mathematics, make and investigate mathematical conjectures, develop and evaluate mathematical arguments and proof, and select and use various types of reasoning and methods of proof" (NCTM, 2000, p. 56). These calls reflect a larger effort to align classroom work in mathematics more closely with the discipline of mathematics, especially at elementary levels where proof has not historically been emphasized (Ball, Hoyles, Jahnke, & Movshovitz-Hadar, 2002). However, existing research indicates that those preparing to teach elementary mathematics have difficulty recognizing an argument as proof (Martin & Harel, 1989) and interpreting and evaluating students' proofs and arguments (Morris, 2007). Likely due to the historical disconnect between proof and other aspects of K-12 mathematics curriculum (Herbst, 2002), pre-service elementary teachers (EPSTs) may not have had meaningful experiences with proving in their own education. Further, the curriculum materials from which they are to teach do not offer adequate support in the area of reasoning and proving (Stylianides, 2007). For these reasons, it is vital that we understand how EPSTs' mathematics content courses provide to develop understandings of proof and proof-related reasoning. These content courses represent some of the first, and potentially only, opportunities for pre-service teachers to engage with proof-related reasoning prior to being expected to teach it to their students, so these courses have a potentially powerful influence on how EPSTs will eventually teach.

One way to understand the opportunities EPSTs have to learn about proof and proving in their preparation coursework is to unpack such opportunities as written in the textbooks used for elementary mathematics content courses. Though the written opportunities may not align with the opportunities as enacted in a course, textbooks influence an instructors' course design and shape EPSTs' independent learning opportunities (McCrory & Stylianides, 2014). This study, broadly, aimed to understand the view(s) of proof promoted in these texts and the opportunities provided for EPSTs to learn about the nature of proof, proving and proof-related reasoning.

A Vision of Proof Communicated to Pre-Service Elementary Teachers

Numerous scholars have drawn attention to how our conceptions of what proof and argument are, as well as what occurs when we engage in proving and argumentation, shape not only the conclusions researchers make about students' thinking and instructional practice related to proof and argumentation (Balacheff, 1988; Stylianides, Morselli & Bieda, 2016) but also how teachers shape how students think about proof and proof-related activities (Conner et al., 2014). Thus, what kind of conceptions should EPSTs have about proof and argument to support the development of student thinking and mathematical practice as proposed in calls for reform?

Schoenfeld (1991) states, "In real mathematical thinking, formal and informal reasoning are deeply intertwined" (p. 311). Proof can serve several important functions: verification, explanation (that is, offering insight into why a statement is true), systematization of axioms, discovery of new results, communication of mathematical knowledge, construction of an empirical theory, exploration of mathematical statements' meaning or consequences, and incorporation of mathematical truths into alternative frameworks (Hanna, 2000). Thus, if classroom practice should align with disciplinary practice, teachers must be familiar with many of these aspects and be able to link informal, exploratory activities with reading or writing formal proofs in order to help students learn mathematics as they prove. Especially in the early grades, explanation, communication, and exploration will have great importance (Hanna, 2000) and engaging with proof in this way can contribute to a process-oriented view of proof (Schoenfeld, 1991). For EPSTs to support their students to engage in proving in this way, it is important that they experience and understand proof as a process.

Existing literature suggests that it is more common for EPSTs to hold more ritualistic and empirical understandings about proving and argumentation that may stem from experiences with a more static view of proof (Harel & Sowder, 1998; Morris, 2002). Classroom practices in content and methods courses for EPSTs should be structured to promote a dynamic view of proof. In a dynamic view of proof, the process of constructing a proof *is* the vehicle for mathematical learning (Schoenfeld, 1991). In this way, a dynamic view of proof relates integrally with the purpose of explanation promoted by Hanna (2000).

From these assumptions about the aspects of proof that EPSTs must have experience with and promote in their teaching practice, this study investigated the nature of opportunities provided in textbooks for mathematics content courses, guided by the questions: *Within the mathematics content textbooks considered here, which functions of proof are promoted, and by extension, which functions of proof will pre-service elementary teachers become familiar with?*

Methods

The textbooks analyzed were *Mathematics for Elementary Teachers* by Beckmann (2003) and the series *Elementary Mathematics for Teachers* and *Elementary Geometry for Teachers* by Parker and Baldridge (2004, 2008). Both texts are written for use in a year-long mathematics content courses for pre-service elementary teachers, who are assumed to receive no other specialized mathematical training. Both texts were identified by McCrory and Stylianides (2014)'s analysis of tables of contents for elementary mathematics content course texts to provide explicit treatment of proving and argumentation.

Focusing solely on student materials, rather than teacher's guides, I analyzed each textbook page-by-page to identify instances of proof and proof-related reasoning. Analyzing student materials can provide insight into the opportunities to learn about proof regardless of what occurs in class, which is important because textbooks can be utilized in a variety of circumstances

(McCrory & Stylianides, 2014). An analysis of student materials also informs instructor planning around material that might need to be supplemented or particularly emphasized outside of the textbook. I generated categories of proof-related reasoning present grounded from the textbooks themselves, and narrowed to a list of three categories that could be used to code each of the instances I analyzed:

1. Complete general proofs presented somewhat in isolation;
2. Exploration of a topic which the textbook author links explicitly to proof, for example by stating that the mathematical statement can be proved or by assigning a proof to the student as homework; and
3. Exploration of a topic which leads to a complete deductive proof within the text.

For each instance of proof related reasoning, I recorded the forms of reasoning present in exploration and/or final proof as either: written reasoning, symbolic/algebraic reasoning, diagrams/graphs, specific worked examples, and imagined motion (e.g. sliding, turning). I also noted the mode of argumentation (e.g. direct logic, construction of counterexamples, proof by contradiction), to gain insight into how proof is presented overall in these texts. Finally, for any instances of proof-related reasoning that included exploration I noted whether this led to the emergence of a key idea.

To demonstrate this coding process, consider the proof that the base angles of an isosceles triangle are congruent from Parker and Baldridge (2008). Initial exploration for this is presented in discussions of symmetry, where the text invites readers to fold a paper isosceles triangle along its line of symmetry to discover matching parts (Parker & Baldridge, 2008, p. 44). Since at this point this is not a general argument, but pertains to one particular paper triangle, this represents exploration. The forms of reasoning present in this exploration are a diagram (to describe the construction of paper isosceles triangle), a specific example of one isosceles triangle, and a description of imagined motion (folding the triangle). Further along in the text, a general proof that the base angles of an isosceles triangle are congruent is presented to the reader in the form of a "teacher's solution", which closely mirrors a two-column proof (Parker & Baldridge, 2008, p. 92). The forms of representation in the proof were coded as minimal text, a symbolic representation to describe congruent parts of the triangle, and a supporting diagram. The mode of argument was coded as direct because the argument begins with given information and follows a direct chain of logic to the conclusion.

In addition to looking for instances of proof-related reasoning, I noted any statements by the author that included reference to proof or mathematical reasoning, but were not connected to specific instances of proof-related reasoning. These include definitions and descriptions of proof, descriptions of unfamiliar modes of argumentation, advice on how to write proofs, and statements about the nature of mathematical knowledge more generally. I examined these statements and the instances of proof-related reasoning identified in order to learn how a dynamic view of proof is (or is not) promoted in the textbooks.

Findings

Conceptions of What Proof Is and Who Does Proof

My analysis revealed that both textbooks included written descriptions, or definitions, of proof. In Parker and Baldridge (2004), proof is defined as "a detailed explanation of why that fact follows logically from statements that are already accepted as true" (p. 110). This definition highlights the purposes of verification, communication, and construction of an empirical theory. Even though the word "explanation" is included in this definition, the authors do not seem to indicate that this refers to explanation of the underlying mathematics in order to provide insight into the statement's truth. By this definition then, it is enough simply that the statement *is* true. Moreover, this definition highlights proof as a finished object, inconsistent with the process-oriented view of proof that pre-service elementary teachers will need to effectively engage their students in learning through proof.

Similarly, Beckmann (2003) chooses to use the term "explanation" in defining proof: , "Proofs are one of the important aspects of this book too, even if we don't usually call our explanations proofs. A proof is a thorough, precise, logical explanation for why something is true, based on assumptions or facts that are already known or assumed to be true. So a proof is what establishes that a theorem is true" (p. 213). Like Parker and Baldridge (2004), Beckmann's definition highlights the purposes of verification, communication, and construction of an empirical theory. Additionally, though, Beckmann (2003) links proof to other types of mathematical explanation, suggesting proof is useful for gaining insight into the mathematics and connecting between informal and formal reasoning. While this definition of proof discusses its value as a product of verification, it hints at the process involved in its creation.

Beyond exact definitions, these texts include written descriptions of the nature of proof. Beckmann (2003) describes a process-oriented view of proof, stating "…mathematics is about starting with some assumptions and some definitions of objects and concepts, discovering additional properties that these objects or concepts must have, and then reasoning logically to deduce that the objects or concepts do indeed have those properties" (p. 65 of volume 2). However several times throughout the text, pre-service teachers are asked to do exercises that warrant a proof but the text indicates that they are not expected to generate such a proof.

Parker and Baldridge (2004) also indicate that there are many forms of mathematical reasoning involved in and related to proof: "In the classroom the reasoning occurs in explanations and guided investigations, while in mathematics textbooks the reasoning often occurs in formal and informal 'proofs'" (p. 109). In this statement the authors do not specifically present proof as a process, but they do present proof as the domain of mathematics textbooks and the mathematicians who write them, and not an area where teachers are expected to be involved. This suggests that pre-service elementary teachers are expected to read and understand proofs, but not write proofs themselves. However, in the context of geometry, Parker and Baldridge (2008) occasionally present "hints" for how pre-service elementary teachers, and eventually their students, might approach specific proving tasks.

Instances of Proof-Related Reasoning

Within Parker and Baldridge (2004, 2008) I identified 68 total instances of proof-related reasoning and within Beckmann (2003) I identified 25 total instances of proof-related reasoning. This amounts to one instance of proof-related reasoning approximately every seven pages in Parker and Baldridge, and approximately every 32 pages in Beckmann. The number of instances of proof-related reasoning in each category (proof only, exploration only, and exploration and proof) for each text is listed in Table 1.

Table 1. Categorization of instances of proof-related reasoning within each text

	Parker and Baldridge (2004, 2008)	Beckmann (2003)
Proof Only	25	6
Exploration Only	8	7

Exploration and Proof	35	12
Total	68	25

The mode of argumentation for almost all instances of proof and proof-related reasoning from Table 1 is direct, where an initial assumption begins a chain of deductive logic (i.e. modus ponens). The only exceptions to this mode of argumentation are two instances in Parker and Baldridge (2004), where proof by contradiction is utilized. The prevalence of direct proofs within both texts indicates a high value on the transparency and understandability of mathematics. Indeed, Beckmann (2003) describes that a good mathematical explanation, which she uses in lieu of the term proof, is clear and logical, without "[requiring] the reader to make a leap of faith" (p. 11). In addition, Beckmann specifies that these should be convincing and usable for teaching, which highlights the possibility that proofs might give insight into the mathematics, and further, might be useful for learning mathematics.

Within Parker and Baldridge (2004, 2008), 27 of the instances of proof take the form of a "Teacher's Solution" or "Elementary Proof" that are similar to two-column proofs from standard geometry textbooks. These instances provide only the minimum number of highly-abbreviated steps. Further directions on the formatting of these proofs specified, "Do not label the two columns 'statement' and 'reason' (everyone already knows this!)" (Parker & Baldridge, 2008, p. 79). Given the similarity of this form to two-column proofs that highlight verification of facts (Herbst, 2002), these elementary proofs do not readily serve the function of explanation. In addition, their emphasis on abbreviation and standard form may distance the final proof from the process that was conducted to create it.

Conclusion

Through my analysis of both texts, including written statements regarding the nature of proof, the form and modes of argumentation within opportunities to prove, and the emergence of key ideas, I found evidence of both dynamic and static conceptions of proof. The strong presence of direct proving methods that aid in mathematical understanding, as well as inclusion of exploratory processes that lead to general proofs, often with consistent key ideas linking these stages, offer pre-service teachers opportunities to develop a dynamic view of proof. Many written statements and the prevalence of rote modes of argumentation and purely symbolic proving methods, however, offer pre-service teachers opportunities to develop a static view of proof. Given that the two texts analyzed represent strong inclusion of proof-related reasoning compared to other elementary mathematics content texts (McCrory & Stylianides, 2014), it is likely that other texts present potentially a more static emphasis on proof.

These findings suggest the possible value of supplementing these texts with additional opportunities from instructor materials or other sources to emphasize a dynamic view of proof. Classroom enactment, also, should be structured by instructors to build on existing opportunities to see proof as dynamic and directly confront messages that promote a static view of proof. Considering that roughly half of the instances of proof-related reasoning include an entire reasoning process that consists informal exploration and a complete general proof, there is much within these texts that offers opportunities to engage in the process of proof-related reasoning. It is on this that instructors for content courses for pre-service elementary teachers must build.

References

Balacheff, N. (1988). Aspects of proof in pupils' practice of school mathematics. In D. Pimm (Ed.), *Mathematics, Teachers and Children* (pp. 216-235). London: Hodder & Stoughton

Ball, D. L., Hoyles, C., Jahnke, H. N., and Movshovitz-Hadar, N. (2002). The teaching of proof. In L. I. Tatsien (Ed.), *Proceedings of the International Congress of Mathematicians* (Vol. III, pp. 907-920). Beijing: Higher Education Press.

Beckmann, S. (2003). *Mathematics for elementary teachers* (Preliminary ed., Vols. 1-2). Boston, MA: Pearson Education.

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., and Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*, 401-429.

Hanna, G. (2000). Proof, explanation, and exploration: An overview. *Educational Studies in Mathematics, 44*, 5-23.

Harel, G. and Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. In E. Dubinsky, A. Schoenfeld, and J. Kaput (Eds.) *Research on Collegiate Mathematics Education* (Vol. III, pp. 234-283). Providence, RI: American Mathematical Society.

Herbst, P. G. (2002). Establishing a custom of proving in American school geometry: Evolution of the two-column proof in the early twentieth century. *Educational Studies in Mathematics, 49*, 283-312.

Martin, W. G. and Harel, G. (1989). Proof frames of preservice elementary teachers. *Journal for Research in Mathematics Education, 20*(1), 41-51.

McCrory, R. and Stylianides, A. J. (2014). Reasoning-and-proving in mathematics textbooks for prospective elementary teachers. *International Journal of Educational Research, 64*, 119-131.

Morris, A. (2002). Mathematical reasoning: Adults' ability to make the inductive-deductive distinction. *Cognition and Instruction, 20*(1), 79-118.

Morris, A. (2007). Factors affecting pre-service teachers' evaluations of the validity of students' mathematical arguments in classroom contexts. *Cognition and Instruction, 25*(4), 479-522.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Parker, T. H. and Baldridge, S. J. (2004). *Elementary mathematics for teachers*. Midland, MI: Sefton-Ash Publishing.

Parker, T. H. and Baldridge, S. J. (2008). *Elementary geometry for teachers*. Midland, MI: Sefton-Ash Publishing.

Schoenfeld, A. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In Voss, J. F., Perkins, D. N., and Segal, J. W. (Eds.), *Informal Reasoning and Education* (pp. 311-343). Hillsdale, NJ: Lawrence Erlbaum.

Stylianides, G. J. (2007). Investigating the guidance offered to teachers in curriculum materials: The case of proof in mathematics. *International Journal of Science and Mathematics Education, 6*, 191-215.

Stylianides, A. J., Bieda, K. N., and Morselli, F. (2016). Proof and argumentation in mathematics education research. In A. Guttierez, G. C. Leder, and P. Boero (Eds.), *The Second Handbook of Research on the Psychology of Mathematics Education* (pp. 315-351). Rotterdam: Sense Publishers.