Individual and Situational Factors Related to Lecturing in Abstract Algebra

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In this study, we report the results of a national survey of 219 abstract algebra instructors concerning their instructional practices and pedagogical decision-making. Organizing our respondents into groups (Alternative, Mixed, Traditional) based on proportion of class time lecturing, we investigated differences in the prevalence of specific pedagogical practices and the individual/situational factors influential therein. We used the reported teaching practices to generate profiles of the salient features of each instructional type and attempted to explain these differences through a combination of individual/situational factors. Results indicate that while significant differences in teaching practices exist, these are primarily explained by individual factors such as personal beliefs, level of experience, and interest in various scholarly activities. Situational characteristics, apart from institution type as identified by terminal degree, such as perceived departmental support and situation of abstract algebra in the broader mathematics curriculum did not appear to be related to instructional differences.

Keywords: pedagogy, abstract algebra, instructor decision-making

In STEM higher education, and specifically in mathematics, lecture-based pedagogy is the norm. The most current report (2014) from the Higher Education Research Institute found that "more than two-thirds of faculty across STEM sub-fields utilize[d] extensive lecturing in all or most of their courses" (Eagan, 2016). This is despite the growing volume of literature, both from the learning sciences and professional organizations, urging teachers to adopt student-centered practices. Indeed, with the mounting body of evidence (see Freeman, et al., 2014 for a meta-analysis), the question is no longer "should we still be lecturing?" but is instead "why are we?"

Literature on instructional decision-making has long focused on individual characteristics, such as beliefs about teaching and learning (e.g., Calderhead, 1996), knowledge of research-based instructional practices (e.g., Henderson and Dancy, 2009), and professional development (e.g., Belnap, 2005; Speer, Gutman, & Murphy, 2005). While we do not discount the enormous influence of individual characteristics, we also acknowledge that a bevy of other external circumstances factor considerably. Research in science and mathematics education had identified a number of external influences, including expectations of content coverage, time expectations, promotion and tenure requirements, and class size (e.g., Henderson & Dancy, 2007; Hora & Ferrare, 2013; Hayward, Kogan, & Laursen, 2016; Johnson et al., 2013; Turpen, Dancy, & Henderson, 2016).

In this paper, our goal is to better understand the teaching practices of mathematicians and to begin to tease apart how both individual and situational characteristics relate to instructional decision-making. We conducted a national survey informed by findings from small-scale interview studies (e.g., Johnson et al., 2013; Roth McDuffe & Graeber, 2003) and from national surveys of undergraduate science educators (e.g., Henderson & Dancy, 2007). Our analysis of the responses provides a nuanced characterization of instruction, while also identifying individual/situational characteristics that are (not) associated with different instructional profiles.

Literature Review and Theoretical Perspective

Case studies of mathematics instructors have uncovered multifaceted beliefs and goals that inform instructional practices (e.g. Johnson et al., 2013; Lew et al, 2016; Weber, 2004). As to be expected, beliefs about teaching and learning appear to be quiet varied. For instance, some are convinced of the benefit of non-lecture instruction as were the interview participants in a 2003 study who reported a desire to teach with "constructivist activities where the depth of knowledge is really greater" (Roth McDuffe & Graeber, p.336). On the other hand, there are those that are convinced of the strength of lecture, stating, "I believe students benefit from seeing education embodied in a master learner who teaches what she learned" (Burgan, 2006, p. 32). These beliefs do not exist in a vacuum and are likely informed by personal experiences – both as a student and a teacher – and influenced by activities such as attending workshops and conferences.

Regardless of how they developed, beliefs about teaching and learning alone do not appear to be deterministic of teaching style. For instance, in a previous report, we found that 64% of the respondents at research-focused institutions who think lecture is *not* the best way to teach lecture anyway (Johnson, Keller, Fukawa-Connelly, 2017). A common reason provided to explain the incongruity between how instructors want to teach and how they actually teach is a concern about content coverage (e.g., Johnson et al., 2013; Roth McDuffe & Graeber, 2003). Content concerns represent an interesting intersection of internal and external pressures. As discussed by Turpen, Dancy, and Henderson (2016), coverage concerns are influenced by personal beliefs about what should be included in the course, expectations from others in the department to cover a set curriculum, and more subtle indicators such as textbooks, their experiences as a student, and emotional attachments to topics. In this way, coverage concerns are to some extent self-imposed, yet attributed to external factors such as a common syllabus or need to prepare students for success in future courses. Such responsibilities to the discipline and the department illustrate that, while university instructors do control their own courses, "the influence of various external factors diminishes their perceived control over their teaching" (Lea & Callaghan, 2008, p.174). Similarly, time constraints are often cited as a strong factor influencing instructional practice. Turpen et al. (2016) noted interviewees discussing both "a broad sense of feeling overwhelmed with the responsibilities and demands on their time" and "specific aspects of their job description or institutional situation that led to them being stretched too thin" (p. 7).

This literature highlights the importance of instructors' beliefs and experiences and institutional and departmental context on instructional decision-making – both of which are central to Henderson and Dancy's (2007) model for predicting instructional behavior (Figure 1). This model provides a framework for considering instructional practices in light of the characteristics of both the instructor and his/her department. However, as acknowledged by Henderson and Dancy (2007), this is a "toy model" –

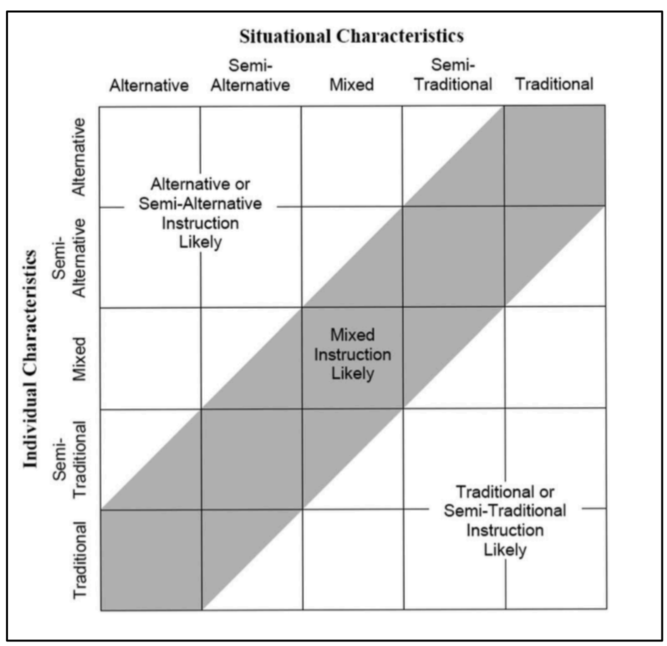

Figure 1. Toy Model

one that simplifies complex systems by highlighting dominant features, without detail about what is important within those domains. Better understanding the complex relationship between instructor characteristics (e.g., background, beliefs, knowledge, goals) and situational context (e.g., departmental norms, departmental supports, institution type) is the focus of this research. In this report we draw on data collected through a national survey of abstract algebra instructors in order to first characterize instruction and then to identify individual and situational factors that are associated with different teaching profiles. Specifically, we investigate three research questions: 1) What is the range and distribution of reported instructional practices in an upper division mathematics course (in this case, abstract algebra), as interpreted as traditional, mixed, and alternative instruction? 2) What individual characteristics (background, beliefs, knowledge and goals) are associated with instructors characterized as traditional, mixed, and alternative? 3) What situational characteristics are associated with instructors characterized as traditional, mixed, and alternative?

Study Context, Data, and Methods

Study Context

We focus on abstract algebra for the following three reasons. First, the major professional organizations have released a joint course-guide for abstract algebra calling for increased activity on the part of the students during the course meetings. Second, the research base in abstract algebra, including curricular innovations, is significant (at least in comparison to other proof-based courses). Finally, the course is often a small class taught by tenure-stream faculty. We have argued that these factors position abstract algebra, of all the required courses in the undergraduate mathematics plan of study, as the course best positioned to be taught with significant non-standard pedagogy.

For this report, we are drawing on two rounds of data collection, both of which used the same survey. In the first round of data collection the target population was instructors at universities that offer a graduate degree in mathematics. In the second round of data collection, we chose to target instructors at institutions not offering graduate degrees in mathematics. This decision was made upon considering the extant literature on teaching practices and observing that this research is primarily conducted by faculty at research-intensive universities about faculty teaching at such universities. Thus, even though the literature includes claims like: “lecture is overwhelmingly the dominant pedagogical technique both in terms of percentage of instructors claiming to use it and percentage of class time they report devoting to its use” (Fukawa-Connelly, Johnson, & Keller, 2016), it is possible that the lack of substantial representation of faculty at teaching-focused institutions may be a problem in terms of understanding the collected instructional practices of mathematicians in proof-based courses. For instance, the types of individuals who seek employment at research universities may have different beliefs about teaching and learning than their counterparts at teaching colleges. This, coupled with the disparate demands on time use, might influence the instructors’ willingness to adopt non-traditional pedagogies in different ways.

Survey Design

The survey was designed to solicit information about the teaching practices, beliefs, and situational context of abstract algebra instructors. This survey was informed in part by both Henderson and Dancy’s physics-education survey (Henderson & Dancy, 2009) and the Characteristics of Successful Programs in College Calculus surveys. Our survey had sections to address: basic demographics and course context, teaching practices, beliefs and influences (including perceived supports and constraints), and knowledge of/openness to non-lecture practices. In the second round of data collection, the target population changed, but the information we were soliciting did not. For that reason, it was methodologically important that the items under investigation remain largely unmodified. While a few supplemental questions were added, the majority of the items were a subset from the previous survey with formatting intact. For the purposes of this paper, only those items unchanged by version were considered for analysis.

Participants and Data

The present data set is the result of two independent sampling attempts. The first data collection period was conducted in 2015. Survey requests were sent to departmental administrators at approximately 200 institutions. We received 126 responses, 91% of which represented instructors teaching abstract algebra at an institution offering at least a Master's degree in Mathematics. The second data collection period took place in summer 2016. In this follow-up, a random sample of 400 institutions was drawn from the IPEDS list, targeting specifically Bachelor's-granting schools. This sample yielded 112 responses, 91 of which were complete. For the purposes of this paper, all responses on applicable items have been combined into a single data set and disaggregated by instructional type for all future analysis. In total, 219 respondents were retained: 96 from Bachelor's-granting institutions, 44 from Master's-granting institutions, and 79 from PhD-granting institutions.

Methods

The purpose of this study was to describe the range and distribution of instructional practices as reported by abstract algebra instructors interpreted to be implementing a traditional, mixed, or alternative approach; and, furthermore, to investigate similarities/differences in the individual and situational factors influencing those practices. The characterization of instructional type was made using the prompt, *Please indicate the approximate percentage of class time that you are lecturing*, for which we coded the respondents as "Alternative" for responding *Never* or *0-25%*, "Mixed" for responding *25-50%* or *50-75%*, and "Traditional" for responding *75-100%*. This classification resulted in the following distribution of respondents: 17% Alternative (38/219), 57% Mixed (125/219), and 26% Traditional (56/219).

To address the first research question – the range and distribution of instructional practices - three survey items were analyzed. In each instance, the prompt instructed respondents to indicate the prevalence (instances per term / instances per class meeting / percentage of class time) of specific classroom activities/pedagogical practices utilized in their classrooms. To address the second research question – the specific individual factors characteristic of each instructional type – five survey items were analyzed. The first two items gathered demographic information on the teaching experience of the respondents and the latter three items polled respondents as to their beliefs about students, beliefs about teaching, and interest in professional activities as measured by a 4-point Likert scale. To address the third research question – situational factors characteristic of each instructional type – eight survey items, divided into two sets, were analyzed. The first were those that situated abstract algebra within the broader mathematics curriculum and the second subset were those intended to capture respondents' perceptions of departmental support for, and institutional constraints on, innovative teaching.

Group mean scores for each sub-item were computed by instructional type and compared using inference testing procedures such as ANOVA, Chi-square, or the Kruskal-Wallis test, as applicable to the data, with post-hoc testing for pairwise comparisons therein; within each item, the Holm-Bonferroni correction was applied to control for the family-wise error rate affiliated with multiple comparisons when appropriate.

Results

Research Question #1 – How can we conceptualize "traditional", "mixed", and "alternative" instruction in upper division mathematics courses?

In order to conceptualize the classroom experience of each instructional approach, the mean reported prevalence of a variety of pedagogical practices was computed and used as a means of comparison. Sorting respondents based on proportion of time spent lecturing highlighted 14 instructional practices that varied significantly between at least two of the three categories, with 8 practices being significantly different (family-wise error rate < .05) on all three pairwise comparisons. Considering reported teaching practices altogether generated these profiles:

- **Alternative** instruction is characterized by spare lecture, with class time split class time (fairly evenly) between showing students how to write proofs, having students work in small groups, having students give presentations, having students work individually, lecturing, holding whole class discussions, and having students explain their thinking. For alternative instruction, when compared to the other instruction profiles, it is less likely for instructors to pause and ask questions, use visual representations, diagrams, and informal explanations. Students in these classes are frequently asked to make presentations to the class and develop their own conjectures and proofs and are sometimes asked to develop their own definitions.
- **Mixed** instruction is characterized by moderate use of lecture lecturing, with significant class time devoted to showing students how to write specific proofs, pausing to ask students questions, and using diagrams, visual representations, and informal explanations to help students with formal ideas. Additionally, there is some class time devoted to students working alone and in small groups, giving presentations, and explaining their thinking. Students in these classes are pretty frequently asked to develop their own proofs, and are sometimes asked to present their work to the class and develop their own conjectures.
- **Traditional** instruction is characterized by heavy use of lecture. During lectures instructors report they are showing students how to write specific proofs and pausing to ask students questions. These lectures often include diagrams to illustrate ideas and informal explanations of formal statements, but are the least likely to discuss why material is useful and/or interesting amongst the three categories. Students in these courses are sometimes asked to develop their own conjectures or proofs.

We do not claim it is surprising that, with less time devoted to lecturing, Mixed and Alternative instructors are spending more time engaging students in mathematical activity (e.g., developing proofs) and in peer-to-peer activity (e.g., working in small groups and giving presentations). Rather, we offer these results to justify using the amount of time spent lecturing as a viable means for differentiating instructors as Traditional, Mixed, and Alternative.

Research Question #2 - What individual characteristics are associated with instructors that report traditional, mixed, and alternative instruction?

Keeping in mind the goal of elaborating on the "toy model" presented in Figure 1, our investigation of research question #2 has yielded information that draws distinctions on individual characteristics between types of instructors. In particular, at least one significant pairwise comparison existed for 12 of the 17 items under consideration: teaching experience

(2/2), beliefs about teaching and learning (7/10), and interest in various types of scholarly activities (3/5). Traditional instructors (i.e., those reporting lecturing more than 75% of class time) are the most experienced group, hold the strongest beliefs in favor of the appropriateness or necessity of lecture and the most pessimistic views on their students' abilities, and have a stronger interest in mathematical research than educational research. Conversely, alternative instructors (i.e., those reporting lecturing less than 25% of class time) tend to be the least experienced, hold the strongest beliefs in favor of non-lecture activities and the most optimistic views on their students' abilities, and prefer research in teaching and learning to that of abstract algebra. For all items, the Traditional and Alternative groups always occupied the extreme positions on the continuum with the Mixed group in between. This provides further evidence to suggest that separating instructors based on a single characteristic (i.e. proportion of class time lecturing) does result in meaningful categorizations.

Research Question #3 - What situational characteristics are associated with instructors that report traditional, mixed, and alternative instruction?

Our investigation of research question #3 failed to provide distinctions on situational characteristics between types of instructors, at least as we have defined them. At least one significant pairwise comparison existed for only 2 of the 8 items under consideration: institution type and time pressure. When considering the distribution of institution type by instructional approach, we found that Traditional instructors are significantly more likely to reside at a PhD-granting institution than either the Mixed ($p < .001$) or Alternative ($p = .001$) groups. Nearly 60% of Traditional instructors are at PhD-granting institutions, whereas only 18% of Alternative instructors are. Furthermore, while we can see that the modal class for all institution types is Mixed instruction, we do see a gradual rise in incidence of Traditional instruction as the terminal degree escalates from Bachelor's (12.5%) to Master's (25%) to Doctorate (41.77%). Collectively, these results indicate a dependency between institution type and instructional approach.

When considering abstract algebra in the broader mathematics curriculum, we found little to suggest that the instructors in the various groups experienced different departmental circumstances. The distribution of responses on both the prerequisite course and follow-up course items revealed some interesting trends, but the lack of statistical significance indicated that these items are likely independent of instructional approach. Alternative instructors tend to be the most likely to work in a department to require a proof-based prerequisite (79%) and the least likely to work in one that offers a subsequent algebra course (58%). Additionally, we observed that the requirement of a proof-writing prerequisite seems to be inversely related to the proportion of time spent lecturing (Alternative > Mixed > Traditional) and that the existence of a follow-up algebra course appears to be directly related (Alternative < Mixed < Traditional); however, there were no significant pairwise comparisons in either case.

The second sub-set of items analyzed included those questions intended to capture respondents' perceptions of departmental support for, and institutional constraints on, innovative teaching. These items focused on departmental expectations and content pressure, the availability of time for teaching and course redesign, travel support for professional development, and freedom to make changes to their abstract algebra course. We found that, not only were there no statistically significant differences for many of these items, the distributions were nearly identical. The lone exception being the question: *Do you feel like your job requirements allow you to spend as much time as you would like on teaching…?* Here we see that about 70% of

Alternative and Traditional instructors responded in the affirmative, whereas slightly less than half of the Mixed instructors felt that way.

Discussion

The goal of this research study was to investigate the range and distribution of reported instructional practices in abstract algebra instruction and how different individual and situational characteristics are associated with instructors who report different types of instructional practices (see Figure 2). This analysis has allowed us to provide further insight into the "toy model" that Henderson and Dancy (2007) developed. In particular, while prior research suggested the importance of a variety of individual and situational characteristics, our work suggests that a relatively small collection of individual characteristics may be actually be the most important for teaching practices, at least in terms of the broad-strokes characterization of instruction we use here.

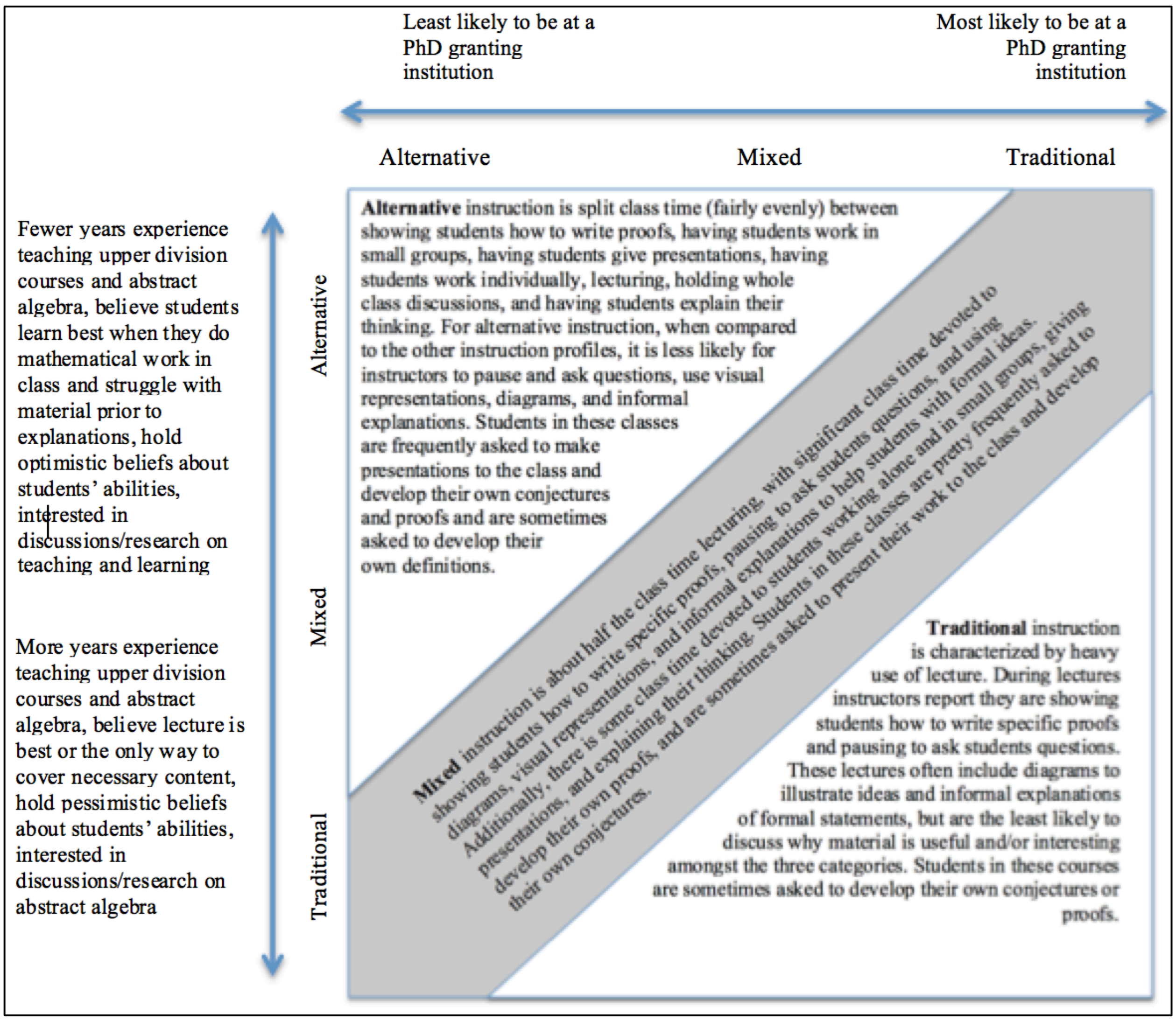

Figure 2. Individual and Situational Factors and Instruction Profiles

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