Informal Content and Student Note-Taking in Advanced Mathematics Classes

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This study investigates four hypotheses about calculus instruction: (i) that lectures include informal content (ways of thinking and reasoning that are not captured by the formal symbolic statements), (ii) that informal content is usually presented orally but not written on the blackboard, and (iii) that students do not record the informal content that is only stated orally but do if it is written on the blackboard, and (iv) that professors often most want students to learn the content they state informally. Via interviews, we also explored why professors chose to write on the board, or not, content. We recorded 5 calculus mathematics lectures and photographed the notes of 78 students. We found that informal content was common, although most informal content was presented in a written form. Typically students recorded formal content while not recording informal content.

Keywords: Lecture, Calculus, Student Learning

Many, if not all STEM majors will be exposed to a calculus course in their study, even if they intend to major in something other than mathematics. In the Fall semester of 2010 alone, over 300,000 students were taking a calculus course at the undergraduate level (Blair, Kirkman, & Maxwell, 2013). It's estimated by the Department of Commerce that STEM jobs will increase by 17% from 2008 to 2018, as opposed to 9.85 for non STEM fields, such an increase in the job market requires an equal increase in new STEM majors (Langdon, et al, 2011). Yet, relatively few students begin their undergraduate careers as STEM majors and very few students transfer in. In particular, Green noted that "not only do the science have the highest defection rates of any undergraduate major, they also have the lowest rates of recruitment from any other major" (1989, p. 478), meaning, there is (nationally) a net loss of students over the undergraduate program (Hilton & Lee, 1998). Often Calculus is both a stepping-stone and barrier into these majors and in order to increase the number of STEM majors, the number of students who succeed in calculus must increase as well. The failure rate and rate at which even successful Calculus 1 students who do not go on to Calculus 2 is high. The lack of success and persistence limits a student's opportunity to pursue a STEM career.

A major recent research project has explored the impact of various characteristics of calculus classes and how they influence student success (Mesa & Burn, 2011). They have described the typical curriculum as including limits and continuity, derivatives, integration, sequences and series. Faculty tended to focus their instruction on procedural fluency, with other aspects being less common. More, they found that lecture was the most common mode of instruction (82% of the time), and that typically presentations involved symbolic manipulations with some graphical representations as well. Yet, research on note-taking (and recall) as well as advanced mathematics classes suggests that further investigation of the actual instruction of calculus could help explain some of student's frustrations and difficulties with the class. Mathematics professors certainly hope that students gain more than procedural fluency from their classes—they want (and believe) that students need to gain understanding. While they may not have the same types of definitions for understanding that mathematics educators do, they certainly include being able to flexibly use procedures and be able to explain why the procedures work. This report seeks to explore the seeming difference between what professors intend students to gain and what students believe that classes focus on.

A pair of previous studies in proof-based mathematics classes motivated this exploration. A qualitative study (Lew, Fukawa-Connelly, Mejia-Ramos, & Weber, 2016) found that students might fail to learn what the professor intended from a lecture, even when the professor repeatedly emphasized the point. In particular, if the professor's main learning objective was about informal ways of thinking, the professor typically spoke it aloud without writing it on the board and the students failed to notice it during the lecture and did not record it in their notes. A subsequent quantitative study (Fukawa-Connelly, Weber, & Mejia-Ramos, 2017) of 11 lectures and 96 students showed that informal mathematics is regularly part of lectures in advanced mathematics, that such content is typically presented orally without the professor recording anything on the board, and that students do not typically record orally presented content. While the study did not test the claim that failure to record content in their notes lead to students being unlikely to learn it, or the converse, that recording lead to learning, there is substantial literature on the efficacy of note-taking as a learning tool. In particular, students normally forget content they do not record in their notes (c.f., Einstein, Morris, & Smith, 1985; Kiewra, 1987).

Following the previous studies of calculus and proof-based mathematics, we investigated three hypotheses and one question:

1. When lecturing about calculus instructors regularly discuss informal aspects of mathematics. Specifically, these lecturers represent mathematical concepts using informal representations, discuss methods that can be useful for completing related mathematical tasks, give informal explanations of concepts and processes, and give heuristics for approaching different types of problems.
2. When lecturers discuss informal aspects of mathematics, they usually make their comments orally and do not record them on the blackboard. The blackboard is reserved for formal mathematics, most prominently, worked examples, as well as definitions, theorems, and proofs.
3. When lectures make these comments orally, students usually do not record these comments in their notes.

The question that we explored is, "what rules and heuristics guide instructor's choices about recording content on the board (or not)?" We wanted to better understand instructor's decision-making in order to understand the conditions of instruction, and, possibly, what might be malleable about their practice.

Methods

Participants

We recruited participants by sending e-mails to every instructor teaching a calculus course at an institution granting doctorates in mathematics (TAs leading recitation sections were not considered instructors; for simplicity, we subsequently refer to all participants as professors, even if that is not the person's actual job). Our email asked the instructor if we could record one of their lectures and invite their students to participate via a researcher photographing their notes. A subset of professors was also asked to participate in a short post-class interview about their instruction. There was no selection process for professor interviews. Professors were not told the purpose of the study. The professors were also asked not to let the students know that we would be conducting research on their note-taking during the lecture (the professors did often want to announce that a researcher would be present and studying the class). 5 different professors participated, and the content of their course is summarized in Table 1 below.

Table 1. Overview table of instructor, class and content

Instructor	Overarching Course-content	Description of content in the lesson we recorded
M1	Calculus 2	Ratio and Root tests for series convergence
M2	Calculus 1	Fundamental Theorem of Calculus
M3	Calculus 1	Fundamental Theorem of Calculus
M4	Calculus 1	Fundamental Theorem of Calculus
M5	Calculus 2	Representation of a function as a power series

Data collection

For each participating, a member of the research team attended a class meeting in which an exam was not given. The researcher audiorecorded the lecture, while transcribing everything that the professor wrote on the blackboard in the researcher's notes. We also attempted a rough-count of the number of students in the class, although because we sat near the front of the room in order to have good-quality audio recordings, it was impossible to ensure that the counts were completely accurate. Each lecture was scheduled to be 70 minutes, although some were shorter than 70 minutes because the professor chose to return exams and then address individual questions. At the end of the lecture, the researcher made an announcement to the class inviting students to share their notes with the researcher, even if their notes were not of high quality or the student did not take notes at all. Collectively, 96 students across the 5 lectures agreed and the researcher photographed the notes that the students took for that lecture (no students indicated that they took no notes). Each class had between 20 and 30 students in attendance. For those professors who were interviewed, the lead author would meet with the professor outside of class and asked the professor to explain what the most important learning goals for the class were and why the professor chose to convey the ideas in the way that he or she did.

Analyzing the lectures

Each lecture was transcribed. The authors coded the lecture for every time one of the following were presented: definitions, propositions, proofs, examples, heuristics, pictures or graphs (these required further analysis), rules, charts (e.g., tables), conceptual examples, and contextual (real-world) examples and described the mathematical content of the lecture (often by referring to the title of the section the professor was presenting on). Any disagreements between the two researchers were resolved by discussion. We developed our categories primarily via the literature, using Fukawa-Connelly, et al's (2017) codes related to proof, and, adding codes related to the presentation of procedures given the different focus of the calculus courses. We added codes for procedures (general statement of procedures) and examples of procedures (e.g., illustrating a process or algorithm). We applied each code at the sentence-level, aggregating sets of sentences in order to capture the complete instance of a particular code. Sometimes different codes would be interspersed; for example, the professor might begin an example, give a heuristic, and then complete the example. In such a case, we would aggregate the example sentences into one unit and the heuristic sentences into another. No sentences were double-coded. In the text below, we refer to definitions, propositions, and proofs as formal mathematics because the previous literature (c.f., Davis & Hersh, 1981) has called them such. Moreover, because the calculus class is focused on procedures (Mesa & Burn, 2017), we also include any step-by-step instructions of how to complete a procedure or algorithm as formal mathematics. We refer to the other content that we coded for as informal mathematics. For space reasons, we concatenate our description of coding schemes, noting that the coding scheme for *definitions, propositions,* and *proofs* were taken directly from Fukawa-Connelly, et al (2017) and that for *informal representation* was adapted to be appropriate for calculus. Because we expected two different kinds of examples; those that illustrate definitions (or concepts) and those that illustrate processes, we differentiated between them, adopting Fukawa-Connelly et al's definition and coding of *example* as our *example of a definition*, while adding a coding structure for an *example of a process*. We used the same rule as Fukawa-Connelly, et al to code whether content was written (either on the board or in student notes).

Analysis of professor's claims

Prior to the interviews, we identified what we believed to be the primary goals that the professors had for student learning and prepared questions about them. There were two primary types of claims that the professors made; the first, described their intended learning goals for the students. For each of the learning goals that a professor stated and the related descriptions of when and how they attempted to convey that content. We aligned those with our instances of coded content. We indicated what type of content was coded for and the mode of presentation. Because we had correctly identified the professor's learning goals from our attendance of the lecture, we were able to ask what motivated their choice of presentation mode for the content. We used open-coding to develop summary codes for decisions, and then summarized them as to whether the reasoning relied on large-scale beliefs about students, or content or structure of the course, or factors specific to the intended content.

Results

Table 2 presents the number of instances of each category, the percentage of instances that were written on the blackboard or only printed orally, and the percentage of possible instances that these comments appeared in students' notes (for example, there could have been up to 383 total recorded instances of Oral Heuristics collectively in student notes, only 30 instances were recorded).

Table 2. Summary of content and recording in notes

		Instances in all lectures	**Recorded in students' notes**
Definition		Total: 2	
	Oral	0 (0% of all instances)	0% (0 out of 0)
	Written	2 (2%)	82
Rule		Total: 18	
	Oral	9 (%)	11% (20 out of 180)
	Written	9 (9%)	24% (45 out of 185)
Example of process		Total: 23	
	Oral	6 (6%)	0% (0 out of 69)

	Written	17 (17%)	80% (219 out of 276)
Informal Rep.	Total: 14		
	Oral	8 (8%)	10% (17 out of 168)
	Written	6 (6%)	14% (13 out of 89)
Proof	Total: 1		
	Oral	0 (5%)	0% (0 out of 0)
	Written	1 (95%)	64% (9 out of 14)
Graph	Total: 2		
	Oral	0 (0%)	0% (0 out of 20)
	Written	2 (2%)	78% (14 out of 18)
Heuristic	Total: 33		
	Oral	7 (7%)	8% (30 out of 383)
	Written	26 (26%)	40% (34 out of 85)
Theorem	Total: 8		
	Oral	0 (0%)	0% (0 out of 20)
	Written	8 (8%)	83% (105 out of 126)

These data largely confirm the first two hypotheses that we test in the paper. First, there were 356 instances of mathematicians presenting mathematical methods, conceptual content, modeling mathematical behaviors, and examples across the 11 lectures, or over 32 instances per lecture. This corroborates the growing body of research that mathematicians do not solely focus on formal mathematics in advanced mathematics. Second, for method, informal representation, and modeled mathematical behaviors, most of these comments were made orally and not written on the blackboard. The presentation of examples was an exception. Examples usually were written on the blackboard; we believe that this is because this allowed the mathematics professors to perform formal calculations and derivations with the examples. Third, when professors presented their comments orally, these comments rarely were recorded in students' notes. However, if they wrote their comments on the blackboard, they usually were recorded in students' notes. When the formal content was not written on the blackboard, the students do not record it. This suggests that what students record in their notes is determined primarily by the mode of presentation, rather than the type of content being presented.

How the Professor Conveyed Content and Why

We illustrate three aspects of instruction; what the professor hoped to convey to students, how it was conveyed and why the professor conveyed it that way, and whether the students recorded it. The most important idea that the professor wanted to convey was, "that the anti-derivative is the same thing as the definite integral … that these two processes are inverses of each other." We interpreted the second statement, "that these two processes are inverses of each other" as relating differentiation and anti-differentiation. Thus, we took the professor's statement to mean that she wanted students to take away that anti-differentiation and the definite integral are the same thing and that differentiation and anti-differentiation are inverse processes.

To convey that differentiation and anti-differentiation are inverse processes, the professor described them that way orally once, without recording it on the board. She also described anti-differentiation as "going backwards" to the original function four separate times, always orally. The professor also referred to "undoing" a process another 4 times, in describing differentiation. None of the students recorded any of the orally-stated claims. More, she twice drew illustrations of the relationship in general form. One such instance was after presenting the FTC Part 1 (see *Figure 1*). She added a line noting, “that is, integration is identical to anti-differentiation.”

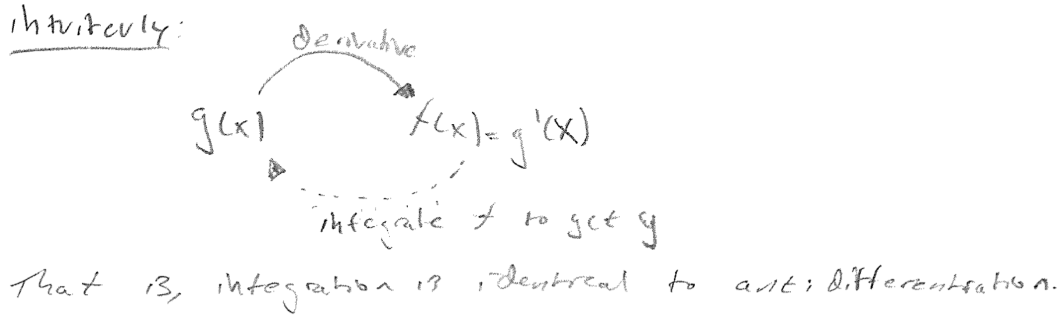

Figure 1. Diagram after the FTC Part 1

This diagram carried both of her intended claims and 7 of 9 students recorded the diagram while one student recorded the sentence without the diagram. The professor also drew a third illustration of the relationship for a specific pair of functions within the context of an example of a process. While 6 of the 9 students in the class recorded the example in their notes, none of them drew the diagrammatic representation included in the example. Thus, 7 of 9 students made any recording that captured the second of the two primary ideas that the professor attempted to convey out of 13 possible times that she expressed the idea, and 8 of 9 captured at least one instance of the first primary idea she wanted to convey. This was the only time she stated the first primary idea informally.

The professor gave a general explanation of her thinking related to writing content on the board. She said:

> The students who tend to be at the top of the class, I can take care of their questions by just saying it without writing it down. And then students who tend to get overwhelmed by the details and maybe perform a little bit lesser on different quizzes and things, the more I ... I have to be careful about what I write down because if I write down too much information that tends to overwhelm them and they're not able to separate the forest from the trees. Does that make sense?

We interpreted the professor here as making a collection of claims; first that the best students in her class would acquire the ideas even if they are only ever stated orally. Second, that the weaker students need something different than the strong students as “too much information tends to overwhelm them.” The idea of “separating the forest from the trees” we interpreted as giving priority to certain types of mathematical proficiencies. In particular, because the professor always wrote algebraic examples of procedures on the board as well as formal mathematical statements, she appears to be claiming that these examples and formal statements are the “forest” that the students must apprehend while the informal ideas are the “trees”—which she claims “might overwhelm them.” That is, we interpreted her as claiming that it is not necessary that the weaker students come to understand the more informal ideas.

Discussion

Our findings offer support for the generality and validity of the following claims:

1. When lecturing about calculus instructors regularly discuss informal aspects of mathematics. Specifically, these lecturers represent mathematical concepts using informal

representations, discuss heuristics that can be useful for completing mathematical tasks, give informal explanations of concepts and processes.

2. When lectures make these comments orally, students usually do not record these comments in their notes.

We did not support the third claim. Instead, instructors recorded 26 out of 33 heuristics on the board. A corrected version of the claim is that:

- Lecturers recorded all definitions, theorems and proofs that were included in the class on the board. They also recorded 74% of examples and 79% of heuristics. Most content recorded on the board (43 of 71 instances, 61%) consisted of examples and heuristics.

Yet even though heuristics were generally recorded on the board, they were not often included in student's notes. We note that examples were recorded in student's notes at an 80% rate. As a result, we suggest two, related, concerns; first, that we have potentially mis-cast examples as informal content. As the calculus class focuses on procedural fluency, examples of processes represent an important category of content that may deserve a different categorization. Another possibility is that the notion of formal and informal is inappropriate for a calculus class; rather, the focus should be on content that describes, illustrates, and justifies procedures. When such content is presented in 'entirely mathematical' text (e.g., text for which a standard mathematics definition/meaning exists) this should be considered the 'formal' corpus of the calculus course. This data suggests that students are differentiating between the types of content that they chose to record in their notes. More investigation is needed to explore their decision to record or not record content in their notes, and, what, if any mathematical or presentation cues they use.

Finally, we have investigated why professors chose to present content in the way that they do. Our investigation reveals that instructors are thoughtful about even this level of detail in their lectures. In particular, we showed an instructor who had considered the range of students in the class and the relative importance, for the students, of the different types of content. The professor we showed here indicated two different main ideas that she wanted the students to take-away from the class, both stated using informal language. She repeatedly stated one them during the class, including drawing three diagrams that illustrated the relationship. She only presented the second 'main' idea one time, but did so in writing. We note that none of her conversation here was specific to the content presented, instead, focusing on the nothing that some students would understand the orally presented content and some needed a written presentation that focused on the "forest," which we understood to be the most important ideas. This appears to be slightly at odds with her claim that these were the most important ideas that she wanted to convey. More investigation is warranted into the decision-making, but, we caution that attempting to force instructors to specify conditions specific to particular pieces of content might lead to post-hoc justifications when the decision was not made consciously, perhaps being habit or culture.

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