Our Mathematical Ideas are Part of Our Identity

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This paper explores the notion that our mathematical ideas are part of our identity. This notion, which was a significant result of a qualitative dissertation study, will be explored in depth through an examination of data in connection with the educational research related to identity. The story of Binary, a first generation college student completing a transition-to-proof course in his final semester of college, provides the context in which we explore this complex notion.

Keywords: Identity, Nature of Mathematics, Mathematician's Practice, Transition-to-Proof

One day in Foundations of Higher Mathematics, a student called Binary[1] says, "I think no one really understands my idea. I think people are just so focused on $2k + 1$ that they are not looking at the bigger picture here." This comment came on the second day of classroom discussion, the bulk of which consisted of students debating the merits of Binary's suggestions regarding a proof of the claim that "the sum of any two odd numbers is even." Notice the language Binary uses: "I think no one really understands *my* idea." Binary identifies the idea as his own. In this paper, we explore the notion that the idea is part of his identity.

Theoretical Frameworks On Identity

Identity is a powerful construct used to examine and understand the interactions between learners and mathematical content and practices (e.g. Boaler & Greeno, 2000; Langer-Osuna 2015). Gee (2000) defined identity as "being recognized as a certain 'kind of person' in a given context" (p. 99). Identity is related to how an individual views her or himself, and how others view an individual. Drawing on work done by Boaler and Greeno (2000) and Martin (2000), Cobb and Hodge (2010) distinguished between core identity, normative identity, and personal identity. Observing student behavior, the authors noted that core identity refers to how students "viewed themselves and who they wanted to become" (p. 187). Normative identity does not refer to how students view themselves or one another as individuals, but rather, the focus is on how students become a "mathematical person" or "doer of mathematics" (p. 187). An important component or normative identity is mathematical competence (Cobb, Gresalfi, and Hodge 2009). What students perceive as necessary to become a doer of mathematics may be in conflict with who they are and want to become (their core identity). Personal identity is developed as students "participate in (or resist) the activities of particular groups and communities, including those of the mathematics classroom" (Cobb & Hodge, 2010, p. 187).

To understand the overlapping but separate notions of personal identity and normative identity, we can use an additional framework called figured worlds. A figured world is "a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland, Lachicotte, Skinner, & Caine, 1998, p. 52). In mathematics education, this

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[1] Students in the course completed an assignment in which they chose a number type that best represented themselves. For this assignment, the student we call Binary, a computer science major, wrote this description of why he chose Binary as his number type: "Very simple once you get to know me. But can be very confusing if you don't."

framework has been used to understand how students navigate one figured world of the mathematics classroom and the figured worlds of their other identities, such as being an athlete, under-represented minority, etc. Boaler and Greeno (2000) described an inquiry-based mathematics classroom as a different figured world than a traditional classroom, the latter of which may be much more misaligned with promoting the development of students' normative identities. Nasir, Hand, and Taylor (2008) examined the lack of intersection between students' figured worlds of the mathematics classroom and the basketball court. Langer-Osuna (2015) extended this notion to examine the challenges of under-represented minority students in mathematics, as the figured world of a mathematics classroom overlaps much more with that of dominant cultures versus that of non-dominant cultures. In all of these cases, the figured world of mathematics is misaligned with the various figured worlds of students outside of the mathematics classroom, thus leading to unsuccessful learning.

Methodology and Analysis

The theoretical frameworks of identity will be used to shed light on a finding from a qualitative dissertation study. The importance of identity emerged from the data, and the construct was not used to frame the original study. The original purpose of the study was to create a humanistic framework for the nature of pure mathematics. Two questions guided the study: What is the nature of pure mathematics? And what should undergraduate students in a transition-to-proof course understand about the nature of pure mathematics? In seeking to answer these questions, the researcher incorporated the methodological framework of heuristic inquiry (Moustakas, 1990). This form of inquiry has roots in humanistic psychology, and it leverages the researcher as instrument. Douglass and Moustakas (1985) wrote, "It is the focus on the human person in experience and that person's reflective search, awareness, and discovery that constitutes the essential core of heuristic investigation" (p. 42).

To understand the nature of mathematics and consider what students should understand about the nature of mathematics, the researcher (first author) collaborated with a graph theorist and co-taught an undergraduate introduction-to-proof course. In regards to this discussion on identity, to the most relevant data collected come from the transition-to-proof course and the researcher's reflective journal. The data gathered from the course included audio recordings of discussions the researcher had with a co-instructor, audio of whole-class discussions, student homework, classwork, exit tickets, and all other class materials. Twenty-three students from the course agreed to participate in the study.

The researcher employed the processes of heuristic inquiry and other qualitative analysis techniques to arrive at several possible characteristics of the nature of mathematics that may be valuable for students to know and understand. The end of heuristic inquiry is what Moustakas (1990) called the creative synthesis:

> Finally, the heuristic researcher develops a creative synthesis, an original integration of the material that reflects the researcher's intuition, imagination, and personal knowledge of meanings and essences of the experience. The creative synthesis may take the form of a lyric poem, a song, a narrative description, a story, or a metaphoric tale. In this way the experience as a whole is presented, and, unlike most research studies, the individual persons remain intact. (p. 51)

The creative synthesis for this study includes the IDEA Framework for the Nature of Pure Mathematics and ten stories that illustrate key characteristics of the nature of mathematics.

Results

A main result of this study is the IDEA Framework for the Nature of Pure Mathematics and ten corresponding stories that illuminate the characteristics of the framework. The IDEA framework consists of four foundational characteristics: Our mathematical ideas and practices are part of our ***I***dentity; mathematical ideas and knowledge are ***D***ynamic and forever refined; mathematical inquiry is an emotional ***E***xploration of ideas; and mathematical ideas and knowledge are socially vetted through ***A***rgumentation. In this paper we present a story, *If Nobody Agrees With You*, highlighting the notion that our mathematical ideas are part of our identity.

If Nobody Agrees With You

One day in Foundations of Higher Mathematics some small groups are working to create group proofs for different theorems and presenting their proofs of those theorems to the class. The Yellow Team create and present the poster shown in Figure 1.

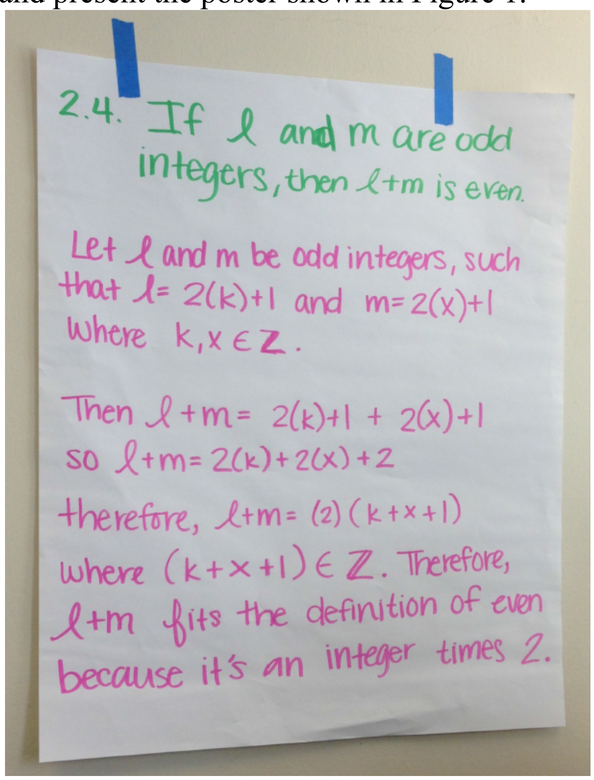

Figure 1. Yellow Team's Poster

In general, the class as a whole likes the group's argument and does not have many questions. Infinitely Repeating Decimal's comments are representative of the class: "It is pretty straightforward, it doesn't get more concise than that." However, Binary does have an important question. He asks, "If instead of having the x, if you did put k, would it make the argument less strong?" We see Binary is referring to the original delineation of the odd numbers l and m as $l = 2k + 1$ and $m = 2x + 1$ where k and x are integers. Would the argument be less strong if we simply defined $l = 2k + 1$ and $m = 2k + 1$? One of the presenters, Odd, responds "Oh, so like k here and k here?" (pointing to the k and x in the original delineations). "Yeah," Binary replies. Odd explains, "Yeah, because basically what you did was, you didn't say that it's any two odd integers, you said the same integers. So you basically just said $l + l$." Binary tries to explain that he thinks k is sufficient, by appealing to the definition of odd number; but he is interrupted as another student, Whole, interjects: "If you want to stick with k you could be like k subscript 1 and k subscript 2." The dialogue continues:

Binary: I just feel like if they both were k it would still make a strong argument.

Odd: If l equals $2k+1$ and m equals $2k+1$, as k increases the other k is increasing so l and m would always be equal.

Binary: But it's still going to be odd. That's what I am saying.

Odd: It's an odd number but it's the same odd number. So it doesn't cover any combination of two odd numbers. So basically you would only be able to say like 3+3 or 6 or 7+7. But with this, because these are different, we could say let l be equal to 3 and m be equal to 7.

Infinitely Repeating Decimal: If you really wanted them to be k's you could use subscripts like k_1 and k_2.

Odd: Yeah.

Infinitely Repeating Decimal: You just need to show that they are not the same integer.

Binary: Yeah. I am just trying to figure out if I just left that as k, and he had that. Would I get less points?

At this point several members of the class laugh and some chime in that indeed the argument would receive less points. Dr. Amicable and Surreal[2] (the co-instructors) bring the class's attention back to the mathematics. Dr. Amicable asks for a show of hands to see how many students understand Odd's explanation, and asks Infinity to explain in her own words.

Infinity: Yeah. If you made both the variables, the k and the x the same. If you did make them both k then they would be the same number. So you would get the same outcome of l and m. And so your numbers wouldn't vary. So, like he was talking about how the answer would be consistently the same throughout. … If you make both of those k's in the equation where x is. Then you are going to get the same exact number.

Dr. Amicable: Hmm. So is it right, if I were to summarize what you are saying Infinity; would it be right to say that essentially we are changing this condition to if l and m are the same odd integers? Then $l + m$…

Infinity: Yea. Because you would be plugging in…. You would be using the same variables.

Dr. Amicable: Okay. Binary what do you think?

At this point the instructors, Dr. Amicable and Surreal, expected Binary to jump on board with the class consensus. But he stands firm.

Binary: I am just saying. In general if you were to use examples, then yeah. But just in general I feel like no matter what you put in, by the definition of an odd number, it's going to come back to the exact same thing.

Whole (interrupting): You have to have some kind of variance in it when you are adding two.

Binary: But not in the way he did, maybe in another proof, yes. But…

Whole: It's the same principle though.

Composite: Okay. But if you do use k in both l and m… If you say $2k + 1$ is equal to l and $2k + 1$ is equal to m. You are going to go to the next step where it adds, and you will have $2k + 1$ plus $2k + 1$. And then instead of being $2k + 2x$ it's going to be $2k + 2k$ which equals $4k$. And you are going to have $4k + 2$ which is not going to be something that looks like the definition of an even number.

Binary: [emotional] Yes you will. You pull out 2.

[2] Surreal is the researcher and first author.

Odd: It would still be even.

Binary: It would still be even.

Binary stated “It would still be even” with conviction. Then several people in the class begin to discuss. Of course, the result $4k + 2 = 2(2k + 1)$ is in the form of an even number and several members of the class agree. But they also stress that by using $l = 2k + 1$ and $m = 2k + 1$, the result is “more narrow” because “you’ve got to cover the spectrum.” Subsequently the students and instructors begin to consider examples that may serve to change Binary’s mind. Binary still does not. He says, “I understand what ya’ll are saying. It makes perfect sense to me what ya’ll are saying. Don’t get me wrong. What ya’ll are saying is 100% correct. But I’m saying that this way still satisfies everything to me for an odd number.” Time runs out for the class and the students are required to write one big idea for the day and one question they have for their exit tickets. Binary’s big idea was “If no one agrees with you, you’re wrong.”

After class, Surreal and Dr. Amicable were very happy about the discussion students had regarding Binary’s question. As students left the room, Even asked if she could stop by Dr. Amicable’s office. Later on, Dr. Amicable and Surreal had this e-mail exchange:

Dr. Amicable: … We will need to chat about Even’s concern. In brief, she felt that Binary was under attack today in class and it made her feel very uncomfortable. She recognizes that Binary may not have felt under attack, but she felt that for him. I appreciated her coming to share her feelings. Perhaps we can address the best ways to critique and also remain professional in our classroom setting (at the beginning of next class), but I'd like to chat with you about your thoughts.

Surreal: Okay we can chat. I did not think Binary was under attack. Only his idea was under attack! But I also recognize that students have never experienced mathematical argumentation and so it may be hard for some of the students to deal with it. Although I do not have immediate thoughts about what we would tell students, I think engaging in a dialogue with students may be productive.

Dr. Amicable: I agree. I did not see it as an attack on Binary either, but it wouldn't hurt to talk with the students about critiquing an idea rather than a person.

Notice that Surreal’s initial thought was that Binary was not under attack. “Only his idea was under attack!” But are our ideas not also our selves? When we criticize another person’s ideas, are we not criticizing the person as well?

The next day in class Surreal began by asking students to talk about their big ideas and questions from the previous class. Many of the students said they had conversations outside of class about Binary’s idea. Others said they were trying to think of new ways to convince Binary of their point of view. Infinitely Repeating Decimal’s big idea was “I’m really struggling to figure out a different way to represent that $l = 2k + 1$ and $m = 2k + 1$ only satisfies $l = m$. There has got to be a way though!” He expanded upon this idea during class.

> Infinitely Repeating Decimal: You know, to me, the discussion we had on Tuesday, it was very clear that set of restrictions only satisfies $l = m$, but to someone else if it is not clear—like they think that can be interpreted differently. I think you have an obligation to make sure that everyone is on the same page. Whether one person or another changes their position, I think it is very important that everybody agrees on a given definition or theorem, etcetera. But I couldn't figure out any other way to represent that, to possibly represent it in another way that might make it more clear.

The classroom dialogue continues:

Real: Yeah I think clearly we spent a lot of time in class on it the other day, and it's an important point. Generality, or proving that something is universally true, is more valuable than obviously proving specific cases. I think it is important that we help Binary get to that point. But I just wasn't sure exactly how to persuade him that we needed to differentiate the k's in that specific example to ensure that we have a general case that our proof covers all the bases.

Surreal: So it seems like Infinitely Repeating Decimal and Real are thinking, "We've got this idea and we want to convince Binary of it." And Binary felt, I think; how did you feel Binary? [Recall Binary's big idea: "If no one agrees with you, you're wrong."]

Binary: The big idea is like, yeah I agree with ya'll 100%. But ya'll are not listening to me when I say that. With Infinitely Repeating Decimal, what he's trying to say, I completely agree. But I'm not looking at it as just "k." When I see that definition of odd—For me, I feel like two times any number in the world plus one would be odd. So I'm feeling like, when I see a definition I am taking that definition plussing that definition to get this new definition. So when I see that I just take k and I make it like $z$or like $2z + 1$ equals odd, and that's how I'm seeing it. [3] So even though the k only satisfies $l = m$. To me, I feel like just the definition alone, no matter the variable, is enough to prove the theorem.

The dialogue continues. Dr. Amicable and Surreal discuss the nature of mathematical argumentation for mathematicians, and the importance of criticizing ideas rather than people.

Dr. Amicable: I know when I have been to math conferences, and mathematicians are presenting their work, sometimes it gets fairly heated in the room, right? … I have seen things that were similar to what we saw in class on Tuesday where it's back and forth like "I'm not sure I understand why you can say that because I see it this way." … Actually there is a lot of emotion involved in mathematics. … we need to keep thinking about one another's ideas, and really try to understand the other idea. The more we can understand someone else's idea, the deeper our own understanding will become. Alright?

Binary: I think no one really understands my idea. I think people are just so focused on 2k+1 that they are not looking at the bigger picture here.

Whole (interrupting): Well earlier before class today, Complex brought it up and we were talking about this ... And he brought it up that what you were saying break it down by the definition of variable—what the actual definition of variable is …

Over the course of two days, this was at least the fourth time that Whole interrupted Binary. Complex shares an idea, and Surreal, wanting to move on to other course content, asks Binary to explain his idea one last time.

> Binary: For me when I see k, I pretty much in my head, I put an odd number times 2, and I put the z which means any real integer, plus 1. So I know if I see $2z + 1$ that represents any possible odd number. So I put $(2z + 1) + (2z + 1)$ equals an even number. That's what's in my head. So when I see z I know I can put in any number imaginable and get an even number. And for this example I feel like that was enough proof. You didn't need an example. You didn't need any other variables, and that is the idea that I had.

A couple weeks later one of the instructors, Surreal, in his research journal, wrote:

> Emotions today. I remember Binary hasn't talked the last two class periods. … We said we were not criticizing Binary, just his ideas. But Even took it as criticisms of him. Our ideas are our selves. Mathematics involves criticism of people's ideas and argumentation.

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[3] It is unclear if Binary is referring to z as an integer or ℤ as the set of integers.

> Students are not ready for a class in which their ideas (and hence their selves) are criticized against the "objective" standard. … I guess what typically happens is that students are told the "right" ideas. Take away the creative act. … I have a vision of pre-service teachers afraid to speak in class. Mistakes are okay! Push our thinking forward as a community. Courage and humility.

Discussion

For many students, the mathematics classroom and the learning of mathematics represent different and non-overlapping figured worlds with their other identities (Boaler 2000, Langer-Osuna 2015). In many of these cases, students do not see themselves as mathematical persons and struggle to understand the relevance of the mathematical ideas to other parts of their lives (Nasir 2008). Such failure in mathematical learning stems from the students' divergent normative and personal identities. Contrary to the examples in existing literature, we identify and examine in this paper a unique case study, where the student's (Binary) figured worlds of mathematics and his other identities seem inseparable (at least in the eyes of the student Even). The data revealed that Binary did not speak in whole class discussions for the two class periods following those described here. The convergence of Binary's normative and personal identities, instead of leading to successful mathematical learning, is also the key in hindering Binary's progress in his progression toward being a mathematician in the transition-to-proof course.

Binary emphasized that "no one really understands *my* idea" and "if no one agrees with you, *you're* wrong." When his ideas are challenged, a classmate Even also points out that Binary himself is being attacked. In these students' minds, the notion of mathematical idea, and thus an individual's normative identity, are one and the same as the individual's personal identity. There is no separation. However, for a professional mathematician (or mathematics educator as in the case of Dr. Amicable and Surreal), the separation is clear. Surreal points out that "[he] did not think Binary was under attack" and that "only his idea was under attack," and Dr. Amicable agrees. When discussing this separation with students, Dr. Amicable brings up her experiences at professional conferences where "mathematicians are presenting their work, [and] sometimes it gets fairly heated in the room." Dr. Amicable also explains to students that "there is a lot of emotion involved in mathematics" and that "[students] need to keep thinking about one another's ideas, and really try to understand the other idea." The focus of Dr. Amicable's point is clearly on the ideas rather than on individuals.

Reflections and Questions

What can undergraduate instructors do so that students come to see mathematical ideas as part of their own identity in a productive manner? In terms of identity, we believe we can learn from a study of school mathematics done by Magdelene Lampert 27 years ago. Drawing from the work of Póyla and Lakatos, Lampert (1990) argued that courage and humility are mathematical virtues that students must develop if they are to participate in authentic mathematical discourse. Students must have courage to put forth their own ideas for examination by the classroom community. They must also have humility, and understand that their ideas may need to be revised in light of this public examination. More research is needed to understand how these virtues can be cultivated at the undergraduate level.

Boaler (2015) claimed that, "children are wrongly led to believe that all of the ideas already have been had and their job is simply to receive them" (p. 172). Do we teach undergraduates the same? Or do we support students in being creators of mathematical ideas? If the instructor tells students what the right ideas are, then it is up for the students to conform. Student identities may be in conflict when becoming mathematically competent means submitting to authority in a particular classroom (Cobb & Hodge, 2010). If students are to see mathematical ideas as part of their own identity, then the mathematics classroom needs to be a site of idea-generation rather than a site of indoctrination into what is "right." However, what is normative, and what it means to be mathematically competent, in inquiry-based classrooms is significantly different than what students experience in traditional courses. Students need more opportunities for idea generation in lower-level undergraduate mathematics classrooms so that by the time they reach their upper-level courses, they have more confidence in their own ideas, and hence their selves.

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