Mathematics Graduate Teaching Assistants' Growth as Teachers: An Unexamined Practice

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In recent years, providing teaching professional development for graduate teaching assistants has become more common in mathematics departments in the US. Following this trend, mathematics education researchers have begun to conduct studies on professional development programs and on graduate students as novice teachers. The purpose of this literature review is to examine the current status of research in this field and make recommendations for future research on graduate teaching assistants and professional development. In examining the literature, we utilize an existing framework for collegiate teaching practices and focus on studies that attended to growth. As a result of this literature review, we recommend that researchers begin developing models or theories for how and why graduate students grow as teachers.

Keywords: Post-secondary, professional development, graduate teaching assistants, teaching practices, growth

Introduction

In recent years there have been many efforts made to improve the quality of instruction in first-year undergraduate mathematics courses, which often have low pass rates. For many students, these are the only undergraduate math courses they experience. Also, graduate teaching assistants (GTAs) are often the primary instructors for these courses. In an effort to improve the quality of instruction in these courses, it has become increasingly common for math departments to offer teaching professional development (PD) for their GTAs. In addition, math departments have begun evaluating the impact of these PD efforts, but the results have been mixed.

At the K-12 level, researchers have found that PD interventions aimed at improving teaching tend to be more effective when they are theory-based and use an explicit model accounting for features of the PD, school contexts in which teachers work, and how the PD is intended to shape the practices of teachers participating in it. In a similar vein, it is reasonable to expect that attempts to improve teaching among GTAs would benefit if informed by models of how these novice college mathematics instructors learn to teach.

With this in mind, we reviewed the research on how mathematics GTAs learn to teach. Our intended goal was to identify and characterize the models and theories that have informed studies of GTAs' growth as teachers. While there are many aspects of GTA teaching in which growth might occur, we chose to focus explicitly on teaching practices in this paper because they directly relate to the professional work that GTAs engage in. We sought to take stock of what is known about improving GTAs' teaching, what gaps there may be, and how to move forward. To do this, we asked:

1. *What GTA teaching practices do researchers attend to?*
2. *How do researchers attend to GTAs' growth as teachers over time?*
3. *What models or theories of growth do researchers use?*
4. *What stances do researchers take regarding GTAs' growth as teachers?*

In our literature review, we searched three major research databases from 2005 to 2016: Education Resources Information Center (ERIC), PsycINFO, and Web Of Science, the RUME proceedings from 2010 to 2016, and the AMS Notices from 2005 to 2016. We chose these as the foundational sources because undergraduate-focused math education research is often published in these sources.

We found there is little empirical or theoretical research that explicitly or implicitly describes GTAs' growth. Here we define growth as the process of changing along an identifiable trajectory. For something to be considered growth, it must be true *that* something has changed and that exactly what has changed can be *identified.* We take our definition of teaching practices from our theoretical framework, which will be taken up again later. In later sections, we will review the results of our search, summarize the few results we did find, and discuss how researchers attended to GTAs' growth as teachers. After providing some background and the results of the literature search, we propose a refinement of Speer, Smith, and Horvath's (2010) framework for collegiate teaching practice. The refinement emerged from our analysis in order to address more recent research on GTAs and their teaching. Finally, the central claim that we make is that *GTAs' growth as teachers is a largely unexamined practice*.

Background

In many departments, GTAs are assigned to teach first-year undergraduate courses, including remedial math, college algebra, pre-calculus, calculus, and mathematics for pre-service K-12 teachers. Since students are stakeholders in instruction, we first highlight ideas from research published about student experiences in lower division undergraduate courses. In calculus, student experiences vary greatly (Bressoud, Mesa, Rasmussen, 2015; Burn, Mesa, & White, 2015). Students traditionally under-served by *status quo* K-12 education continue to be at a disadvantage in post-secondary settings (Bahr, 2010; Cuellar, 2012; Kena et al., 2016; Nuñez, Hurtado, & Calderón Galdeano, 2015). Difficulty passing initial college mathematics courses has a negative impact on persistence of STEM-intending students (Thompson, Castillo-Chavez, et al., 2007). While these phenomena are influenced by a variety of factors, instruction is a key element in student success. For that reason, research on how professional development can help improve the quality of instruction is an important facet of research on undergraduate mathematics education.

While providing teaching professional development for GTAs has only recently become more common, there is a wealth of research on professional development for pre- and in-service K-12 teachers (Chen & McCray, 2012; Desimone, 2009). However, measuring effective PD can be difficult and results are mixed and influenced by many external factors (Guskey & Yoon, 2009). Moreover, these studies focus primarily on professional development for teachers who have earned at least a bachelor's degree in education, stressing the importance of discipline-specific scaffolds for teacher learning. In contrast, GTAs often have a great deal of experience in doing mathematics but little to no formal training in teaching and instruction. Consequently, they are a different audience for professional development than K-12 teachers.

Frameworks

To guide our analysis of teaching practices, we drew on Speer et al.'s (2010) framework for examining teaching practices at the collegiate level. Speer et al. define *teaching practices* to be the instructional judgments, decisions, and actions employed by instructors in and outside of the classroom. Note that this is distinct from *instructional activities*, which are activities used to organize student learning and stimulate student engagement with classroom resources (e.g., using group work). Teaching practices and instructional activities are interwoven and the distinction between them is often not made clear, or even mentioned, in research publications. Both are important teaching elements examined by researchers and were present in the articles we analyzed. See Table 1 for a full description of the teaching practices identified in Speer et al.'s (2010) framework.

Table 1. Framework for collegiate teaching practice of Speer et al. (2010)

Teaching Practice	Description
Allocating time within lessons	Deciding how much time to allocate among topics and within individual class periods
Selecting and sequencing content within lessons	Choosing and sequencing the mathematical content presented in an individual class period; for example selecting problems and deciding which theorems to present
Motivating specific content	Introducing, motivating, and providing a rationale for sequencing topics, specifically to promote student engagement
Posing questions, using wait time, and reacting to student responses	Deciding what questions to ask, how long to wait for a response, and how to respond to students' answers
Representing mathematical concepts and relationships	Deciding which mathematical ideas to present in the classroom and how to present them
Evaluating and preparing for the next lesson	Reflecting and evaluating on individual class periods, and using (or not using) this information to inform the next lesson
Designing assessment problems and evaluating student work	Developing assessment problems by considering content coverage, expected difficulty level, sequencing of problems, and relevance to particular elements of content

Methodology

All articles considered for inclusion in the review were peer-reviewed and contained at least one search term from each of four categories: teaching, domain, level, and participants (see Table 2 for exact search terms). This yielded 1,889 articles. We read each abstract to determine if an article could reasonably address our research questions. We double-coded until we reached consensus on the criteria for inclusion, with an inter-rater reliability of 97%. Our intent was to focus on mathematics GTAs, but we also decided to code for STEM fields in general that way we could keep track of articles that might be relevant if we decided to extend our literature review. After discussion, we agreed to seven articles that were relevant. To capture other relevant research on this topic, we then read the abstracts of the RUME proceedings for the years 2010 through 2016 (we restricted our time frame due to infrequency of relevant articles), again coding for inclusion, and found 17 relevant articles. Finally, we searched the AMS Notices using an advanced Google Scholar search for the years 2005 through 2016 (using the same search terms in Table 2, excluding the "domain" category). This yielded an additional two articles, which gave us a total of 26 articles relevant to our research questions.

Each article was then open coded for teaching practices, attention to growth over time, use of an explicit or implicit model or theory of growth, and stances on growth. Six articles were double-coded, at which point the team discussed preliminary findings and how to adjust the coding procedure. After consensus was reached, the rest of the articles were coded. Using the Speer et al. (2010) framework, we conducted a second cycle of coding that categorized our open codes to fit into the given framework.

Table 2. Search Terms

Category	Terms
Teaching	teach*, instruct*, "professional development", PD, training, TD
Domain	STEM, math*, science, physics, chemistry, biology, statistics, engineering
Level	undergrad*, collegiate, tertiary, college
Participants	"graduate student", GST, GSI, GI, novice*, "future faculty", beginning, GTA, TA

Findings

While all 26 articles focused on GTA teaching, the participants involved in the studies still varied. The majority included graduate student participants who were currently teaching or who would be teaching in upcoming semesters. In addition, one mixed-methods study included "novice college math instructors," who were defined as instructors with less than seven years of teaching experience and included non-graduate students. A few studies explicitly stated that the graduate students were in their first or second year as instructors, but many studies did not specify how long the GTAs had been teaching. In addition, the researchers gathered data in a variety of ways, including interviews, classroom observations, observations of GTA PD classes, and surveys. Details of our findings are given below and summarized in Tables 3 and 4.

Teaching Practices and GTA Growth

Of the 26 articles, we were able to utilize Speer et al.'s (2010) framework to categorize the teaching practices studied in 14 of the articles. Each of the specified teaching practices was addressed in at least one article, suggesting that their framework is consistent with the current research efforts surrounding GTA PD. However, we suggest two refinements to the framework based on our review. First, we suggest adding “anticipating student thinking” to the framework. We found eight articles out of the 26 that examined this teaching practice. This teaching practice is implicitly part of “selecting and sequencing content within lessons” and “motivating specific content,” but we suggest that it be explicitly stated. Also, anticipating student thinking plays a central role in facilitating productive mathematical discussions (Stein, Engle, Smith, & Hughes, 2008) and is something that novice teachers often struggle with. Second, we suggest that the practice of “representing mathematical concepts and relationships” be refined to include verbal descriptions. One of the articles we coded with this teaching practice specifically focused on “speaking with meaning” (Musgrave & Carlson, 2016). Although Speer et al.’s (2010) description does not explicitly exclude verbal representations, it emphasizes *what* is shared rather than *how* it is shared. This refinement also reflects recent work on teachers’ coherent mathematical meanings (Thompson, Carlson, & Silverman, 2007).

Table 3. Number of articles addressing the teaching practices in adapted Speer et al. (2010) framework

Teaching Practice (* Adapted)	Number of Articles
Anticipating student thinking*	8
Allocating time within lessons	2
Selecting and sequencing content within lessons	4
Motivating specific content	1
Posing questions, using wait time, and reacting to student responses	5
Representing mathematical concepts and relationships including how concepts are described in words*	3
Evaluating and preparing for the next lesson	4
Designing assessment problems and evaluating student work	3
Does not examine any specific teaching practice	12

After coding for teaching practices, we then coded to identify articles that focused on how GTAs grow over time, provided models or theories of growth, and took stances regarding growth. These are discussed in the next few subsections.

Table 4. Number of articles that attended to growth

	Number of Articles
Focused on growth over time	11
Used models or theories of growth	3
Took a stance regarding growth	13
Did not attend to growth	9

Growth over time. We found 11 articles out of the 26 that addressed growth in GTAs' teaching practice over time. For example, Raychaudhuri and Hsu (2012) conducted a longitudinal study to explore how beliefs and pedagogical approaches evolve over the span of a year. Based on their preliminary analysis, Raychaudhuri and Hsu present stages of GTA beliefs regarding students moving from teacher-centered knowledge to student-centered knowledge. Musgrave and Carlson (2016) studied GTAs' descriptions of average rate of change before and after one semester of PD. They found that graduate students who participated in the PD described average rate of change more conceptually than their counterparts, but still struggled to verbalize the meaning of average rate of change. In another study, Duncan (2016) used a teaching experiment methodology to examine how one GTA's mathematical meanings and instructional planning decisions changed while creating a hypothetical learning trajectory (HLT) on angles, angle measure, and the radius as a unit of measurement. Duncan's results suggest that having GTAs work through tasks in a researcher generated HLT can cause changes in what GTAs identify as being important to teach.

Models or theories of growth. We found three articles out of the 26 that employed a specific model or theory of growth. Beisiegel (2011) utilized Lave and Wenger's (1991) theory of legitimate peripheral participation to describe the process by which GTAs gain knowledge and understanding about the practice of teaching post-secondary mathematics. In particular, Beisiegel studied how "the attention to legitimate peripheral participation in a mathematics department [might] prevent graduate students from adopting alternate modes of teaching" (p. 20). In a study examining the teaching philosophies of GTAs, Nepal (2014, 2015) used a cognitive apprenticeship model, which stems from situated cognition and Vygotsky's (1978) sociocultural theory. Nepal applied this model to explain how and why GTAs' teaching philosophies change as they "accumulate knowledge about teaching and learning gradually through the interaction with other people and their own teaching experiences" (2015, p. 5). Some papers mentioned a model or theory of growth but did not incorporate it as a key aspect of the study. For example, Yee, Rogers, and Sharghi (2016) claimed that reflecting, revising, and collaborating help GTAs "actively engage with teaching theories" (p. 1458) and "develop a community of practice" (p. 1459). However, an explicit model or theory of growth was not referenced or used.

Stances regarding growth. Thirteen articles took stances on teaching quality that referenced knowledge for teaching, cognitive demand, pedagogies, and student achievement. Firouzian (2014), Speer and Firouzian (2014), and Firouzian and Speer (2015) cited the large body of work on mathematical knowledge for teaching (MKT) as evidence for why it is important for GTAs to develop MKT. Roach, Noblet, Roberson, Tsay, and Hauk (2010) cited Smith and Stein's (1998) work on cognitive demand to describe the (limited) variety in cognitive demand in the questions TAs asked. Finally, Yee et al. (2016) drew upon *Principles to Actions* (National Council of Teachers of Mathematics, 2014) as evidence for why specific teaching practices are associated with effective teaching.

Discussion

Based on our findings, we argue that GTAs' growth as teachers is a largely unexamined practice. We assume that the purpose of most, if not all, GTA PD programs is to foster growth as teachers, but were surprised to find that only a small percentage of the research on GTA PD addresses growth. Of the 26 articles we reviewed, nine of them did not focus on growth at all. Only three of them provided explicit models or theories of growth, but none of these focused on teaching practices. Current research gives a general sense of what GTAs may be doing in the classroom, but how their teaching practices change as they gain experience and participate in PD is understudied. It should also be noted that the four studies that address growth demonstrate that growth is possible, so further research is warranted. As a result, we suggest that the field would be greatly enhanced by additional longitudinal studies exploring how GTAs grow as teachers.

In particular, it would be beneficial to begin developing an accepted definition of GTA growth. We argue that part of this process is being clearer on our stances as a research field on teaching quality and how these relate to models or theories of growth in teaching. Moreover, most studies assume a common understanding of the term "teaching practice" rather than attempting to define it. This leads to a lack of clarity about which aspects of teaching are analyzed in research as well as the researchers' stances on teaching quality. It is striking that there were no articles that were explicit both about their stance on teaching quality *and* a model or theory. We call for future research to take an explicit stance on teaching quality and how teaching quality is related to models and theories of GTA growth.

Finally, our findings suggest a need for explicit models or theories of growth in teaching linked to stances on teaching quality. There has been some progress on this (e.g., development of MKT by Thompson, Carlson, & Silverman, 2007), but more development is needed. We call for the research community to begin developing the models of growth that will allow research on GTA PD to grow into a rich body of literature such as exists in the research literature on K-12 teacher PD.

Future Directions

Although we did not find many articles on mathematics GTAs' growth as teachers to include in our literature review, there was a larger body of work on STEM GTAs in general. Since there are many similarities between mathematics and other STEM disciplines, it would be fruitful to see how researchers have attended to STEM GTAs' growth as teachers. In particular, if studies on STEM GTAs attend to teaching practices or define GTA growth or use explicit models or theories of growth, then we could build upon this in the RUME community. However, it is also important that we attend to the ways in which the teaching and learning of mathematics differs from other STEM disciplines.

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