Graphing as a Tool for Exploring Students' Affective Experience as Mathematics Learners

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Researching affective issues can be difficult in education; methods like interviews and surveys can place artificial categories on participants' experience and exert biased influence. This lack of tools to study affect calls for better methods. We explore graphing as a potential tool with affordances for studying affect, by reporting results of three separate studies at different timescales where undergraduates graphed affective phenomena like confidence or emotion: two in an introduction to proof course and one in a pre-service teacher content course. By systematically describing each study and looking across the three, we argue that graphing can be a useful technique for representing experience. Its utility lies in aligning research goals with the structure imposed by the temporal axis. More structure along the temporal axis allows researchers access to what a student experiences at predetermined temporal points and less structure allows access to what students themselves find to be salient events.

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Research is beginning to appreciate the deep importance of affect in the experience of learning mathematics (Ainley, 2006; McLeod, 1992). However, despite this increased recognition of the importance of affect, the field lacks methodological tools to investigate students' non-cognitive, affective, emotional experience during cognitive activity. This methodological paper reports on the approach of affect graphing during learning experiences (building upon the work of McLeod, Craviotto, & Ortega, 1990; Smith & Star, 2007; and Smith, Levin, Bae, Satyam, & Voogt, 2017). We explore the use of affect graphing across three recent studies within undergraduate mathematics education (two studies situated in the context of an introduction to proof course and one study situated in the context of a number and operations content course for pre-service elementary teachers). Particularly notable is how different time scales were engaged in each context: the scale of reflection on work on a single problem, a single class discussion, and finally the scale of reflection was an entire course.

Interviews remain a commonly used method that takes as its object of inquiry the experience of the individual. However, studying affect on the sole basis of verbal protocols is problematic. For example, while interviewers can prompt a subject to report how they are feeling in the moment, one needs to be aware that is an intervention in the experience and may change or shape the perception of the experience. Asking subjects to report their experience in a completely open way can lead to subjects focusing in on very particular moments and not supporting reflection across an entire time interval of interest to the researcher. Lastly, interviews are dependent on interviewees being able to articulate their emotions and feelings in words. Depending on the amount of direction given, participants may need to interpret and respond to categories given to them as opposed to describing their own experience and its ebb and flow in their own terms.

A second potential contrasting approach to studying affect involves surveying participants about their affective experience. Positive implications of such an approach include the ability to generate a larger volume of data with prompts that serve as proxies for experience, beliefs, and participant feelings during problem solving. However, such methods are less responsive to participants' own categories of experience, forcing subjects to again fit their experience into the pre-conceived categories of the researcher. Surveys are also conducted in a way that does not allow for the temporal, moment-by-moment recording of an experience.

While the above discussion of contrast methods is not meant to suggest that adaptations of such methods cannot ameliorate some of the constraints of those methods of data collection, it is meant to point out that other methods (such as the graphical approaches we discuss here) may have advantages in addressing such questions. The field needs better tools for studying affect.

Framework

This paper differs from the typical empirical report in that we explore the affordances of an innovative methodology. The goal of our paper then is to analyze the ways in which graphing was productive. We therefore present a framework for how we describe each context and compare them to each other. For each context, we provide (1) a *description* of the overall study, (2) the *purpose* of graphing as a tool in this context, (3) what the graph *measured*, (4) *features* of the graph such as timescale, axes, labels, (5) *results* from analyses and suggestion for potential analyses that could be done, and finally (6), a prototypical *example*.

The Three Contexts

We describe three separate studies in which the approach of graphing was used. Two of the contexts were research projects and one was from a course, as seen in Table 1.

Table 1. Features of the Three Studies Using Graphing as a Methodological Tool.

Course	Population	Axes	Graph Measures
Intro to Proof	Math majors and minors	1st quadrant	Students' confidence
Numbers & Ops	Pre-service elementary teachers	1st quadrant	Students' confidence
Intro to Proof	Math majors and minors	1st & 4th quadrant	Students' emotion

The first was a study of undergraduates' confidence reflecting on a completed intro to proof course. The second project examined elementary pre-service teachers' confidence levels in reaction to a class discussion in a numbers and operations course. The third context was also in the intro to proof course but focusing on students' emotions while working on a proof. Each of these studies was conducted by different subsets of the authors of this paper.

We have chosen to order contexts by timescale, from broadest to shortest. We do this to make salient the variations in how the graph was used when the time scale shrinks. In addition, the first and second contexts use graphing to measure the same construct (confidence), while the last context measures emotion; emotion may include confidence but can be multi-dimensional.

Context 1: Introduction to Proof Undergraduates Graphing Confidence over a Semester

The focus of the first study in which we explore the use of graphing was understanding undergraduates' experience in an introduction to proof course. The population ($N = 14$) consisted of math majors and minors who had just completed the introduction to proof course. In a prior study, Smith et al., (2017) interviewed them about their view of the nature of the course in contrast to past math courses, their sense of success, and how their view of and work on proof tasks may have changed over the course.

The researchers also wanted to tap into the affective dimension of experience – how they felt at different points – so they asked students to graph their confidence in the course across the semester. Interviewers left the room while students drew their graph and when they returned, asked students to talk through their graph. The x-axis measured time, from before the semester started (to account for expectations of the class prior to its start) to right after the final exam. There were otherwise no fixed tick marks along the x-axis, because of the interest in not only how students' confidence changed but *what* influenced shifts in confidence. The y-axis measured confidence. Tick marks on the y-axis were given, corresponding to low, medium, and high confidence. The majority of students drew continuous graphs. The task was kept relatively open to allow students to represent their experience however they chose to, away from our judgment of what may be important milestones in the course.

One analysis is to categorize the *shape* of the graph, to identify common patterns of confidence over the semester. We found five categories of shapes, with two graphs as outliers to categorization. The most common ($n = 4$) shape for confidence was a "W" shape: initial high confidence in the course with a quick drop early in the semester, then an increase over time, followed by a decrease and then a final increase to the end of the course (see Figure 1).

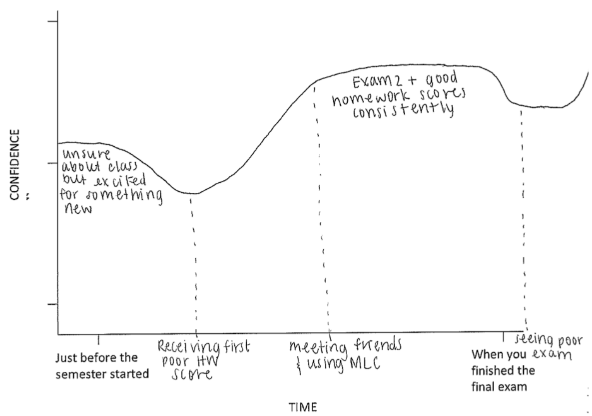

Figure 1. Example of the "W" shape, the most common pattern for confidence over time. This figure also shows the kinds of x-axis markers students included as places where their confidence shifted.

The other four shapes in our set of 14 graphs were: (a) continuous increase, (b) concave up parabolic shaped graph, (c) initial increase followed by a sinusoidal wave for the rest of the semester, (d) initial increase followed by decrease, with a final confidence level that was lower than their initial level. The W pattern as the most frequent makes sense given the different nature of a proof-based work and the introduction to advanced mathematics (analysis, linear algebra, number theory) after the half-way point of the course.

Graphing was insightful here in that it gave students a vehicle through which to reflect across an entire semester (half a year). The act of drawing the graph served as a way of recalling and organizing how they felt, in a way that localized interview questions did not capture. In addition, the openness of the x-axis meant students could tell us what events corresponded with rises and falls in their confidence, as opposed to our assumptions that it would revolve around exams for example. With this, we could identify the events that stood out as pivotal moments, relative to the entire experience.

Context 2: Pre-service Teachers Graphing Confidence over a Class Period

The second context that used graphing involved a study of pre-service teachers' response to an orchestrated classroom discussion. Two of us have been involved in developing and revising a one-semester course on number and operations for pre-service teachers with an emphasis on justification, specifically on developing our prospective elementary teachers' (PTs') abilities to analyze and critique the work of others. These goals are not easily achieved, as there are issues that arise in orchestrating such class discussions, particularly those that capitalize on incorrect patterns of reasoning that PTs themselves may generate (e.g., Chamberlin, 2005; Silver, Ghousseini, Gosen, Charalambous, Strawhun, 2005). Toward this end we have been developing a strategy for orchestrating discussions for enabling PTs to consider and analyze divergent thinking on mathematical tasks.

The strategy has 4 main stages: (1) Engagement: PTs generate their own ideas for a solution; (2) Interruption, juxtaposition, and re-focusing: The teacher posts two different answers and asks PTs to determine a solution path that would lead to each answer; (3) Articulating the reasoning of another: PTs present ideas for the chain of thinking that led to an answer. This discussion focuses on understanding the reasoning (without bias), along with establishing common ground; (4) Validity: PTs consider how to determine the validity of one approach (and thus why the other approach is not valid). The goal of this orchestration was to destabilize PTs' thinking.
Prior analysis of class videotapes indicated this approach's efficacy, but we wanted tools for tracking individual PT thinking at key stages of the orchestration strategy, along with evidence of their understanding at the conclusion of the activity. Graphing was used for this purpose.

Immediately following the activity, PTs were asked to rate their confidence level in their own thinking at each stage on a 5-point scale from low to high, thus creating a "confidence graph" for the activity. At the end of the activity, students were asked to explain the valid strategy in a way that would help someone with the invalid strategy understand why it was invalid. Our data suggest that the orchestration strategy used in this case was successful, both in establishing cognitive dissonance, and also, importantly, in allowing students to come to resolution.

The main patterns in student confidence graphs were the same across two different classrooms with different instructors: the lowest confidence level was at Stage 2b (2.7 out of 5, or 54%) and the highest confidence level was at the end of the activity (4.7 out of 5, or 95%). With respect to the overall shape of PTs confidence graphs, the majority of the students (83%) had at least one point in the activity at which their confidence took a downward turn (indicating some degree of destabilization in their thinking). Analysis of the written student work that accompanied the generation of the confidence graphs, in conjunction with students' professed levels of confidence at the end of the activity, indicated that students came away from the activity with a heightened understanding of the mathematical content of the activity.

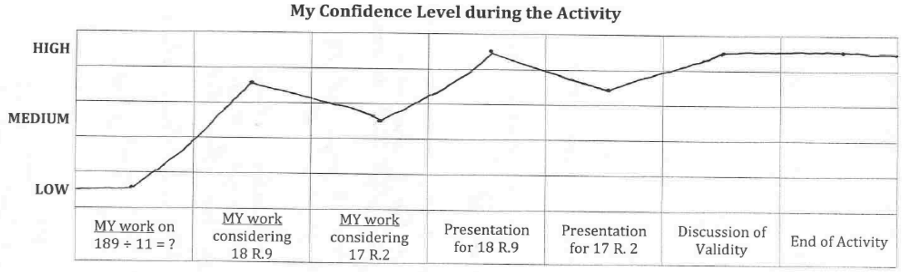

Figure 2. Sample confidence graph produced by a student reflecting on their confidence over the course of the class discussion of 189 divided by 11.

Collecting PT's reflections on their affect across the discussion allowed us to capture data that would be difficult to get from other methods like class observation and videotape. Though video reveals an "impression" that at least some of the PTs were deeply engaged in discussions, the confidence graph activity gives researchers (and teachers) a tool for measuring where the entire class is in their understanding and how this shifts over the course of the discussion.

Context 3: Introduction to Proof Undergrads Graphing Emotion over a Single Problem

The third study we discuss tracked students' emotions while working on a proof construction task. N = 11 undergraduate students (some math majors, some not) were interviewed four times across the semester, while taking an intro to proof course. In each interview, they were given two proofs to work on for a maximum of 15 minutes each, which were picked intentionally to be challenging, hence problems. Students were encouraged to "think-aloud" while working. After each task, students were asked to describe their process, choose from a given set of emotion words to describe what they felt and then draw a graph of their emotions during that problem.

Graphing as a technique was chosen (a) as a talking aid, to help students articulate their emotions, which in general can be difficult and (b) to succinctly compare patterns of emotion across participants on the same problem and that of a single participant within a problem. The level of intensity in how they felt at various points could be better seen and compared visually.

The graph measured emotion, signed (positive or negative) intensity without specifying the exact emotion. The x-axis was time, from when the student started working on the problem to when they stopped. The y-axis represented emotion, with a tick mark above the x-axis denoting positive emotions (e. g. satisfaction or excitement), a tick mark below the x-axis denoting negative emotions (e. g. frustration or panic), and the axis itself being neutral with no particular emotion, i.e. one's "resting state." All students drew continuous graphs, a line graph across the page. They also marked on the graph reason(s) why emotions shifted.

In sorting the graphs, the analysis revealed 6 general profiles of graphs: (1) overall positive, (2) overall negative, (3) flat, (4) concave down, (5) concave up, and (6) other. Of the 88 graphs, 40% were concave up, 18% were overall positive, 13% were overall negative, 10% were flat, 8% were concave down, and 11% fell into the other category. Concave up graphs suggested that the student overcame struggle(s), whereas overall positive graphs had no issues impacting emotion. Flat graphs showed experiences with little variation in emotion, whether completely flat and above the x-axis because the problem was easy, or right at the x-axis because the student stayed unsure the entire time. Concave down graphs were experiences that started well but where the student got stuck and could not resolve it.

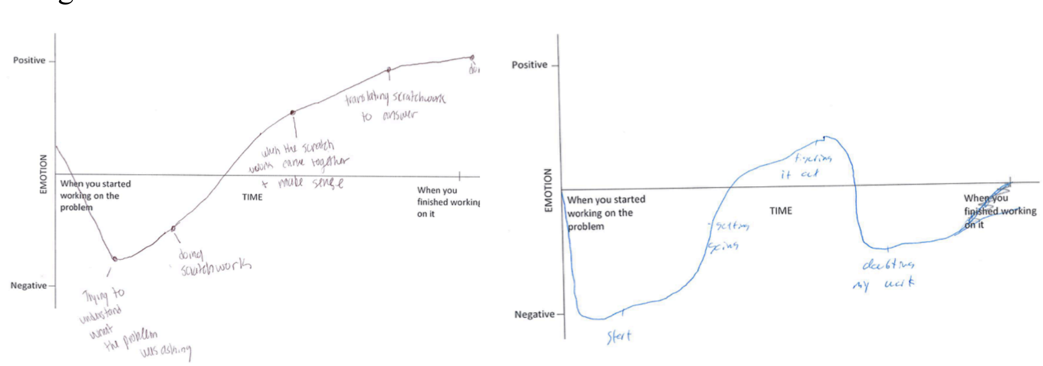

Figure 3. (Left) Prototypical example of concave up, the most common graph type. *(Right)* An example of a graph in the Other category, showing rises and falls in emotion.

The other category consisted of graphs with many changes in emotion (such as a “W” shape), large rises and falls, and states of confusion. This other category is a collection of volatile problem solving experiences, due to the size and number of rises and falls in emotion.
The results showed that there were a number of experiences where students successfully worked past a struggle. The graphs were useful in quickly and visually identifying whether students engaged in problem solving behavior - the existence of a struggle.

The choice to collect graphs at multiple timestamps for each student allows for temporal analyses as well. For example, one could look at how students’ emotions while problem solving change over time, as seen in Figure 4.

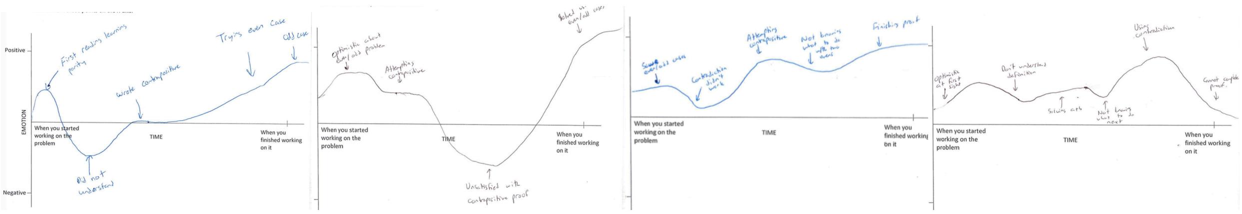

Figure 4. Graphs of emotions for one participant over 4 points in time.

Overall, graphing was most useful as a way for students to communicate their problem solving experience in a temporal fashion. Like in the first context, keeping the x-axis unstructured meant students talked about (and annotated on the graph) the events that caused their emotions to shift. The identification of these events and how students interpreted them, and how this changed over multiple points in time especially, was valuable.

Discussion

We now turn to comparing the use of this new analytic tool across the three contexts and point to directions for future use of the tool. We focus our discussion on the affordances of this tool across the three contexts.

In all three contexts, graphing an aspect of affective experience during learning and problem solving (e.g., confidence, emotion) was used to glean information that would have been challenging to gather using traditional methods such as verbal interview, video of problem solving or class discussion, or survey methods. In all cases, we considered the affective variable over time and assumed that the experience (problem solving, taking part in a class discussion, engagement in a course) influenced the graph that was produced. All three contexts ask participants to reflect on their experience and represent it graphically. As discussed earlier, the produced graphs tap into the utility of this approach for understanding participants’ experience of events of differing time scales: work on a single problem, engagement in a class discussion over an entire period, participation in a course over a semester.

The presentation of the studies and results where this tool was used demonstrates the wide applicability of this methodological tool. While we used the tool with both undergraduate mathematics majors and with pre-service elementary teachers, we can envision that this tool would be possible to use with an even broader range of participants. Mathematics majors were more familiar with graphing and more able to interpret and adapt the tool (e.g., including their own points of salience along the x axis). For the pre-service teacher study, we included more structure along the x axis and grid lines to allow participants to either create a continuous graph or to simply mark whether they had high, medium, or low affect at each of the pre-specified points within the discussion. Many students chose this way of interacting with the given template, creating a bar graph as opposed to a line graph. From a theoretical standpoint, we see that the act of drawing the graph serves as a form of *rendering* for the student, i.e. making sense of an experience. It locates but also provides temporal structuring, allowing students to organize their experience temporally, which helps them communicate their experience to us.

At a general level, there are several affordances to this approach. The method is easy to administer. Participants have a range of agency in terms of what they draw and how (completely open in the case of undergraduate math majors; more structured in case of PSTs). A methodologically attractive feature of the graphs produced is that they capture the students' reflection on their affect over the entirety of the experience as opposed to the interviewee focusing in on one particular part of the experience that was more salient to them. For this reason, giving participants the opportunity to explain their written graphs can allow researchers to elicit data on the relationships between participants' affect at different points in time over the entire experience and also what parts of the experience were most salient. The graphing activity encouraged a negotiation in the representation of particular focal experiences/feelings and a global sense of the experience. Having participants discuss their graph also gave insight into participants' views of the driving forces or reasons behind shifts in confidence or affect. The shared artifact to talk over seemed to help participants craft a narrative not only of how their experience shifted but what was behind those shifts.

While there are numerous affordances of the method, the approach, like any qualitative approach focuses on self-report. However, because our interest is in participants' models of their own experience, it is less critical for us to judge whether or not participants' confidence or emotions actually did increase or decrease in the ways they reported. The important data for us is participants' perception of their own experience, captured very well by the graph. One constraint with capturing data on participants' perception of their experience is that the most vivid data about participants' perceptions comes as close as possible to the experience itself. In the study of problem solving and the study of students' experience of the classroom discussion, the reflections took place immediately after the experience. We felt this was the "best possible" timing so that participants would not be simultaneously reflecting on their experience while also engaging in the focal task. The post-hoc interviews of confidence over the experience of the course were more challenging in this respect because, necessarily, more time had passed between the experience and the participants' reflection on it.

Graphing as a tool has implications for other purposes besides research too. While we focus here on the use of graphing as a research tool, it also works as an in-class tool, as a form of formative assessment. It can function as a support for student reflection or for teachers to check-in with students, as was done in the pre-service teacher context here. We believe graphing is useful for other populations of students also. We focused on the use of this tool with college populations and admittedly, there is reason to believe it is especially useful there because they undergraduates are familiar with graphing as an activity. However, a more structured approach like in the PST context could translate well to less mathematically sophisticated populations.

References

Ainley, M. (2006). Connecting with learning: Motivation, affect and cognition in interest processes. *Educational Psychology Review*, *18*(4), 391-405.

Chamberlin, M. T. (2005). Teachers' discussions of students' thinking: Meeting the challenge of attending to students' thinking. *Journal of Mathematics Teacher Education*, *8*(2), 141-170.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. *Handbook of research on mathematics teaching and learning*, 575-596.

McLeod, D.B., Craviotto, C., & Ortega, M. (1990). Students' affective responses to non-routine mathematical problems: An empirical study. In G. Booker, P. Cobb, & T. de Mendicuti (Eds.), *Proceedings of the Annual Conference of the International Group for the Psychology of Mathematics Education with the North American Chapter 12th PME-NA Conference* (pp. 159–166). Mexico. http://doi.org/ED419696

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. T. F. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *The Journal of Mathematical Behavior*, *24*(3), 287-301.

Smith, J. P., Levin, M., Bae, Y., Satyam, V. R., & Voogt, K. (2017). Exploring undergraduates' experience of the transition to proof. In Weinberg, A., Rasmussen, C., Rabin, J., Wawro, M., & Brown, S. (Eds.). *Proceedings of the 20th Annual Conference on Research in Undergraduate Mathematics Education.* San Diego, CA.

Smith, J. P. & Star, J. R. (2007). Expanding the notion of impact of K–12 Standards-based mathematics and reform calculus programs. *Journal for Research in Mathematics Education*, *38*, 3–34.