

## **Surveying Professors' Perceptions of Incorporating History into Calculus I Instruction**

*The goal of this study is to document undergraduate mathematics professors' perceptions of incorporating the history of mathematics into their Calculus I instruction. Although research has been documented on benefits of incorporating history into mathematics teaching and learning, little has been documented on undergraduate professors' beliefs and how they may incorporate history into Calculus I. To address this question, we created a survey based on Schoenfeld's (1999) theoretical framework of knowledge, goals, and orientations to capture perceptions about instructional decisions related to history incorporation. Calculus I professors in a southeastern state were surveyed to gain an understanding of perceptions on the importance of history and how they incorporate history. The majority of professors (80%) view history as important for Calculus I learning for a variety of reasons and incorporate it in different ways. Implications for supporting undergraduate Calculus I teaching and learning are shared along with questions for further research.*

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Numerous educational theorists agree that learning the history of a discipline is essential to learning the content of that discipline (e.g. Frederick & Katz, 1997; Munakata, 2005). Regarding mathematics, Berlinghoff and Gouvea (2002) explain, "Learning about math is like getting to know another person. The more you know of someone's past, the better able you are to understand and interact with him or her now and in the future" (p. 1). While a number of researchers have investigated the usefulness of incorporating history into mathematics instruction, we found little research on undergraduate mathematics professors' views on this topic. Understanding professors' perceptions is important to the field of undergraduate mathematics education. Smestad (2011) points out that teachers' views on incorporating history into mathematics instruction must be taken into account before attempting to influence their teaching.

To better understand the views of Calculus I professors, the study documents aspects of professors' perceptions on usefulness of mathematics history and how they may employ history. The research begins to fill the particular gap in the literature regarding the perceptions of Calculus I professors. The results of this work may better inform the field of undergraduate mathematics education on how to best support professors seeking to enhance Calculus I teaching and learning through incorporating history.

### **Background and Literature**

The National Science Foundation (NSF) has supported a number of scholars in their attempts to disseminate research and resources on the benefits of incorporating history into undergraduate mathematics. Frederick and Katz (1997) established the NSF supported endeavor, Institute in the History of Mathematics and its Use in Teaching. The institute was associated with the Mathematical Association of America, and one of its many aspirations was "to increase the presence of history in the undergraduate mathematics curriculum" (p. 1). Another NSF supported project, TRansforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS), narrows the focus employing history in undergraduate mathematics through

developing curricular materials based on primary historical sources. Barnett, Clark, Klyve, Lodder, Otero, Scoville, and White (2017) explain:

That faculty with an interest in primary sources can use them in their teaching is well and good, but we are convinced that there are so many benefits derived from their use that we would like to see them available to all instructors of university mathematics (p. 1).

A relevant question emerges of what type of historical resources professors use, primary or otherwise, and how they perceive them as useful.

Ferreira and Rich (2001) reviewed literature on incorporating history to conclude that using history improves perceptions of mathematics and increases enthusiasm for learning. While these benefits have been established, Calculus I professors' perceptions of incorporating history and how they use history in their instruction are largely unknown. Furthermore, while most undergraduate mathematics textbooks include historical anecdotes throughout the text (e.g. Stewart, 2015), it is also unknown to what degree such anecdotes influence instruction. Research on undergraduate mathematics education must account for Calculus I professors' perceptions in order to adequately guide curricular resource development and instructional practice.

### **Theoretical Framework**

While there is general consensus that the history of mathematics is important to mathematics teaching and learning (e.g. Frederick & Katz, 1997; Fauvel & van Maanen, 2000), we found little research on undergraduate faculty perceptions of incorporating history into their Calculus I curriculum. A gap exists in the field of undergraduate mathematics regarding the beliefs, knowledge, and goals professors exhibit on the inclusion of history to teach Calculus I. Research questions were formed and analyzed with the lens of Schoenfeld's (1999) framework of knowledge, goals, and orientations as these three dimensions capture professors' perceptions about both short and long term instructional decisions. The following research questions were addressed: 1) How important do Calculus I professors view the incorporation of history into their instruction and why? 2) What kinds of pedagogical practices do professors use to incorporate history into Calculus I and why?

### **Methods and Data Analysis**

Surveying methodology provided an overall picture of professors' perceptions of using the history of mathematics in their Calculus I instruction. Rea and Parker explain, "[t]he ultimate goal of survey research is to allow researchers to generalize about a large population by studying only a small portion of that population" (1997, p. 2). After identifying all undergraduate mathematics professors in the state of North Carolina that reported Calculus I in their professional teaching portfolio, we sent each a mailed letter and emailed invitation to participate in an online survey. Of the 599 survey invitations, we received 96 completed surveys. This provided us a 16 percent response rate.

Survey questions were designed to gain an understanding of the demographic of the responding professors (survey items 1-3), their beliefs about incorporating history (survey items 4-6), how and why they use mathematics history in teaching Calculus I (survey items 7 & 8), and interest in learning more about history incorporation (survey item 9). Survey items six and eight were open ended, and the others were multiple choice. Figure one below summarizes survey data for these categories.

<b>Survey Item</b>	<b>Responses with Percentage</b>	
1) Years teaching at a four year	0-2 years: 6%	3-6 years: 11%

undergraduate institution	7-10 years: 16%      11-15 years: 10% 15+ years: 57%
2) Number of undergraduate Calculus I courses taught	0-5 courses: 33%      6-10 courses: 15% 11-15 courses: 12%      16-24 courses: 14% 25+ courses: 26%
3) Ever incorporated history into Calculus I instruction	Yes: 71% No: 29%
4) Believe incorporating history into Calculus I instruction may benefit students	Yes: 80% No: 20%
5) Importance of incorporating history into Calculus I instruction for teaching and learning (Scale from 0 to 5 with 5 being highest)	0: 6%      1: 18% 2: 20%      3: 24% 4: 19%      5: 13%
6) If believe history is beneficial, how will incorporating history into Calculus I instruction benefit students?	See math as <i>human endeavor</i> : 28% Make <i>connections</i> to history/other disciplines/real world: 27% Increase <i>motivation</i> : 27% Increase <i>understanding</i> : 14% <i>Broadens perspective</i> : 4%
7) If you use history, how often do you incorporate history into Calculus I instruction during a one semester course?	Never: 12%      Rarely: 19% Once: 11%      2-4 times: 35% 5-9 times: 14%      10+ times: 9%
8) If you include history, describe way(s) you use history and why?	Mentioned during <i>lecture</i> : 36% Assigned outside historical <i>readings</i> : 30% <i>Projects</i> with historical component: 11% <i>Discussion</i> on mathematics history: 10% <i>Videos</i> about mathematics history: 8% <i>Papers</i> with historical component: 5%
9) Interest in learning more about incorporating history into Calculus I instruction (Scale from 0 to 5 with 5 being highest)	0: 6%      1: 11% 2: 14%      3: 36% 4: 20%      5: 13%

Figure 1. Summary Survey Responses

To analyze the data, summary statistics were calculated for all numeric data along with thematic analysis of open response data. The thematic analysis work was guided by Creswell's (2012) recommendations for analyzing qualitative data to document emergent themes in responses. We applied thematic analysis to items six and eight with the key word of the theme italicized for clarity in Figure 1 above. A subset of survey participants responded to items six and eight with 75 responding to six and 67 responding to item eight. To address the first research question, we examined summary statistics on professors' beliefs as related to their reporting of how often they incorporate history and the degree to which they are interested in learning more about incorporation. To address the second research question, we examined summary statistics on how history was used and why they reported incorporating it into their instruction. While claims will be generalized to professors in North Carolina, study findings can reasonably be used to inform knowledge of professors' perceptions outside of this state.

## Results

## Beliefs on Incorporating History

Recall the first research question. How important do Calculus I professors view the incorporation of history into their instruction and why? The majority of professors (80%) indicated that history is useful for students learning in Calculus I however only 71% reported using history in their teaching. On the importance scale from zero to five (see item five), the majority of respondents were clumped towards the middle with 63% providing ranks from 2 to 4. Professors reported multiple reasons for viewing history as beneficial to the Calculus I learner. The top three reasons were the following: 1) See math as *human endeavor* (28%); 2) Make *connections* to history/other disciplines/real world (27%); and 3) Increase *motivation* (27%).

Figure 2 below provides a small sample of the explanations professors offered for viewing the incorporation of history into Calculus I instruction as beneficial to the student.

Categories of Benefits of Incorporating History	Sample Response in Each Category
See math as human endeavor (28%)	Students benefit greatly from being helped to see that mathematics is, like any other human invention, the product of human activity, often social activity.
Make connections to history/other disciplines/real world (27%)	At face value, this can be a dry subject for students who do not see the bigger picture. Incorporating history gives students another connection to the material, and helps to position mathematics.
Increase Motivation (27%)	I have found that by placing results within context students tend to be more engaged in the topic.
Increase Understanding (14%)	Giving the context of the discovery of l'Hospital's rule, I believe, helped students retain information.  Gives an understanding of why there are multiple notations for the same things.
Broadens Perspective (4%)	It broadens student perspectives, and gives students a break from the "normal" routine.

Figure 2. Sample Responses on Beliefs

In summary, the majority of professors saw incorporating history into Calculus I instruction as beneficial for reasons that were categorized into five groups. The reasons given largely align with research on incorporating history. For instance, increased motivation has been documented as a result of incorporating history (e.g. Ferreira & Rich, 2001). One interesting finding is that the reason, increase understanding, did not make it into the top three reasons. Finally, the majority of professors indicated a substantial interest in learning more about using history in Calculus I instruction.

## How and Why History is Used

Professors reported teaching Calculus I with history in a variety of ways with varying frequencies. Regarding frequency of use in a one semester course, the bottom three categories (never, rarely, and once) account for 42% of respondents. About a third (35%) reported using history two to four times during the semester, and the remaining 23% use history five or more times. Some professors reported using history in multiple ways while others reported only one. The most common ways reported were in lecture and reading representing 66% of responses. See Figure 3 for a sample of reasons professors gave for incorporating history corresponding to a particular instructional method.

<b>How History is Used</b>	<b>Description/Why</b>
Mentioned during <i>lecture</i> : 36%	To compare the methods of either Fermat or Barrow to modern methods of finding tangent lines
Assigned outside historical <i>readings</i> : 30%	To compare the past with the present
<i>Projects</i> with historical component: 11%	[T]he biggest project, which I include EVERY time I teach Calc[ulus] I, is a re-enactment of the Newton-Leibniz controversy, in the form of a civil trial that the students plan for about a month and then stage in class. The project, often initially received skeptically, is almost always UNIVERSALLY acclaimed by students by the end.
<i>Discussion</i> on mathematics history: 10%	In discussions I give short explanations on the history and showcase where/why/how the math in their textbook was developed.
<i>Videos</i> about mathematics history: 8%	So students can see the evolution of concepts
<i>Papers</i> with historical component: 5%	To show students the history lets them know that mathematics is a dynamic subject and reinforces the idea that the concepts are most essential so they can apply Calculus to the applications that will arise in the future

Figure 3. Sample Responses on How History is Used

Professors reported augmenting their instruction with history in multiple ways. One dominant theme was that of professors finding that their students were more engaged in the class and found historical ideas interesting. Similar to responses on benefits of incorporating history (item six), few professors mentioned increased understanding in response to item eight (see Figure 3). In categorizing reasons why professors use a particular method of incorporating history, we found no mention of primary historical source use.

### Findings and Implications for Future Research

Two findings stood out. First, while the majority of professors have used history in their Calculus I teaching, even more professors believe that their students would benefit from its use. This study identifies a need to assist the segment of professors who feel that incorporating history into Calculus I is important, even though they reported incorporating such history no more than once per semester. Furthermore, the majority of professors indicated a substantial interest in learning more about incorporating history. Study findings on how history is currently incorporated may assist those concerned with developing curricular supports that enhance Calculus I curriculum with history.

The second primary finding is that only 14% of professors reported increased understanding as a perceived benefit of incorporating history. It is possible that many professors viewed increased understanding as a secondary benefit of an initial benefit they reported. For instance, those that listed increased motivation (28%) may believe this increase leads to increased understanding. Another possible explanation for the low percentage of professors listing increased understanding is their lack of reporting using primary sources to teach Calculus I content. More refined studies are needed to parse out perceptions in these regards.

As further understanding of professors' perceptions on incorporating history into Calculus I instruction is established, the field of undergraduate mathematics education will be better prepared to assist those seeking to harness the potential of using history to teach mathematics. We see at least three relevant research questions for the RUME audience. How can RUME researchers develop adequate resources to assist Calculus I professors to incorporate history in meaningful ways? What are professors' perceptions of using primary historical sources to teach Calculus I? How might RUME researchers add to our knowledge of using primary sources?

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