Prerequisite Knowledge of Mathematics and Success in Calculus I

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This preliminary report describes how prerequisite content knowledge is related to success in a first semester calculus course. Data collected included adaptive assessments administered in both Pre-Calculus and Calculus I, standardized test scores, prior enrollment in Pre-Calculus, prior enrollment in Calculus I, and final grades in Calculus I. Analysis revealed that (1) standardized metrics such as ACT, SAT, and placement test scores did not reliably predict students' success in Calculus I, (2) passing Pre-Calculus directly impacted students' prerequisite content knowledge which in turn led to a stronger performance in Calculus I, and (3) students lost a significant amount of knowledge between the end of Pre-Calculus and the beginning of Calculus I. Lastly, in an effort to identify how deficits in specific knowledge domains impact student performance in Calculus I, additional analysis revealed that students' ability to graph trigonometric functions was most predictive of their performance in Calculus I.

Keywords: Prerequisite Content Knowledge, Pre-Calculus, Calculus I

Despite national reports calling for additional Science, Technology, Engineering, and Mathematics (STEM) degrees over the next decade, students are choosing to leave STEM programs of study, in part because of their inability to pass Calculus I (Bressoud, Camp, & Teague, 2012). Although research teams have explored reasons why students struggle with college level mathematics and some have even pinpointed specific topics for which students lack sufficient prerequisite knowledge (e.g., concept of function, composition of functions, quantitative reasoning) failure rates in Calculus I remain problematic nationally (Breidenback, Dubinsky, Hawks, & Nichols, 1992; Carlson, Madison, &West, 2015).

Mathematics instructors in higher education have been regularly contending with students who are unprepared to take college level courses. Porter & Polickof (2011) have found that as many as 20% of students at PhD granting institutions and 60% of community college students are required to take remedial courses before they are permitted to take college level courses. Although, many students have high school credit for precaclulus and calculus courses, Bressoud et al. (2012) added that students who pass high school calculus courses are not necessarily better prepared for success in college level calculus courses (Bressoud et al., 2012).

Most colleges and universities utilize some type of placement procedure with their first year mathematics students. The purpose of a placement procedure is to assess students' prerequisite knowledge and subsequently place them in a course that is commensurate with that knowledge. A land-grant university in the Mid-Atlantic Region of the United States implemented the following placement procedure for all mathematics students taking Calculus 1 during the Spring 2017 Semester. New students were placed in Calculus 1 via their ACT Math score, SAT Math score, scores on a math placement exam, or successful completion of a pre-calculus course. Regardless of this placement process, failure rates (students earning a D, F or withdrawing from

the class) in Calculus 1 have remained high. The failure rates from Fall 2015, Spring 2016, Fall 2016, and Spring 2017 were 44%, 55%, 34%, and 50% respectively.

In an effort to better understand the relationship between students' prerequisite knowledge and their performance in Calculus I, this preliminary report will specifically address the following research questions:

1) How does students' prerequisite knowledge influence their success (earning an A, B, or C) in Calculus I?
2) Are students who take Pre-Calculus more likely to be successful (earning a grade of A, B, or C) in Calculus I than those who does not?
3) How do deficits in specific knowledge domains impact students' success (earning an A, B, or C) in Calculus I?

Method

Data Collection

Data were collected from 118 students who were enrolled in Calculus I at a land-grant university in the Mid-Atlantic Region of the United States during the Spring 2017 Semester. Forty-eight of the 118 students successfully completed the institution's Pre-Calculus Course during the Fall 2016 Semester. All 118 students took an Initial Assessment during the first week of their Calculus I course. The Initial Assessment was part of a commercial software package that uses artificial intelligence to assess the student's current course knowledge by asking him 20-30 questions open-ended questions. Students who took Pre-Calculus prior to taking Calculus I took a Final Assessment similar to the Initial Assessment in Calculus I at the conclusion of their Pre-Calculus course. Both of these assessments measured the students' level of mastery with respect to 21 knowledge domains including: Equations and Inequalities, Quadratic Equations, Rational Equations, Radical Equations, Lines, Polynomial and Rational Functions, Graphs and Transformations, Logarithmic and Exponential Functions, Trigonometric Functions and Equations. For the 48 students who completed both Pre-Calculus and Calculus I, a change score was determined. This change score was calculated by subtracting the Initial Assessment Score in Calculus I from the Final Assessment Score in Pre-Calculus. This was used to help identify which topics students did not retain between the end of the fall semester and the beginning of the spring semester. In addition to the scores from these assessments and the change score between the two assessments, the following data were also considered: standardized test scores (ACT Math, SAT Math, and Math Placement Exam) used for placement into Calculus I, prior enrollment in Pre-Calculus, prior enrollment in Calculus I (number of students repeating the course), and final grades in Calculus I.

Data Analysis

A hierarchal regression analysis was used to explore predictors of students final scores in Calculus I, which included a final sample of $n = 83$ (removing students who withdrew from the course, or who did not have standardized test scores to report). Step 1 of the analysis included students' standardized test scores (converted to ACT units, $M = 25.92$, $SD = 2.73$), prior enrollment in Pre-Calculus (about 49% of the sample), and past enrollment in the Calculus I course (about 25% of the sample). Step 2 included students' overall performance on the Initial Assessment in Calculus I ($M = 51.49$, $SD = 20.62$). The only significant predictor of Calculus I performance from these variables was students' Initial Assessment scores, which explained about

20% of the unique variance in their Calculus I scores (see Table 1). No other predictors contributed any significant or meaningful direct impact at any step in the regression model.

Table 1. Hierarchal regression analysis examining predictors of students' performance in Calculus I.

	B	*SE B*	β	*t*	*p*-value	unique R^2
STEP 1						
Standardized Test Scores	1.10	.943	.140	1.17	.246	.02
Pre-Calculus enrollment	2.32	5.99	.054	.356	.700	~.00
Repeating Calculus I	1.15	6.58	.023	.175	.892	~.00
$F(3,79) = .461$, $p = .710$, $R^2 = .02$						
STEP 2						
Standardized Test Scores	.720	.580	.092	.846	.400	~.00
Pre-Calculus enrollment	-2.19	5.47	-.051	.401	.690	~.00
Repeating Calculus I	1.59	5.90	.032	.270	.788	~.00
Initial Assessment Scores	**48.06**	**10.70**	**.462**	**4.49**	**< .001**	**.20**
$F(4,78) = 5.47$, $p < .001$, Δ, $R^2 = .21$, adj. $R^2 = .18$ Durbin-Watson = .902						

Note. Significant predictors bolded for ease of interpretation.

A follow-up analysis considered the potential for prior enrollment in Pre-Calculus to indirectly impact Calculus I performance through a direct impact on Initial Assessment scores. This analysis showed that prior Pre-Calculus enrollment was a significant direct positive predictor of Initial Assessment scores (β = .25, $p < .001$, explaining about 6% of the variance in those scores), which in turn had a significant positive impact on Calculus I Final Grades (β = .48, $p < .001$explaining about 24% of the variance); there was no direct impact of Pre-Calculus enrollment on Calculus I. As a robustness check (and to provide mean group comparisons), students who had previously enrolled in the Pre-Calculus course scored significantly higher, nearly 10% more ($n = 36$, $M = 56.34$, $SD = 20.18$) than those who had not ($n = 47$, $M = 47.76$, $SD = 20.37$), $F(1,80) = 4.42$, partial η2 = .05(and controlling for standardized placement scores, which has no impact, $p < .334$, partial η2 = .01).

Additionally, to determine a cut-off score on the Initial Assessment score for determining the odds of students passing Calculus I, a receiver operating characteristic (ROC) curve was used to determine the test's predictive utility, with area under curve $C = .751$ ($SE = .53$, 95% CI from .647 to .855, $p < .001$). A minimum score of 40% on the initial exam (sensitivity = .825, specificity = .539) was the lowest score predictive of passing Calculus I. Notably, standardized tests scores (such as ACT scores) had no predictive value in a student's odds of passing Calculus I, area under curve $C = .557$ ($SE = .072$, 95% CI from .416 to .697, $p = .389$).

Finally, as more discrete performance data was available on the Pre-Calculus students who eventually enrolled in Calculus I (n = 48), we did additional analysis on their performance on specific dimensions of the Initial Assessment, as well as their Calculus I performance. First, given that these students would have taken the initial assessment twice (once at the conclusion of Pre-Calculus in December (as the final assessment in that class) and once at the start of Calculus I the following January), we compared the change scores on these two tests between students who passed Calculus I and those who did not—higher changes scores are indicative of a decline in performance on the Assessment. Overall, students who eventually passed Calculus I forgot less information on the Initial Assessment after taking Pre-Calculus (n = 26, M = 21.86, SD = 24.18) than students who failed Calculus I (n = 22, M = 44.43, SD = 22.45), $t(46) = 3.33$, p = .002, R^2 = .19—students failing Calculus I had a nearly 20% higher discrepancy score between the first and second iteration of the initial assessment that those who passed the course.

For the 21 specific knowledge domains, three tests were conducted. First, we compared tests scores on each domain directly. There was a universal and significant drop in knowledge retention on all domains. The lowest drop was observed with Equations and Inequalities at 10% (p = .003, $R^2$10= .05); all other domains experienced significant (p's < .001) and substantial (R^2 ranging from .09 to .75) drops of at least 20% (Slopes) to as much as 67% (Polynomial Functions). Second, we compared change scores for students who passed Calculus I to those who failed the course. In nearly every case, students who passed Calculus I retained more information—the smallest significant discrepancy being Unit Circle Trigonometry with a 20% discrepancy, $t(46) = 2.02$, p = .049, R^2 = .07) and the largest being Right Triangle Trigonometry with a nearly 36% discrepancy, $t(46) = 4.14$, $p < .001$, R^2 = 26. Domains that did not observe significant differences in knowledge retention were Composite, Polynomial, and Rational Functions (three separate domains), and the domains of Graphing Trigonometric Functions, Inverse Trigonometric Functions, Trigonometric Identities, and Trigonometric Equations (four different domains)—the average knowledge loss for these domains was M = 50.37, SD = 24.38).

Finally, we used a hierarchal regression to determine which knowledge domains seemed to be most predictive of performance in Calculus I, controlling for standardized test scores. As with the earlier analysis, standardized scores had no impact on Pre-Calculus students' performance in Calculus ($R^2 \sim .00$). The collective addition of the 21 knowledge domains increased R^2 by about 16%—of which, nearly half was explained by Graphing Trigonometric Functions (R^2 = .08). Notably, significance levels were not interpreted due to the small sample size for this post-hoc analysis.

Discussion

The analysis showed that although the institution utilized standardized metrics such as ACT, SAT, and placement test scores, these metrics did not reliably predict if a student will pass his or her first calculus course. Interestingly, the Initial Assessment that was given to all students as they entered Calculus I was a better predictor of course performance than any other predictive variable utilized. The score on the Initial Assessment explained 20% of the overall variance in the final course grade in Calculus I. In other words, students who demonstrated weak prerequisite skills began the class two letter grades behind those who exhibited a strong prerequisite knowledge base.

Since the Initial Assessment was a strong predictor of student success in Calculus I, additional analysis was used to determine a cut-score capable of predicting the odds of students passing Calculus I. The analysis revealed that students who obtained a score of at least 40% on

the Initial Assessment had an 80% chance of passing Calculus I. Although 40% appears to be a low score, the overall average on the Initial Assessment was only 51.49%. The Initial Assessment has the potential to be quite valuable when assessing students' prerequisite knowledge along with students' ability to be successful in Calculus I.

Students who completed Pre-Calculus successfully were not more likely than their counterparts to be successful in Calculus I. While the analysis revealed that no connection between the two courses existed directly, it did reveal a connection indirectly. Students who passed Pre-Calculus scored higher on the Initial Assessment in Calculus I than their counterparts and subsequently students who did well on the Initial Assessment were more likely to pass Calculus I. This was a significant finding as the Initial Assessment was the only predictive variable for students' final grade in Calculus I. Thus, passing Pre-Calculus directly impacted students' prerequisite content knowledge which in in turn led to a stronger performance in Calculus I.

Furthermore, it was found that students who enrolled in Pre-Calculus had a significant decrease in content knowledge in all 21 knowledge domains between the Final Assessment in December of 2016 in Pre-Calculus and the Initial Assessment in January of 2017. Despite this loss, Pre-Calculus students still outperformed their counterparts by 10 percentage points. Furthermore, students who lost the least amount of knowledge between semesters performed significantly better in Calculus I. On average, students who failed Calculus I had a nearly 20% higher discrepancy between the two assessments than those who passed Calculus I.

Finally, in an effort to identify which prerequisite topics had the most significant impact on students' performance in Calculus I, change scores between the Final Assessment in Pre-Calculus and Initial Assessment in Calculus I were analyzed. Although, the analysis revealed that students' ability to graph trigonometric functions was most predictive of students' success in Calculus I, other interesting findings emerged. First, students retained the most prerequisite knowledge in the domain: Equations and Inequalities (10% drop). In fact, it was the only knowledge domain with no significant knowledge drop. Second, students lost on average 67% of the content knowledge related to polynomial functions. Last, students who failed Calculus I, had a significant knowledge drop in seven out of 21 knowledge domains.

Conclusion

It is troubling that some of the predictive metrics utilized in the institution's placement process did not accurately predict who would be successful in Calculus I. Many institutions solely use standardized tests to place students into mathematics courses. If these metrics are not providing an accurate snapshot of students' prerequisite knowledge, then perhaps colleges and universities should consider adjusting their placement procedure to include adaptive assessments such as the Initial Assessment discussed in this report.

Another concern raised in this report is the significant decrease in content knowledge between the end of Pre-Calculus and the beginning of Calculus I, only one month later. Researchers and instructors alike must find ways to mitigate this knowledge loss. These results should inform teaching decisions in pre-calculus courses especially, as students are clearly not retaining topics critical to their understanding of calculus. Lastly, it is important to acknowledge the small sample size used in this preliminary research project. Further research should be conducted to see if these outcomes are replicated in future semesters.

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