Productive Failures: From Class Requirement to Fostering a Support Group

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Mistakes occur frequently in mathematics. In two classes (Abstract Algebra and Calculus II), mistakes were brought to the forefront in the form of a "productive failure." Through five interviews with students, we initially looked for affectual responses to the pedagogical allowance and student-led demonstration. Many of the responses, both benefits and drawbacks of the productive failure, were interpreted by the research group to resemble peer-led support groups such as Alcoholics Anonymous. Descriptions of both productive failure and support groups, as well as quotes from the students, aim to shed light on psychological benefits of valuing mistakes.

Keywords: productive failure, affect, inquiry-based learning

Introduction

At one point in their life every student will reach a mathematical impasse when attempting to solve a problem. What students do after such an impasse might define how they view mathematics as a process. Additionally, what instructors do to cultivate such a process may further (and perhaps ultimately) influence students' thoughts about mathematics. The present investigation focused on the pedagogical action of allowing students to demonstrate their problem-solving impasses and explain their struggle positively. We call this struggle a "productive failure." At first, we investigated affect in students' interview responses to productive failures. However, we conjecture that many of the affectual responses may also be found as benefits and drawbacks in peer-led support groups such as Alcoholics Anonymous. While the two do not equate on a societal level, the characteristics and effects seemed to align. This proposal describes what a productive failure is, gives background on affect and support groups, and argues the resemblance of a support group to the demonstrations of a productive failure.

Background Literature

Productive Failures

The notion of using mistakes, difficulties and impasses as productive has been discussed in many capacities, often with success. However, both what constitutes "productive" and what kind of difficulty arises, varies in the literature. For example, Granberg (2016) defined productive failure as a "result in the restructuring of mental connections in more powerful, useful ways through which the problem at hand would make sense and new information, ideas and facts would become assimilated" (p. 34). Granberg, again, stated that errors play a large part: "It appears that making, discovering and correcting errors may generate effort that can engage students in productive struggle" (p. 34). However, productive to Granberg meant to obtain a correct solution, whereas the authors mean productivity in how students learned about their own problem-solving methods. What must occur for a student to be productive in their failure is a recognition of the failure or mistake (the "checking" phase of Carlson and Bloom's (2005) problem-solving process), subsequent recovery or additional approach (the cycle back to "planning" and "executing" phase of Carlson and Bloom (2005)), and the metacognitive awareness of modifying their approach for future problem solving. Research has suggested that during productive struggle, students activate prior knowledge and intuitive ideas (Kapur & Bielaczyc, 2012; Kapur, 2014). Furthermore, the more problem-solving methods that students construct during their struggles, the more prior knowledge is to be activated (Kapur, 2014).

As a pedagogical tool, there is an indication in previous literature that an environment structured for utilizing failures or mistakes can be successful in refining students' problem-solving skills. For example, an explicit incentive to correct their mistakes can be an effective formative assessment tool (Black & Wiliam, 2009). This incentive could be points or other credit in the course: "Offering grade incentives to diagnose and correct mistakes can go a long way to close the performance gap between struggling and high-performing students" (Brown, Singh, Mason, 2015, p. 4). A by-product of this pedagogical action is that it can create "failure tolerance" (e.g., Clifford, 1984; 1988), turning potentially negative occurrences into positive outcomes. Tulis (2013) stated that research into pedagogical actions on failure and mistakes is scarce: "little is known about adaptive classroom practices for dealing with errors and the reciprocal effects of students' and teachers' attitudes towards learning from mistakes" (p. 56). These effects on attitude led us to search for affect in our project, which will be described next.

Affect

McLeod (1992) stated that the definition of affect "refers to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). He goes on to state that there are three general categories to the affective domain: beliefs, attitudes, and emotions. While others have added categories to the domain (namely, values, motivation and engagement (Attard, 2014)), for the purposes of this project, the focus will be on these three categories, and on affect as a whole. Beliefs are "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). For example, an instrumentalist view of mathematics may state that mathematics is all about rules and procedures. Attitudes are "develop[ed] from several similar and repeated emotive responses to an event or object" (Grootenboer & Marshman, 2016, p. 19). Emotions are more visceral and momentous. Positive emotions include AHA! moments (Liljedahl, 2013), while negative emotions involve frustration. Negative emotions can largely contribute to how students approach problem solving tasks: "Furner (2000) suggested that two-thirds of Americans either hate or loathe mathematics" (Grootenboer & Marshman, 2016, p. 21).

The difficult part about affect is that it can be influenced by a variety of factors, some that can be controlled by pedagogical actions. For instance, Grootenboer and Marshman (2016), citing Pajares (1992), stated that "because central beliefs have been developed through experience, new activities giving rise to positive experiences and reflection upon those experiences is critical to belief change" (p. 17). Therefore, while affect is personal, and can influence cognition and learning, it is difficult and lengthy to foster or change in students. Nevertheless, a demonstration on the productivity of failure may be an influence students' affect.

Research Question

What are the effects of demonstrations of a productive failure on a student and the classroom? In particular, what changes in affect occurred during and after a productive failure demonstration?

Methods

This investigation focused on two classes: an undergraduate/graduate abstract algebra course using TAAFU (Teaching Abstract Algebra For Understanding) materials (Larsen, Johnson, and

Bartlo, 2013); and calculus II, covering from the definition of definite integral to integration techniques. The algebra class was in Fall 2015 with 32 students, and the calculus class was in Fall 2016 with 137 students. Demonstrating a productive failure in front of the class accounted for 5% of the final grade, with 2% extra credit in the calculus course if the student demonstrated in front of the large lecture instead of the discussion sections.

Productive failures generally occurred in the same manner. The instructor asked if any students had a productive failure, and if one did, the instructor would ask them to come to the document camera and demonstrate it. Students would describe their mistakes and were encouraged to reflect on them. Unless already mentioned, they were typically asked why it was productive for them. Often the problem or theorem in question was an entry point to discuss the topic for that day. These demonstrations lasted for an average of five minutes. After questions from other students and the instructor, the presenting student would walk back to their seat while their peers applauded.

The first author was the instructor for both courses and taught using inquiry-based learning (IBL) (Cook, Murphy, & Fukawa-Connelly, 2016). The second author researched the calculus course, taking observation notes of daily classes, interviewing four students (including the fourth author), and conducting an online survey (different than the end-of-course evaluations). All interviews were conducted and transcribed by the second author. One question in the interview focused specifically on productive failures and their presentations. Due to space, both the full interview questions and survey questions are omitted.

The third author was a student in the abstract algebra course and presented a productive failure after the second test, which occurred on week 10. The fourth author was a student in the calculus course, and presented her productive failure to the large class before the first test. Both were asked to participate in a reflection session with the first author about 7 months after their demonstrations, where they discussed the demonstration of the failure, the reactions that they had during the time, and future effects. The first and third authors watched the video of the third author's productive failure presentation (collected for another project), and discussed instances together in an unstructured group reflection. The third author then transcribed that discussion. The first and second author analyzed and coded utterances using affect, and then discussed the importance and significance of those codes. While coding for affect, the authors then found resemblances between the responses given and characteristics of support groups.

Results

These math classrooms, when incorporating the presentation of productive failure, can be viewed as analogous to a support group. It is prudent to reiterate that this is an analogous relation only and that it is not the intent of the authors to imply that the support that these students are receiving is of the same magnitude to other formal support groups. By the theory of Schopler and Galinsky (1995), support groups have certain characteristics that include:

- "organizational sponsorship or be the creation of an innovative practitioner" (p. 4)
- being member-centric, with members providing experiences, information, advice, and occasionally leadership within the group.
- leaders sharing authority with the members, having their legitimacy often being based on training
- providing a supportive environment and a means for developing coping abilities

The instructor implemented the productive failure requirement in his courses beginning Spring 2016, but was influenced by the IBL community (e.g., Yoshinobu, 2014) and previous literature about impasses (e.g., Savic, 2015). This wasn't necessarily the "creation of an innovative practitioner," but a practitioner that created the productive failure requirement influenced by an innovative community. All productive failures were done by the students, and frequently ended with a round of applause from the majority of the students, hence were "member-centric." The third author stated in the follow-up interview, "We have to clap! This person did such a good job! I was so excited for anyone to get up there and do it that even if it was horrible." The instructor shared the class time (and the power) with the students, and was trained to teach IBL, therefore satisfying the third requirement. As for "providing a supporting environment and a means for developing coping abilities," both may be apparent when discussing the positive and negative effects of the productive failure.

Positive Effects

Positive effects of social groups can include "greater social resources, increased knowledge about the focal concern, a sense of relief and reassurance, and enhanced skills for coping" (Schopler & Galinsky, 2014, pp. 6-7). In the interviews, each benefit seemed to align with multiple affectual quotes from the students, which are portrayed in Table 1. The affect code in the quotes is interspersed as normal font.

Table 1: Benefits with Student Responses

Benefit	Student Response
Greater social resources	"I just remember a lot of people having the same questions that I did and following down the same path that I did [Belief], so I didn't feel all that bad about having a failure and a lot of people [Emotion], I feel, benefited from me going up, because a lot of people were making the same mistake as I was. And we all got to work together to figure out the right way to do it as a class, which felt awesome. It made the class a lot more interactive and I felt that I learned a lot from my first presentation. [Belief]" – Calculus Student 4, Follow-up Interview
Increased knowledge about focal concern	"For example, in BC Calc, I really struggled with integration by parts; it never really made sense, I didn't know where it came from, but this year, integration by parts, now that I actually understand all of the background to it, makes so much more sense and it comes so much easier now. And it's because I had that opportunity to try and then fail and then see where it came from [Belief]." – Calculus Student 4, Initial Interview
Sense of relief and reassurance	"So, I went to the board and presented my productive failure and, I didn't feel bad [Emotion], which was odd because, you know, most classes when you make a mistake, people just look at you like 'wow, she's so dumb' and not in this class. They value when you make a mistake and then you realize why you made the mistake [Attitude] and you can fix it because then you're not gonna forget it, you're not gonna make the mistake again [Belief]." – Fourth Author, Initial Interview
Enhanced skills for coping	"[W]hat I learned from that was I try, really try not to fail [Attitude], but I'm not afraid of it anymore [Emotion]. So, now whenever I'm doing homework or whatever, I'm not thinking about 'I'm not going to get this right.' I think about, 'What can I do to not fail and get it right?' [Attitude] … Like, if I fail, well I fail. I just restart again [Attitude]." – Fourth Author, Follow-up

Negative Effects

There were students that stated negative effects of productive failures. This is also reflective of the support group research literature; Schopler and Galinsky (1994) found that participants felt "pressure to conform, stress related to group obligations, feeling overwhelmed and less adequate, learning ineffective and inappropriate responses, embarrassment, and overconfidence" (Schopler & Galinsky, 2014, p. 7). Calculus Student 2, in her initial interview, stated that productive failures are "terrifying," and preferred a large class because she could "hide with all those people," both are affectual responses that can be categorized as *pressure to conform* and *feeling overwhelmed*. Calculus Student 4, in his follow-up interview, stated that he enjoyed productive failures but did not find that it would transfer to his major in medicine, where he hoped to specialize in cardiovascular surgery. A student evaluation of the course stated that "I don't feel like the productive failures are effective cause it's a hit or miss whether they'll explain it well," which can be categorized as *learning ineffective and inappropriate responses*.

Discussion

The pedagogical action of a productive failure demonstration seemed to create a support-group environment. Therefore, we expect similar benefits to support groups. Although we have not found any evidence of this yet, Brown, Tang, and Hollman (2014), citing Brown (2009), stated that "Part of [support groups'] strength lies in their empowering nature, where participants help each other as equals rather than taking on dependency roles where they rely on the advice of professionals" (p. 84). Therefore, in addition to inquiry-based learning, demonstrations of productive failures may help shift power to create a more equitable classroom (Tang et al., in press).

The socio-mathematical norm (Yackel & Cobb, 1996) of learning from mistakes has effects on students' approaches to future problems. For example, the fourth author stated in her interview that she is "not afraid of failing," thus her self-efficacy may have increased for subsequent courses. Finally, this study was first conducted in order to figure out affective as well as cognitive and metacognitive shifts due to productive failures. Thus, there may be many metacognitive gains for students when demonstrating a productive failure. The third author stated in her demonstration interview that "I really do think [the productive failure] impacted me. I don't know if it impacted other people but I think that specific instance has changed how I perceive problems when I see them. I had a lot more success in Abstract Algebra 2 I think because of it."

Conclusion

Productive failure demonstrations allow mistakes to be open and psychologically constructive instead of damaging, give a platform and power that otherwise may not be available, and may influence both the presenter and their peers affectively. The intention is to investigate and collect further data, especially for the gains in problem solving. A conjecture is that as failures tend to be recast, more students will persist in their problem solving. Time and effort may improve their mathematical skills, and allow them to grow to be more content with their abilities. Encouraging productive failures in a classroom can give students the affectual support to grow as practicing mathematicians.

Questions for the readers:

1. What other pedagogical actions can create environments where mistakes are valued?
2. What other pedagogical actions can create support groups?

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