Epistemological Beliefs About Mathematics and Curriculum Goals in the Cognitive Domain: a Case Study of Preservice Secondary Mathematics Teachers

Tamara Lefcourt Ruby
Efrata College of Education

Linor Hadar
Beit Berl College

Beliefs have long been recognized as a "hidden variable" in mathematics education. Epistemological beliefs are an inherent, although often implicit, component of curriculum goals in the cognitive domain. Connections between acquiring and accessing higher order cognitive strategies and epistemological beliefs are gradually becoming better understood. Israeli guidelines for mathematics teacher preparation emphasize of view of mathematics as a complex body of knowledge and knowing mathematics as a dynamic process. We present a case study of Israeli preservice secondary mathematics teachers' epistemological beliefs about mathematics, assessed via concept maps at the beginning and end of undergraduate studies. A mixed-methods approach was used to analyze maps. Results suggest that students' beliefs shifted to align with Israeli goals. Implications for STEM curriculum design are discussed.

Keywords: Preservice Secondary Mathematics Teachers, Epistemological Beliefs, Concept Maps, Curriculum Design

Introduction:

Complex interactions between cognition, metacognition and epistemological beliefs impact selection and application of cognitive strategies (Hofer & Sinatra, 2010). Epistemological beliefs, therefore, play an inherent role in education policy; they are implicit, often hidden (Leder, Pehkonen, & Torner, 2002), components of curriculum objectives in the cognitive domain.

We present a case study assessing alignment of students' beliefs with goals for the preparation of secondary mathematics teachers in Israel. During the three-year course of studies, beliefs that mathematical knowledge is simple and disconnected became less central; beliefs that mathematics is absolute knowledge of procedures evolved to a nuanced system of beliefs about a mathematics of procedures, concepts and processes.

Theoretical Background:

Learning objectives in undergraduate STEM education include retention of knowledge over time, the application of knowledge to solve unfamiliar problems, and commitment to lifelong learning (Fairweather, 2008) which are associated to higher order cognitive processes (Fink, 2003). Acquisition of higher order thinking skills is a necessary, although not sufficient, component of solving complex problems and responding innovatively to changing circumstances (Binkley et al., 2012). Students must also internalize a deep approach to learning (Bromme, Pieschl, & Stahl, 2010) as an active, self-driven process of deep thought and construction of knowledge and understanding (Marton & Saljo, 1976).

Students' epistemological beliefs about the nature of knowledge, knowing and learning (Hofer & Pintrich, 1997; M. Schommer, 1990) are connected to their adoption of surface or deep learning approaches (Vermunt, Van Rossum, & Hamer, 2010) Beliefs that knowledge is absolute--facts with or without accompanying understanding--correspond to a surface, outcome-oriented approach to learning. Beliefs that knowledge is dynamically constructed and context-dependent aligns with a deep-learning, process-oriented approach (Vermunt et al., 2010).

Epistemological beliefs and metacognition are connected (Schraw & Moshman, 1995); the ability to access the cognitive tools needed to engage in critical thinking and problem solving is mediated through beliefs about knowledge and knowing (Hofer & Sinatra, 2010; Kuhn, 1991). Holding non-availing beliefs impedes acquisition of higher cognitive skills (Schraw, Crippen, & Hartley, 2006) and limits the range of cognitive strategies that are accessed (Hofer, 2004; Louca, Elby, Hammer, & Kagey, 2004). Since beliefs impact the acquisition and application of higher order cognitive skills (Hofer & Sinatra, 2010; Schraw et al., 2006), they are an inherent, although often hidden (Leder, Pehkonen, & Torner, 2002) component of curriculum objectives.

In mathematics, higher order cognitive skills include discovery, making connections and building understanding, which characterize (deep) *conceptual knowledge* (Hiebert & Lefevre, 1986; Star, 2005). Mathematics-related beliefs are strongly related to the cognitive processes of mathematics (McLeod & McLeod, 2002). For example students who believe that knowledge is simple, isolated facts (M. Schommer, 1990) are less likely to use higher level cognitive strategies and have lower levels of achievement (Cano, 2005); a belief that knowledge is simple affects self-regulation in learning (Muis, 2007) and has been shown to negatively impact student achievement in remedial mathematics courses in university (Briley, Thompson, & Iran-Nejad, 2009). Fostering dynamic beliefs about mathematics as process of discovery (Ernest, 1991; Grigutsch & Törner, 1998) is an inherent component of cognitive curriculum goals; neglecting epistemological beliefs as a component of mathematics education goals is a potential source of inequity of access to career and educational opportunities for which mathematics is a gateway (Leder, Pehkonen, & Törner, 2002).

Epistemological beliefs of mathematics education students have been widely studied (e.g., Ball, 1990; Schmidt et al., 2008) because mathematics teachers' beliefs impact their teaching practice (Beswick, 2005; Blömeke & Delaney, 2012; Thompson, 1992), student learning (Staub & Stern, 2002) and student achievement (Tatto et al., 2008). While some countries, for example the United States, explicitly include belief-related goals in guidelines for mathematics teacher preparation (CBMS, 2012), the guidelines of other countries, *e.g.,* Israel do not contain explicit belief-goals (Gutfreund & Rosenberg, 2012).

Despite the absence of explicit belief-related goals, the importance of addressing belief-development in programs for mathematics teacher preparation has long been recognized (Brownlee, Purdie, & Boulton-Lewis, 2001; Wilkins, 2008). For example, a cross-country study of 23,000 pre-service mathematics teachers from six countries examined the structure of mathematics teacher training programs and the mathematics-related beliefs of the programs' students at the end of their studies. Following Grigutsch (1998) beliefs were characterized as static/absolute and/or dynamic. End-of-program beliefs varied by country and by program across all six countries graduates held dynamic beliefs, but the extent to which graduates also held absolute beliefs varied by country (Schmidt et al., 2008). In addition, preliminary research indicates that the number and type of mathematics and mathematics education courses in mathematics teacher preparation programs impact beliefs (Blömeke, Buchholtz, Suhl, & Kaiser, 2014) with more opportunities to learn mathematics education courses leading to more dynamic beliefs. The connection between programs' explicit or implicit belief goals and graduates' beliefs was not examined, raising the question of the role implicit or explicit expectations play in belief development.

Israeli guidelines for preparation of secondary mathematics teachers (Gutfreund & Rosenberg, 2012), referred to as "Gutfreund guidelines" in what follows, do not specifically address beliefs, however they include content goals characterizing mathematics as a process of knowledge development, which are aligned to dynamic beliefs about mathematics (Grigutsch & Törner, 1998). Pedagogical goals address higher order cognitive skills; also aligned with dynamic beliefs, and, additionally, a belief that mathematical knowledge is complex, rather than simple (M. Schommer, 1990). Developing epistemological beliefs of mathematics as a complex body of knowledge and a dynamic process of inquiry is, therefore, an implicit goal of the Gutfreund guidelines.

Research Goal

The previously unexplored connection between belief expectations and students' end-of-program beliefs provided a rationale for studying the epistemological beliefs about mathematics held by Israeli mathematics education students at the beginning and end of the program of studies and analyzing these beliefs in terms of their alignment to the Gutfreund guidelines (2012).

Methodology

This case-study was conducted within the framework of a regulated B.Ed. program mathematics education at an Israeli college of education; graduates are certified to teach Israeli secondary mathematics.

Sample

Twenty-five students began the mathematics education program in the 2014-15 academic year. Data was collected before students began the course of studies. A second set of data was collected 3.75 years later at conclusion of the program. Twenty-two of the 25 students completed the program. All students who completed the program agreed to participate in the study. The initial data of the students who did not complete the program was excluded.

Data Collection

Various methods have been employed to collect data on beliefs of pre-service mathematics teachers, including interviews and classroom observation (e.g., Ball, 1990) and Likert-type surveys assessing level of agreement with statements reflecting a pre-determined set of mathematics-related beliefs (e.g., Tatto et al., 2008). Interviews and observations provide an in-depth picture of beliefs, but they are time intensive in terms of both data collection and data analysis. Likert surveys are an important tool for gathering and analyzing large data sets, but they cannot access beliefs that are not included in the survey (Grigutsch & Törner, 1998) and may yield "false-positive" agreement with some beliefs (Philipp, 2006).

We employed concept maps (Novak & Gowin, 1984), which visually represent abstract knowledge and understanding, to collect students' beliefs about knowing mathematics. Primarily used to assess knowledge and understanding of content, concept maps have also been used to capture meta-cognitive views about thinking (Ritchhart, Turner, & Hadar, 2008). We adopted Ritchhart's methodology to collect students' epistemological beliefs: they were asked to reflect on what it means to know mathematics; to generate a list of words and phrases that came to mind; to arrange their ideas in a hierarchy of importance/centrality to the notion "knowing mathematics"; to connect related ideas with lines and to briefly describe the connections. Students created concept maps before beginning the course of studies and again 3.75 years later at the completion of the program course-work.

Data Analysis

We used a two-step process to analyze the items (words and phrases) on the two sets of concept maps. First-stage qualitative analysis used a constant comparative paradigm; students' responses were read and reread to discover commonalities and recurring themes (Strauss & Corbin, 1990). This inductive process uncovered a structured system of categories describing different ways of knowing mathematics. Student maps included many items. When a single item seemed to relate to more than one way of knowing, it was categorized under each of the appropriate categories. Each item on a map was assigned a rank from 1 to *n* indicating its distance from the center of the map, with items closest to the center assigned a rank of one. The items were coded by category and rank.

The coded maps were used to define "beginning of program" and "end of program" matrices. Each category was assigned to a variable and each map was assigned to two rows in the appropriate matrix; each item on a map corresponded to two matrix entries-- the first denoting its category and the second, its rank. The ranking was used to assign a weight to each item; On a map with *n* ranks, rank *n* items (those items furthest from the center) were assigned a weight of $\frac{1}{n}$, rank *n*-1 items were assigned a rank of $\frac{2}{n}$, items closest to the center (rank one) received a weight of one.

For each map, the sum of the weights of the items in the category was computed, labeled as the [*category name*]-belief score. A [*category-name*] average weight was computed for each map by dividing the belief score by the number of map items in the category. Ratios of each belief score to the sum of all belief scores were computed both for the complete set of categories and for various subsets. Subset ratios will be described in the findings. The belief scores, average weights and belief ratios are dependent variables of the maps. For each student, differences between beginning-of-program and end-of-program values were computed, defining dependent variables of the students in the program.

Findings

Students related to knowing mathematics in complex and varied ways. Two main ways of knowing mathematics, *i.e.*, epistemological beliefs, emerged from the categorical analysis of the maps: knowledge of *mathematical content* comprised of topics (such as algebra) or skills (such as addition); and *attitudes* toward mathematics. Students' attitudes were expressed in terms of cognitive, behavioral and affective components, *e.g.*, perseverance (behavioral), satisfaction (affective), and success (cognitive). The categorical structure is shown in Figure 1.

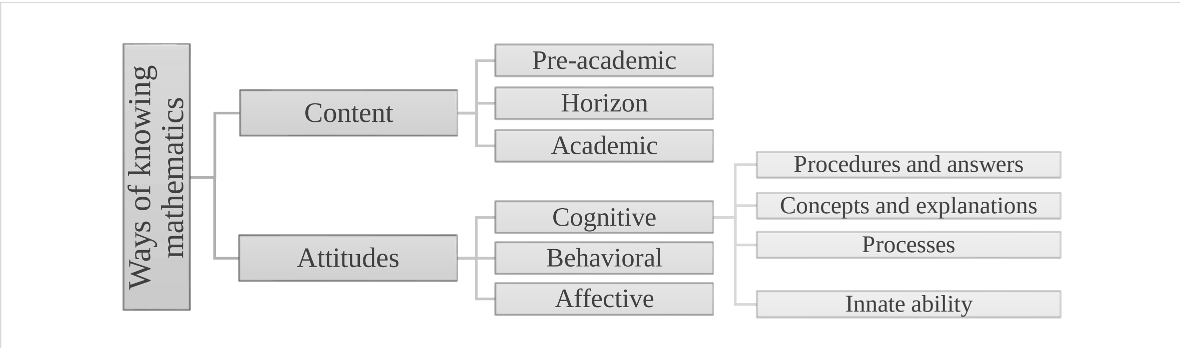

Figure 1. Categorical structure of beliefs

In the initial stage of quantitative analysis, four belief scores were computed corresponding to the content category (*content*) and the three components of the attitude category (*affective, behavioral, cognitive*.) The ratio of each score to the sum of the four scores was then computed. Mean ratios at the beginning and end of the program are shown in Table 1.

Table 1. Differences in belief ratios

	Belief Ratios			
	Content	Affective	Behavioral	Cognitive

Beginning of Studies	$\mu = 0.355$ $\sigma = 0.245$	$\mu = 0.033$ $\sigma = 0.060$	$\mu = 0.118$ $\sigma = 0.171$	$\mu = 0.494$ $\sigma = 0.217$
End of Studies	$\mu = 0.149$ $\sigma = 0.189$	$\mu = 0.044$ $\sigma = 0.096$	$\mu = 0.099$ $\sigma = 0.140$	$\mu = 0.708$ $\sigma = 0.195$

Students at both the beginning and end of the program related to knowing mathematics as attitudes towards the subject more strongly than knowing specific mathematical skills and topics. Due to the small sample size and wide variation between students, effect size and statistical significance do not provide meaningful data. None-the-less, the results suggest that trend was stronger at the end of the program; the content ratio at the beginning of the program was 0.355 ($\sigma = 0.245$) and the content ratio at the end of the program was 0.149 ($\sigma = 0.189$). Same-student comparisons confirm this finding; the content ratio of 77% (*n*=17) of the students decreased over the course of studies.

Mathematical Content

Mathematical content was included on most maps: 86% (*n*=19) of the beginning-of-program maps and 64% (*n*=14) of the end-of-program maps. There were three categories of mathematical content: *pre-academic content* such as geometry, solving equations and order of operations; *horizon content*, which in Israel bridges secondary and post-secondary mathematics, such as three-dimensional geometry and vectors; and *academic content* such as cyclic groups and infinity. (Note: In Israel, high school encounters with infinity, such as horizontal asymptotes, are algorithmic and are not associated to the symbol or concept of infinity.) Four students included horizon content and four students included academic content. One of the four included both horizon and academic content. For only one student did the number of academic items exceed the number of pre-academic items. This finding suggests that program graduates do not relate to academic mathematics, *i.e.*, the mathematics of mathematicians (Beswick, 2011).

Students did not connect different areas of mathematics on their maps, suggesting that at both the beginning and end of the program students' viewed mathematics content as a disjoint set of skills and topics rather than as a connected system (e.g., Beswick, 2005), *i.e.*, they had a simple knowledge belief about the structure of mathematics (M. Schommer, 1990).

The mean weight of the content items changed over the course of studies. The mean weight was 0.68 ($\sigma = 0.32$) on the initial maps and 0.32 ($\sigma = 0.26$) on the final maps. Items closest to the center of a map have a weight of one, therefore the results suggest that content became less central to students' views about knowing mathematics over the course of studies. Same-student comparisons confirm this finding; for 82% (*n*=18) of the students, the mean content weight of the second map was less than the mean content weight of the first. These findings suggest that the strength of students' belief in simple mathematical knowledge decreased over the course of studies.

Attitudes toward mathematics

Students' attitudes toward mathematics included cognitive, affective and behavioral components (Figure 1). Statistical analysis indicated that and behavioral components of attitudes became less central to students' epistemological beliefs about mathematics over the course of studies; the centrality of cognitive beliefs was stable.

Cognitive Beliefs

Four distinct sub-types of cognitive beliefs emerged from the data: three categories of cognitive processes associated to knowing mathematics and one relating to who knows mathematics. The three sub-types of cognitive processes are listed below:

- *Procedures and answers:* Mathematical knowledge consists of procedures; it is absolute. Knowledge is demonstrated by implementing mathematical procedures and achieving correct outcomes. "Correct" presentation is (sometimes) included.
- *Concepts and explanations*: Mathematical knowledge consists of understanding the concepts underlying procedures. It is absolute. Knowledge is demonstrated by an ability to explain or understand explanations of procedures and concepts.
- *Processes*: Mathematical knowledge is based on concepts which can be intuited, and discovered (or rediscovered). New knowledge can be constructed from existing knowledge. Knowledge is demonstrated by connecting concepts, relating mathematics to real life, asking and answering questions about mathematics, and creating (or recreating) new ways to solve problems.

The fourth sub-type cognitive belief, labeled *innate-ability*, describes a belief that knowing mathematics requires a certain type of intelligence (C. Dweck, 2006; C. S. Dweck, Chiu, & Hong, 1995). Thirty-six percent of the students (n=8) began the program holding a belief that knowing mathematics requires "intelligence" or a "mathematical head." No students expressed this belief at the end of the program, indicating that these future teachers had, indeed, internalized the idea that intrinsic ability is not a pre-requisite for learning mathematics.

Fostering dynamic beliefs is implicit in the guidelines for Israeli mathematics teacher preparation (Gutfreund & Rosenberg, 2012). We therefore separately analyzed map items in the three categories of cognitive processes that emerged from our data, computing procedures-and-answers, concept-and-explanations and processes beliefs scores as well as the ratio of each of these belief scores to the sum of the three scores. Differences between beginning- and end-of-studies mean ratios are shown in Figure 2.

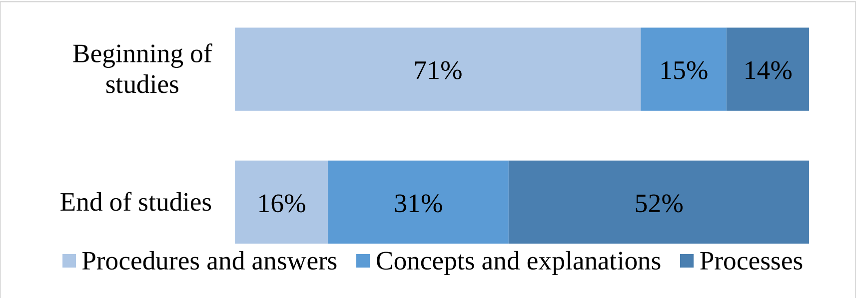

Figure 2. Cognitive process ratios

Cognitive beliefs expressed by beginning students were overwhelmingly focused on procedures and answers. By the end of the program, students' cognitive beliefs had shifted to process; 52% of their beliefs expressed mathematics-as-processes. The mean weights for items in each of the three categories of cognitive processes support this finding. On the initial maps, the mean weight of procedures-and-answers items was 0.775 ($\mu = 0.166$); on the final maps the mean weight was 0.480 ($\mu = 0.316$). Same-student comparison showed that the mean weight of procedures-and-answers beliefs decreased for 77% (n=17) of the students. In contrast, the mean weight across all students of process beliefs increased from 0.338 ($\mu =$ 0.388) to 0.829 ($\mu = 0.152$) and the mean weight of process beliefs increased for 77% (n=17) of the students. These findings indicate that over the course of studies students' views shifted from mathematics as procedures to mathematics as processes.

Discussion

This study analyzed pre-service secondary mathematics teachers' beliefs about knowing and learning mathematics at the beginning and end of their studies, as expressed through concept maps. Our findings document how their epistemological beliefs about mathematics changed over the course of studies, evolving to align with beliefs that support the cognitive objectives detailed in the Israeli guidelines for the preparation of secondary mathematics teachers (Gutfreund & Rosenberg, 2012). Prior research indicates that education impacts beliefs about the structure and stability of knowledge (Marlene Schommer, 1998), therefore the overall picture that emerged supports the idea that the program of studies impacted student beliefs about the complexity of mathematical knowledge and the cognitive components of mathematics. Other factors, such as age, may also have impacted beliefs.

Our results indicate that a view of knowing mathematics as knowing isolated content (simple knowledge) decreased over the course of studies. This has positive implications for these students' abilities to access and apply higher order cognitive strategies (Cano, 2005) and self-regulate their learning (Muis, 2007). The Gutfreund guidelines (2012) highlight these abilities as main components of mathematical literacy, deemed an essential component of teacher preparation. Our findings indicate that end-of-program beliefs align with the goals for mathematical literacy expressed in the guidelines.

Gutfreund guidelines (2012) for content knowledge of mathematics include understanding mathematics as a creative process rather than a finished, polished product; guidelines for pedagogical knowledge guidelines include supporting learning of both low and high order cognitive processes and stress equity of opportunities to learn mathematics vis a vis gender and differing abilities. These goals are aligned with dynamic, process-oriented beliefs about mathematics (e.g., Blömeke & Delaney, 2012; Briley et al., 2009). Our findings document that students' beliefs changed dramatically from an almost exclusive focus on mathematics as procedures and outcomes to a nuanced set of beliefs where mathematics includes procedures, concepts and dynamic processes.

The connection between the structure of mathematics teacher preparation programs and graduates' beliefs have shown that differences in opportunities to learn mathematics education courses impact graduates' beliefs. Our findings indicating post-program alignment with (implicit) belief goals present another avenue of exploration: the connection between program goals and epistemological beliefs. As a first step belief-alignment to the Gutfreund guidelines (2012) of other Israeli mathematics teacher preparation programs should be assessed. Belief-alignment in other countries should also be evaluated, including comparing alignment when belief goals are explicitly stated and alignment when they are implied.

Conclusion

This case study showed that the epistemological beliefs of students completing a three-year undergraduate program in mathematics education are consistent with goals in the guidelines for the preparation of Israeli secondary mathematics teachers (Gutfreund & Rosenberg, 2012) and are aligned with belief expectations implicit in the goals.

The attention paid to the epistemological beliefs of graduates of mathematics education programs (e.g., Blömeke & Delaney, 2012) should be expanded to other undergraduate STEM programs. Connections between acquisition and application of higher cognitive strategies and epistemological beliefs (Hofer & Sinatra, 2010) suggest a role for beliefs in setting and meeting STEM goals of complex problem solving and lifelong learning (Fairweather, 2008).

References

Ball, D. L. (1990). The Mathematical Understandings that Prospective Teachers Bring to Teacher Education. *Elementary School Journal*, *90*(4), 449–66. Retrieved from http://eric.ed.gov/?id=EJ405882

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal*, *17*(2), 39–68. https://doi.org/10.1007/BF03217415

Beswick, K. (2011). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics*, *79*(1), 127–147. https://doi.org/10.1007/s10649-011-9333-2

Binkley, M., Erstad, O., Herman, J., Raizen, S., Ripley, M., Miller-Ricci, M., & Rumble, M. (2012). Defining Twenty-First Century Skills. In *Assessment and Teaching of 21st Century Skills* (pp. 17–66). Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-007-2324-5_2

Blömeke, S., Buchholtz, N., Suhl, U., & Kaiser, G. (2014). Resolving the chicken-or-egg causality dilemma: The longitudinal interplay of teacher knowledge and teacher beliefs. *Teaching and Teacher Education*, *37*, 130–139. https://doi.org/10.1016/j.tate.2013.10.007

Blömeke, S., & Delaney, S. (2012). Assessment of teacher knowledge across countries: a review of the state of research. *ZDM*, *44*(3), 223–247. https://doi.org/10.1007/s11858-012-0429-7

Briley, J. S., Thompson, T., & Iran-Nejad, A. (2009). Mathematical beliefs, self-regulation, and achievement by university students in remedial mathematics courses. *Research in the Schools*, *16*(2), 15–28.

Bromme, R., Pieschl, S., & Stahl, E. (2010). Epistemological beliefs are standards for adaptive learning: a functional theory about epistemological beliefs and metacognition. *Metacognition and Learning*, *5*(1), 7–26. https://doi.org/10.1007/s11409-009-9053-5

Brownlee, J., Purdie, N., & Boulton-Lewis, G. (2001). Changing Epistemological Beliefs in Pre-service Teacher Education Students. *Teaching in Higher Education*, *6*(2), 247–268. https://doi.org/10.1080/13562510120045221

Cano, F. (2005). Epistemological beliefs and approaches to learning: Their change through secondary school and their influence on academic performance. *British Journal of Educational Psychology*, *75*(2), 203–221. https://doi.org/10.1348/000709904X22683

CBMS. (2012). *Conference Board of the Mathematical Sciences. The Mathematical Education of Teachers II (MET2)* (2nd ed.). Providence, RI and Washington, D.C.: American Mathematical Society and Mathematical Association of America. Retrieved from http://www.cbmsweb.org/archive/MET2/met2.pdf

Dweck, C. (2006). *Mindset: The new psychology of success*. Random House. https://doi.org/10.5860/CHOICE.44-2397

Dweck, C. S., Chiu, C.-Y., & Hong, Y.-Y. (1995). Implicit Theories and Their Role in Judgments and Reactions: A World From Two Perspectives. *Psychological Inquiry*, *6*(4). Retrieved from https://hub.hku.hk/bitstream/10722/44536/1/11302.pdf?accept=1

Ernest, P. (1991). *The Philosophy of Mathematics Education* (Vol. 1). Routledge. Retrieved from https://books.google.com/books?id=SnyOAgAAQBAJ&pgis=1

Fairweather, J. (2008). Linking Evidence and Promising Practices in STEM Undergraduate Education. Retrieved from http://otl.wayne.edu/wider/linking_evidence--fairweather.pdf

Fink, L. D. (2003). *Creating significant learning experiences : an integrated approach to designing college courses*. Jossey-Bass.

Grigutsch, S., & Törner, G. (1998). World views of mathematics held by university teachers

of mathematics science. Retrieved from http://duepublico.uni-duisburg-essen.de/go/duett-05272002-102811
Gutfreund, H., & Rosenberg, Y. (2012). *Knowledge and Training of Secondary School Mathematics Teachers*. Jerusalem.
Hiebert, J., & Lefevre, P. (1986). *Conceptual and procedural knowledge in mathematics: An introductory analysis.*
Hofer, B. K. (2004). Epistemological Understanding as a Metacognitive Process: Thinking Aloud During Online Searching. *Educational Psychologist*, *39*(1), 43–55. https://doi.org/10.1207/s15326985ep3901_5
Hofer, B. K., & Pintrich, P. R. (1997). The Development of Epistemological Theories: Beliefs About Knowledge and Knowing and Their Relation to Learning. *Review of Educational Research*, *67*(November 2015), 88–140. https://doi.org/10.3102/00346543067001088
Hofer, B. K., & Sinatra, G. M. (2010). Epistemology, metacognition, and self-regulation: musings on an emerging field. *Metacognition and Learning*, *5*(1), 113–120. https://doi.org/10.1007/s11409-009-9051-7
Kuhn, D. (1991). *The skills of argument*. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511571350
Leder, G. C., Pehkonen, E., & Torner, G. (Eds.). (2002). *Beliefs: A Hidden Variable in Mathematics Education?* Dordrecht: Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_6
Leder, G. C., Pehkonen, E., & Törner, G. (2002). Setting the Scene. In *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 1–10). Dordrecht: Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_1
Louca, L., Elby, A., Hammer, D., & Kagey, T. (2004). Epistemological Resources: Applying a New Epistemological Framework to Science Instruction. *Educational Psychologist*, *39*(1), 57–68. https://doi.org/10.1207/s15326985ep3901_6
Marton, F., & Saljo, R. (1976). on Qualitative Differences in Learning: I-Outcome and Process*. *British Journal of Educational Psychology*, *46*(1), 4–11. https://doi.org/10.1111/j.2044-8279.1976.tb02980.x
McLeod, D. B., & McLeod, S. H. (2002). Synthesis — Beliefs and Mathematics Education: Implications for Learning, Teaching, and Research. In *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 115–123). Dordrecht: Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_7
Muis, K. R. (2007). The Role of Epistemic Beliefs in Self-Regulated Learning. *Educational Psychologist*, *42*(3), 173–190. https://doi.org/10.1080/00461520701416306
Novak, J. D., & Gowin, D. B. (1984). *Learning how to learn*. Cambridge University Press.
Philipp, R. A. (2006). Mathematics Teachers' Beliefs and Affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning:* (pp. 257–315). Charlotte: National Council of Teachers of Mathematics. 1906 Association Drive, Reston, VA 20191-1502. Tel: 800-235-7566; Tel: 703-620-3702; Fax: 703-476-2970; e-mail: orders@nctm.org; Web site: http://www.nctm.org/publications/. Retrieved from https://books.google.com/books?hl=en&lr=&id=Cww16Egbp4oC&pgis=1
Ritchhart, R., Turner, T., & Hadar, L. (2008). Uncovering Students ' Thinking About Thinking Using Concept Maps. *Metacognition and Learning*, *4*(2), 1–27. https://doi.org/10.1007/s11409-009-9040-x
Schmidt, W. H., Houang, R. T., Cogan, L., Blömeke, S., Tatto, M. T., Hsieh, F. J., … Paine, L. (2008). Opportunity to learn in the preparation of mathematics teachers: its structure and how it varies across six countries. *ZDM*, *40*(5), 735–747. https://doi.org/10.1007/s11858-008-0115-y

Schommer, M. (1990). Effects of beliefs about the nature of knowledge on comprehension. *Journal of Educational Psychology*, *82*(3), 498–504. Retrieved from http://cat.inist.fr/?aModele=afficheN&cpsidt=19248583
Schommer, M. (1998). The influence of age and education on epistemological beliefs. *British Journal of Educational Psychology*, *68*(4), 551–562. https://doi.org/10.1111/j.2044-8279.1998.tb01311.x
Schraw, G., Crippen, K. J., & Hartley, K. (2006). Promoting Self-Regulation in Science Education: Metacognition as Part of a Broader Perspective on Learning. *Research in Science Education*, *36*(1–2), 111–139. https://doi.org/10.1007/s11165-005-3917-8
Schraw, G., & Moshman, D. (1995). Metacognitive theories. *Educational Psychology Review*, *7*(4), 351–371. https://doi.org/10.1007/BF02212307
Star, J. R. (2005). Research Commentary: Reconceptualizing Procedural Knowledge. *Journal for Research in Mathematics Education*, *36*(5), 404–411. Retrieved from http://eric.ed.gov/?q=Reconceptualizing+procedural+knowledge&id=EJ764986
Staub, F. C., & Stern, E. (2002). The nature of teachers' pedagogical content beliefs matters for students' achievement gains: Quasi-experimental evidence from. *Journal of Educational Psychology*, *94*(2), 344–355.
Strauss, A., & Corbin, J. (1990). *Basics of qualitative research*. Newbury Park, CA: Sage.
Tatto, M. T., Ingvarson, L., Schwille, J., Peck, R., Senk, S. L., & Rowley, G. (2008). *Teacher education and development study in mathematics (TEDS-M) : policy, practice, and readiness to teach primary and secondary mathematics : conceptual framework*. Amsterdam: Schmidt, Bill (William) Michigan State University. Teacher Education and Development International Study Center International Association for the Evaluation of Educational Achievement. Retrieved from https://eric.ed.gov/?id=ED542390
Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York, NY: Macmillan Publishing Company.
Vermunt, J. D., Van Rossum, E. J., & Hamer, R. N. (2010). *The Meaning of Learning and Knowing*. Sense Publishers. Retrieved from http://dspace.library.uu.nl/handle/1874/43866
Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education*, *11*(2), 139–164. https://doi.org/10.1007/s10857-007-9068-2