The 21st Annual Conference of the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education

Program
(including abstracts)

February 22nd-24th
2018
San Diego, CA

2018 SIGMAA-RUME CONFERENCE SCHEDULE

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Breakfast
Point Loma Terrace</td></tr>
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Note: Theoretical Reports will be presented in the Contributed Report sessions

Day	Time	Session Type
Thursday	**7:30-11:30**	***Working Group***

Working Group 1: Equity in Undergraduate Mathematics Education

Room: Point Loma 2&3

There remains a need to understand equity issues in undergraduate mathematics education and ways that equity-focused perspectives can complement existing research in RUME. To address this need, this working group serves as a collective of scholars and practitioners aimed at advancing the equity agenda in undergraduate mathematics by exchanging constructive feedback on related scholarly work, instructional and curricular resources, and other artifacts from their professional practice. Particularly, the group aims to address the following questions: 1) In what ways can equity considerations conceptually and methodologically enhance the quality of research in RUME?; 2) Alongside the broader policy changes in higher education at large, how do we see issues of equity in the day-to-day teaching and learning experiences across undergraduate mathematics classrooms and other related learning contexts? 3) How can we leverage insights from K-12 mathematics education research to further advance the equity research agenda and inform more equitable teaching and learning opportunities in undergraduate mathematics? Informal and sustained mentorship will be encouraged within the working group considering the variation across members' stages of academic and professional development.

Working Group 2: Research on College Mathematics Instructor Professional Growth

Room: Point Loma 1

This long-standing working group focuses on research on the professional development and growth of college mathematics instructors regardless of their level of experience or expertise. Group meeting time is structured to bring in researchers new to the field through a variety of scholarly activities: explore and discuss the literature, give and receive feedback on research projects that are in progress, brainstorm potential collaborations and mentoring relationships for both long- and short-term studies, and continued discussion of issues central to the field and ways to address them. Research areas include, but are not limited to, factors that shape instructional practices, experiences of instructors as they attend to student thinking in their instruction, and changes in instructional orientations, planning, and practices as teaching experiences accumulate. Researchers need not present their own work to participate in the group or to provide feedback to others. Dissemination from the group is broad, from publications aimed at education research audiences to practice-oriented college mathematics instructor and mathematician communities. What drives the working group is meeting the needs of the group. Regular online meetings during the year sustain collaborations and communication among group participants.

Working Group 3: Research on Community College Mathematics

Room: Private Dining Room

The goal of this working group is to bring together researchers who focus on teaching and learning in the unique and significant context community college mathematics
There is a significant need for research in this domain of mathematics education research that overlaps research in both K-12 and RUME settings, but that has its own unique questions.

Supported through past working group sessions at RUME and committee work within AMATYC, a growing group of researchers has been collaborating to advance a national agenda and create a web of community college mathematics education research. Projects have been funded, collaborative research has been undertaken, and the dissemination of findings is ongoing. We propose to leverage the RUME working group session to continue a discussion about editing a special issue of a mathematics education journal to highlight this work. We welcome new working group participants who are conducting research in mathematics teaching and learning at community colleges and are looking for a venue to publish their findings.

Working Group 4: Education Research at the Interface of Mathematics and Physics: "Thick" Derivatives

Room: Del Mar

The working group focus this year will be on the teaching of the concept of derivative, with an emphasis on numerical, computational, and physical representations, and on attaining a conceptual understanding rather than procedural fluency. The aim of this group is to share research on the learning and teaching of mathematics and physics at the college level through cross-disciplinary discussions between researchers in both fields.
Activities will include a short interactive workshop to introduce participants to different disciplinary approaches to derivatives that involve the use of finite difference approaches. This will be followed by small group discussion of the importance and relevance of different aspects of the concept image of derivative. The session will end with each group sharing their perspective with the working group as a whole.
Participants who wish to present briefly on their own perspective on derivative during the working group should contact the organizers at least two weeks prior to the session.

Working Group 5: Statistics Education Research in RUME

Room: Bay Room

This working group solicits persons interested in learning about and pursuing research on the teaching and learning of undergraduate statistics. This includes, but is not limited to, theoretical analysis or empirical research situated within the introductory statistics sequence, pre-service teacher courses, or advanced undergraduate statistics courses. Additionally, we encourage persons interested in investigating problem spaces that are shared among the undergraduate mathematics and statistics education communities (e.g., function, probability, modeling, etc.) with the hope that such research may inform the practices and research agendas of both communities.

This group will briefly cover the history of stats education and how it relates to RUME, it will recap work done at the 2016 "RUME with a View" mini-conference at Oklahoma University, it will share ongoing projects and open problems, and provide a space for those isolated researchers to work together and plan new projects.

Working Group 6: Supporting Faculty to Improve Student Learning and Success in Precalculus Level Courses

Room: Coronado

This working group session will engage its participants in discussing learning goals for precalculus level mathematics and mechanisms that research has reported for improving student learning and success in precalculus level courses. Working group organizers will begin the session by providing an overview of what has been identified in research as obstacles and promising approaches for shifting precalculus level courses and instruction to be more mathematically meaningful, engaging, and coherent for students. The following topics will be the focus of working group discussions: 1) Setting meaningful learning goals for precalculus level courses that are based in research on learning; 2) Mechanisms for supporting faculty to be more effective in developing students' mathematical practices and understandings of key precalculus concepts, while concurrently developing procedural fluency meaningfully; 3) Novel instructional approaches such as stretch courses, adaptive learning using validated formative assessments, supplemental online learning platforms, etc.; 4) Navigating cultural, departmental, and other administrative obstacles to shifting the content focus of precalculus level courses to be more focused on developing students' mathematical meanings, practices, and confidence.

Working Group 7: Research Opportunities for RUME Researchers in the Context of Mathematics Resource Centers

Room: La Jolla

The 2017 Working Group on Mathematics Resource Centers (MRCs) brought together a group of researchers who were interested in studying students' cognition in the context of a math resource center. The justification of this working group was that much of RUME research focuses on the classroom context and that students' learning of mathematics outside the classroom is currently largely unexplored. The following is a list of research topics that we will continue to work on:

1. Creating research-based tutor training.
2. Analyzing tutors' reflections on their practice.
3. Quantitative and qualitative evaluation of the effectiveness of resource centers.
4. Theoretical work to define "mathematical knowledge for tutoring."
5. Analysis of responses from national survey of resource center leaders
6. Studying the development of GTA's mathematical knowledge for teaching
7. Leveraging interactions with students at the resource center to foster improvements in curriculum and instruction department wide.

Day	Time	Session Type
Thursday	**13:00-13:10**	***Special***

Opening Session

Room: Point Loma 2&3

Day	Time	Session Type
Thursday	**13:10-14:10**	***Special***

Honoring the Work of Ed Dubinsky

Room: Point Loma 2&3

This special session is organized to honor Ed Dubinsky, the founder of RUME - the first SIG of MAA, for his significant contribution to our field. Several colleagues will offer personal reflections on working with Ed and a brief overview of his the most important contribution to the field. The session will conclude with a personal 'message' from Ed.

	Day	Time	Session Type
Session 1	**Thursday**	**14:20-14:50**	***Contributed Report***

Development of the Inquiry-Oriented Instructional Measure

George Kuster, Christopher Newport University; Rachel Rupnow, Virginia Tech; Estrella Johnson, Virginia Tech; Annie Garrison Wilhelm, Southern Methodist University

Room: Point Loma 2&3

In this article we discuss the Inquiry Oriented Instructional Measure (IOIM). The development of the IOIM was a multi-phase, iterative process that required analyzing current research literature and videos of classroom instruction and piloting the measure with both experts and novices. The process resulted in identifying multiple instructional practices that support the successful implementation of Inquiry-Oriented Instruction (IOI) at the undergraduate level, and creating a rubric for evaluating the degree to which one's classroom instruction is reflective of these practices. Our goals with this paper are to share the development process and elaborate on the rubric so as to contribute to the knowledge base regarding the implementation of IOI.

Generating Equations for Proportional Relationships Using Magnitude and Substance Conceptions

Sybilla Beckmann, University of Georgia; Andrew Izsak, University of Georgia

Room: Private Dining Room

We discuss a magnitude conception and a substance conception of fractions and variables that future middle-grades and secondary teachers used when developing and explaining equations for proportional relationships by reasoning about quantities. We conjecture that both conceptions are important for developing equations. The substance conception is useful when a fraction or variable functions as a multiplicand, but not when it functions as a multiplier. The magnitude conception is useful when a fraction or variable functions as a multiplier, but may not be essential when it functions as a multiplicand. Expertise may involve recognizing that the conceptions are distinct and developing a sense of when each conception is useful.

Our Mathematical Ideas are Part of Our Identity

Jeffrey Pair, California State University Long Beach; Stanley Lo, University of California San Diego

Room: La Jolla

This paper explores the notion that our mathematical ideas are part of our identity. This notion, which was a significant result of a qualitative dissertation study, will be explored in depth through an examination of data in connection with the educational research related to identity. The story of Binary, a first generation college student completing a transition-to-proof course in his final semester of college, provides the context in which we explore this complex notion.

Reasoning about Quantities or Conventions: Investigating Shifts in In-service Teachers' Meanings after an On-line Graduate Course

Teo Paoletti, Montclair State University; Jason Silverman, Drexel University; Kevin Moore, University of Georgia; Madhavi Vishnubhotla, Montclair State University; Zareen Rahman, Montclair State University; Ceire Monahan, Montclair State University;

Room: Del Mar

Although pervasive in school mathematics, few researchers have paid explicit attention to the impact graphing conventions have on teachers' meanings for function and rate of change. We examine the role conventions play in in-service teachers' (ISTs') meanings and ways to promote their developing more sophisticated meanings. We provided pre and post surveys to ISTs enrolled in an on-line graduate course specifically designed to promote their development of more sophisticated meanings for function and rate of change via reasoning quantitatively. We prompted them to consider hypothetical student responses about these ideas in unconventional representations. In this report, we characterize ISTs' meanings in relation to conventions commonly maintained in school mathematics and examine shifts in the ISTs' meanings.

Students' Usage of Visual Imagery to Reason about the Divergence, Integral, Direct Comparison, Limit Comparison, Ratio, and Root Convergence Tests

Steven Jones, Brigham Young University; John Probst, Brigham Young University

Room: Coronado

This study was motivated by practical issues we have encountered as second-semester calculus instructors, where students struggle to make sense of the various series convergence tests, including the divergence, integral, direct comparison, limit comparison, ratio, and root tests. To begin an exploration of how students might reason about these tests, we examined the visual imagery used by students when asked to describe what these tests are and why they provide the conclusions they do. It appeared that each test had certain types of visual imagery associated with it, which were at times productive and at times a hindrance. We describe how the visual imagery used by students seemed to impact their reasoning about of the convergence tests.

The use(s) of 'is' in mathematics

Paul Dawkins, Northern Illinois University ; Matthew Inglis, Loughborough University; Nicholas Wasserman, Teachers College, Columbia University

Room: Point Loma 1

This paper analyzes some of the ambiguities that arise among statements with the copular verb is in the mathematical language of textbooks as compared to day-to-day English language. We identify patterns in the construction and meaning of is statements using randomly selected sample statements from corpora representing the two linguistic registers. In particular, for the grammatical form "[subject] is [noun]," we compare the relative frequencies of the subcategories of semantic relations conveyed by that construction. Specifically, we find that this construction – in different situations – conveys a symmetric relation, an asymmetric relation, or an existential relation. The intended logical relation can only sometimes be inferred from the grammar of the statement itself. We discuss the pedagogical significance of these patterns in mathematical language and consider some strategies for helping students interpret the intended meaning of the mathematical text they hear or read.

	Day	Time	Session Type
Session 2	**Thursday**	**15:00-15:30**	***Contributed Report***

Analyzing Narratives About Limits Involving Infinity in Calculus Textbooks

Miroslav Lovric, McMaster University; Andrijana Burazin, University of Toronto, Mississauga

Room: Coronado

We analyze Calculus textbooks to determine to what extent narratives about limits at infinity and infinite limits align with research in mathematics education. As reasoning about limits falls within the domain of advanced mathematical thinking (AMT), we looked for evidence of appropriate treatment of, and support for, AMT: clear and precise narratives, deductive and rigorous reasoning, intuitive development that does not create or enhance students' misconceptions, opportunities for "personal reconstruction" (Tall, 1991), adequate representations, and the appropriate use of definitions. In conclusion, both high school and university Calculus textbook narratives do not place infinity in a precise, well-defined context, thus possibly creating or strengthening (novice) students' misconceptions. We identified very little evidence of the type of support for AMT that we were looking for. This paper concludes with several suggestions for possible modifications of narratives which involve infinity.

Cognitive Consistency and Its Relationships to Knowledge of Logical Equivalence and Mathematical Validity

Kyeong Hah Roh, Arizona State University; Yong Hah Lee, Ewha Womans University

Room: Del Mar

The purpose of this study is to explore how cognitive consistency is related to knowledge of logic and mathematical proofs. We developed a logic instrument and administered it to forty-seven (47) undergraduate students who enrolled in various sections of a transition-to-proof course. The analysis of the students' scores on the logic instrument indicated that students' knowledge of logical equivalence and their knowledge of mathematical validity were somewhat related to one another. On the other hand, cognitive consistency was not closely related to either student knowledge of logic or knowledge of mathematical validity. Based on these findings, we address the importance of cognitive consistency in logical thinking and discuss implications for the teaching and learning of logic in mathematical contexts.

Computing as a Mathematical Disciplinary Practice

Elise Lockwood, Oregon State University; Matthew Thomas, Ithaca College; Anna DeJarnette, University of Cincinnati

Room: Point Loma 2&3

The role of computation continues to be prominent in the STEM fields, and the activity of computing has become an important mathematical disciplinary practice. Given the importance of computational fluency in science and mathematics, we are curious about the nature of such activity in mathematics. To study this, we interviewed six mathematicians about the role of computation in their work, and we identified several aspects of computation that sheds light on the nature of computing as a mathematical disciplinary practice. In this paper, we present examples and applications of computation for these mathematicians, highlight types of computation, provide specific examples of computation in their work, and emphasize how computation relates to mathematics in particular.

Development of the Elementary Algebra Concept Inventory for the College Context

Claire Wladis, BMCC/CUNY; Kathleen Offenholley, BMCC/CUNY; Susan Licwinko, BMCC/CUNY; Dale Dawes, BMCC/CUNY; Jae Ki Lee, BMCC/CUNY

Room: La Jolla

This study describes the creation and validation of the first concept inventory for elementary algebra at the tertiary level. A 22-item multiple choice/multiple answer instrument was created through a combination of literature review, syllabus review, and collaboration with instructors. The instrument was then revised and tested for content, construct and concurrent validity as well as composite reliability, using a circular process that combined feedback from experts (mathematicians, instructors, and mathematics education researchers), cognitive interviews with students, and field tests using both classical test theory and item response theory. Results suggest that the inventory is a valid and reliable instrument for assessing student conceptual understanding in elementary algebra, as conceptualized in this study.

Mind the 's' in Individual-With-Contexts: Two Undergraduate Women Boosting Self-Efficacy in Mathematics

Fady El Chidiac, University of California, Berkeley; Melissa Carlson, University of California, Berkeley; Sakthi Ponnuswamy, University of California, Berkeley

Room: Private Dining Room

Investigations of students' identities are shedding light on the processes that generate differential learning. In this paper, we expand the construct of dispositions to account for what individuals carry through their bodies and beliefs across contexts. While bringing back the attention to individualized dispositions, we avoid the trap of innateness. After we create a three-layered framework of dispositions to explain why and how two undergraduate women, Bettie and Melissa, develop differential confidence in mathematics through a semester-long number theory class. We learn that individualized dispositions can change throughout time, and desensitize or make individuals vulnerable to gendered, racialized, and sexualized stigma.

Using Quantitative Diagrams to Explore Interactions in a Group Work and Problem-Centered Developmental Mathematics Class

Martha Makowski, University of Alabama; Sarah Lubienski, Indiana University

Room: Point Loma 1

Despite low success rates and an academically vulnerable population, classes taught at the pre-college (or developmental) level have rarely been examined by mathematics education researchers. Mathematical Literacy, a recent developmental curriculum innovation, aims to better meet the unique needs of developmental students through group work and problem-centered materials. Using a novel quantitative representation of the classroom and descriptive statistics, this study examines the productivity of developmental mathematics students, their engagement with each other and the instructor, and their access to the curriculum in a Mathematical Literacy implementation. We find that students' engagement is regular, productive, and frequently involves the instructor. However, some students have less access to group discussions. Future implementations should focus on early identification of such students.

	Day	Time	Session Type
Session 3	**Thursday**	**15:40-16:10**	***Contributed Report***

A Study of Calculus Students' Solution Strategies when Solving Related Rates of Change Problems

Thembinkosi Mkhatshwa, Miami University; Steven Jones, Brigham Young University

Room: Del Mar

Contributing to the growing body of research on students' understanding of related rates of change problems, this study reports on the analysis of solution strategies used by five calculus students when solving three related rates of change problems where the underlying independent variable in each problem was time. Contrary to findings of previous research on students' understanding of related rates of change problems, all the students in this study were able to translate prose to algebraic symbols. All the students had a common benchmark to guide their overall work in one of the tasks but no benchmark to guide their overall work in the other two tasks. Three students exhibited weaker calculational knowledge of the product rule of differentiation. Directions for future research and implications for instruction are included.

Connecting the Study of Advanced Mathematics to the Teaching of Secondary Mathematics: Implications for Teaching Inverse Trigonometric Functions

Keith Weber, Rutgers University; Nicholas Wasserman, Teachers College, Columbia University; Juan Pablo Mejia-Ramos, Rutgers University; Tim Fukawa-Connelly, Temple University

Room: Point Loma 2&3

Prospective mathematics teachers are usually required to complete courses in advanced mathematics to be certified to teach secondary mathematics. However, most teachers do not find these advanced mathematics courses as relevant to their teaching. In this paper, we describe a novel way to teach real analysis to future teachers that connects the content of real analysis to the activity of teaching secondary mathematics. We illustrate this method by describing a module that links the study of the relationship of continuity, injectivity, invertibility, and strict monotonicity in real analysis to the teaching about the arcsine function and solving trigonometric equations in secondary mathematics. We describe a teaching experiment in which this module was implemented and present evidence of the efficacy of this instruction.

Early Undergraduates' Emerging Conceptions of Proof and Conviction

Alison Lynch, California State University, Monterey Bay; Ryan Pugh, California State University, Monterey Bay

Room: La Jolla

Before enrolling in an introduction-to-proof course, undergraduates often hold conceptions of mathematical proof that do not align with those accepted by the mathematics community. These conceptions are informed, in part, by past experiences with proof in mathematics and science courses. In this study, we sought to investigate the influence of these past experiences on students' conceptions of proof. We conducted interviews with nine undergraduates in their first or second year in which we asked them to solve number theory tasks and determine the validity of provided number theory statements. In this paper, we report on the various conceptions of proof these students conveyed and the influence of past experiences on these conceptions.

Epistemological Beliefs About Mathematics and Curriculum Goals in the Cognitive Domain: a Case Study of Preservice Secondary Mathematics Teachers

Tamara Lefcourt Ruby, Efrata College of Education/Bar Ilan University

Room: Private Dining Room

Beliefs have long been recognized as a "hidden variable" in mathematics education. Epistemological beliefs are an inherent, although often implicit, component of curriculum goals in the cognitive domain. Connections between acquiring and accessing higher order cognitive strategies and epistemological beliefs are gradually becoming better understood. Israeli guidelines for mathematics teacher preparation emphasize of view of mathematics as a complex body of knowledge and knowing mathematics as a dynamic process. We present a case study of Israeli preservice secondary mathematics teachers' epistemological beliefs about mathematics, assessed via concept maps at the beginning and end of undergraduate studies. A mixed-methods approach was used to analyze maps. Results suggest that students' beliefs shifted to align with Israeli goals. Implications for STEM curriculum design are discussed.

The Creation of a Humanistic Educational Framework for the Nature of Pure Mathematics

Jeffrey Pair, California State University Long Beach

Room: Bay Room

Within the field of mathematics education research, scholars have found that students often have naïve views about the nature of mathematics. Mathematics is seen as an impersonal and uncreative subject. What can educators do to challenge such views, and support students in developing richer understandings of the nature of mathematics? In this paper, I describe my dissertation study, the goal of which was to identify humanistic characteristics of pure mathematics which may be of benefit for undergraduate students in a transition-to-proof course to know and understand. Using the methodological framework of heuristic inquiry, which leverages the researcher as instrument in qualitative research, I identified humanistic characteristics of mathematics by reviewing relevant literature, collaborating with a professional mathematician, co-teaching an undergraduate transition-to-proof course, and being open to mathematics wherever it appeared in life. The main result is the IDEA Framework for the Nature of Pure Mathematics.

The Potential Virtues of Wicked Problems for Education

Jeffrey Craig, University of Arizona; Lynette Guzman, University of Arizona; Andrew Krause, Michigan State University

Room: Coronado

We offer a contribution to a theory of transdisciplinary curriculum based in empirical research of an undergraduate mathematics course in quantitative literacy. By organizing around contexts and developing open, semester-long projects, this course blurred disciplinary boundaries. Fortunately, ignoring debates about where mathematics ends and these contexts begin is well-suited for the goals of general undergraduate courses. We found that the language of transdisciplinary and wicked problems fitted our experiences designing, teaching, and studying the course. We share selected empirical findings, then develop a transdisciplinary curriculum theory for wicked problems.

Day	Time	Session Type
Thursday	**16:10-16:35**	***Break***

Coffee Break

Room: Cabo Courtyard (Weather Permitting)

	Day	Time	Session Type
Session 4	**Thursday**	**16:40-17:10**	***Contributed Report***

A Case of Community, Investment, and Doing in an Active-Learning Business Calculus Course

Abigail Higgins, California Maritime Academy

Room: Private Dining Room

Presented here is a case study of Christina and her goals in an active-learning business calculus course. The larger project, from which this report emanates, involved an instructional redesign of a business calculus course intended to address two main student concerns: (seemingly) irrelevant content and a lack of opportunities to be active in class. Class size mediated Christina's access to community, which she perceived to be a necessary condition for her learning. Additionally, Christina drew a distinction between authentically situated mathematics and pseudo-situated problems that fail to invest her in the problem-solving process. She valued opportunities to do mathematics during class and receive feedback from her instructor and peers. This project has implications for the mathematics education of business students, active-learning in post-secondary mathematics, and situated mathematics problems.

Developing Understanding of the Partial Derivative with a Physical Manipulative

Jason Samuels, CUNY-BMCC; Brian Fisher, Lubbock Christian University

Room: Del Mar

Multivariable calculus education is an area of growing investigation, and in this study we specifically target the topic of partial derivatives. Data was collected on students learning in an innovative curriculum using physical manipulatives. We trace the complex path as students developed both their mathematical knowledge and their use of the artifacts at their disposal, and analyze the interaction between them. Implications for the classroom and for research are noted.

Didactical Disciplinary Literacy

Aaron Weinberg, Ithaca College; Ellie Fitts Fulmer, Ithaca College; Emilie Wiesner, Ithaca College; John Barr, Ithaca College

Room: La Jolla

Undergraduate mathematics students are routinely asked to learn from various "texts" such as textbooks, videos, and lectures. In order to understand how students read and learn from discipline-specific texts, literacy researchers in recent years (e.g. Shanahan & Shanahan, 2012) have begun to direct their attention to disciplinary literacy: the ways that disciplinary experts themselves interpret, create, and critique materials. In this vein, we set out to investigate the disciplinary literacy practices of calculus students and non-mathematics STEM faculty, but found that focusing on this form of literacy alone was insufficient to explain the differences between the students' and faculty members' practices. To address this, we propose a new construct of didactical literacy, provide examples, and discuss its details and ramifications.

Framework for Students' Understanding of Mathematical Norms and Normalization

Kevin Watson, Virginia Tech

Room: Bay Room

Mathematical norms and normalization of vectors are important concepts used throughout the mathematical and physical sciences; however, very little research has been done on students' understanding of these concepts. To remedy this lacuna, this report presents a framework that can be used to model, explain, and predict the ways students reason about and solve problems involving norms and normalization.

Mathematics Graduate Teaching Assistants' Growth as Teachers: An Unexamined Practice

Erica Miller, University of Nebraska-Lincoln; Karina Uhing, University of Nebraska - Lincoln; Meggan Hass, University of Nebraska-Lincoln; Rachel Zigterman, University of Nebraska-Lincoln; Kelsey Quigley, University of Nebraska-Lincoln; Yvonne Lai,

Room: Point Loma 2&3

In recent years, providing teaching professional development for graduate teaching assistants has become more common in mathematics departments in the US. Following this trend, mathematics education researchers have begun to conduct studies on professional development programs and on graduate students as novice teachers. The purpose of this literature review is to examine the current status of research in this field and make recommendations for future research on graduate teaching assistants and professional development. In examining the literature, we utilize an existing framework for collegiate teaching practices and focus on studies that attended to growth. As a result of this literature review, we recommend that researchers begin developing models or theories for how and why graduate students grow as teachers.

Proof Norms in Introduction to Proof Textbooks

Joshua Fagan, Texas State University; Kathleen Melhuish, Texas State University

Room: Coronado

We present a textual analysis of three of the most common introduction to proof (ITP) texts in an effort to explore proof norms as undergraduates are indoctrinated in mathematical practices. We focus on three areas that are emphasized in proof literature: warranting, proof frameworks, and informal instantiations. Each of these constructs have been connected to students' ability to construct, comprehend, or validate proofs. We carefully coded all the proofs and supplemental material across common sections in the textbooks. We found that the treatment of proof frameworks was inconsistent. We further found that textbook proofs rarely used explicit warranting and informal instantiations. We conclude by reflecting on the impact of inconsistent proof norms and unsubstantial focus on supportive proof components for students in ITP courses.

	Day	Time	Session Type
Session 5	**Thursday**	**17:20-17:50**	***Contributed Report***

Developing Strategic Competence With Representations for Growth Modeling in Calculus

Chris Plyley, University of the Virgin Islands; Celil Ekici, Texas A&M University-Corpus Christi

Room: La Jolla

Using inquiry based modules centered around growth modeling, we explored the development of strategic competence and representational fluency in undergraduate calculus. Building on student experiences and using multiple representations with discrete and continuous methods, we discuss the emerging substantial and problematic practices with representational fluency, communication, and strategic competence for modeling growth.

Professor Goals and Student Experiences in an IBL Real Analysis Course: A Case Study

Michael Oehrtman, Oklahoma State; Paul Dawkins, Northern Illinois University ; Ted Mahavier, Lamar University

Room: Coronado

Through an in-depth case study of one real analysis course taught by a very experienced instructor, we gain insight about two goals expressed by advocates of Inquiry Based Learning (IBL) instruction: developing students' persistence in mathematical study and their identity as mathematics learners. The research study was guided by collaborative workshopping research priorities and questions with of a group of experienced IBL instructors. We provide an in-depth characterization of this highly-experienced instructor's conceptualization of his teaching practice in undergraduate Real Analysis; specifically, we identify how his deviation from conventional proof-oriented instruction served to uphold his key goals that students create proofs and overcome challenges. We then use this characterization of his practice to report on students' experiences learning in the course, especially as related to the professor's two goals.

Replacing Exam with Self-Assessment: Reflection-Centred Learning Environment as a Tool to Promote Deep Learning

Juuso Nieminen, University of Helsinki; Jokke Häsä, University of Helsinki; Johanna Rämö, University of Helsinki; Laura Tuohilampi, University of Helsinki

Room: Del Mar

A new blended learning environment encompassing a wide variety of formative assessment was developed for a large undergraduate mathematics course to promote deep learning approach. In order to enhance reflection, the final exam was replaced by students' self-assessment. At the end of the course, a cluster analysis found four student clusters differing in their deep and surface learning approaches. Analysis of open feedback questions suggests that the contextual factor most commonly associated with deep learning approach was innovative assessment. Our findings lead us towards understanding how to foster deep learning approach in different kinds of learners.

Transforming students' definitions of function using a vending machine applet

Milan Sherman, Drake University; Jennifer Lovett, Middle Tennessee State University; Allison McCulloch, University of North Carolina - Charlotte; Lara Dick, Bucknell University; Cyndi Edgington, North Caroline State University; Stephanie Casey, Eastern

Room: Private Dining Room

The purpose of this study is to examine the understandings of functions that students developed and tested while engaging with a Vending Machine applet. The applet was designed to purposefully problematize common misconceptions associated with the algebraic nature of typical function machines. Findings indicate that the applet disrupts students' algebraic view of function and supports their transformation of meaning schemes for the function concept.

When "Negation" Impedes Argumentation: The Case of Dawn

Morgan Sellers, Arizona State University

Room: Bay Room

This study investigates one student's meanings for negations of various mathematical statements. The student, from a Transition-to-Proof course, participated in two clinical interviews in which she was asked to negate statements with either one quantifier or logical connective. Then, the student was asked to negate statements with a combination of quantifiers and logical connectives. Lastly, the student was presented with several complex mathematical statements from Calculus and was asked to determine if these statements were true or false on a case-by-case basis using a series of graphs. The results reveal that the student used the same rule for negation in both simple and complex mathematical statements when she was asked to negate each statement. However, when the student was asked to determine if statements were true or false, she relied on her meaning for the mathematical statement and formed a mathematically convincing argument.

		Day	Time	Session Type
Session	**Poster Session 1**	**Thursday**	**17:50-18:50**	***Poster***

Active vs. Traditional Learning in Calculus I

Beth Cory, Sam Houston State University; Taylor Martin, Sam Houston State University

T22

In this poster, we describe an ongoing study on the effect of active learning in Calculus I. We compare the achievement gap between underprepared and prepared students in the active versus traditional setting. Data comes from 16 sections of Calculus I during the 2017 – 2018 academic year, targeting the concepts of limits, continuity, differentiability, and area. We present our study design and initial findings; we look forward to feedback as we enter the latter half of our project.

Adjunct Instructor Learning Through Implementing Research Based Curriculum

Zareen Rahman, Montclair State University; Eileen Murray, Montclair State University; Amir Golnabi, Montclair State University

T6

Due to high demand, part-time adjunct instructors play an increasingly important role in introductory classes at many higher education institutions. As part of a project to support adjunct instructors teaching Precalculus, we are exploring the impact of course coordination and support on content knowledge and instructional practice of instructors.

An APOS Perspective of Meaning in Mathematics Teaching

Ahsan Chowdhury, Virginia Tech

T31

The 'meaning of' mathematics can be thought of as mathematical understandings whereas the 'meaning for' mathematics can be understood as understanding the significance of math for non-mathematical purposes. Studies have suggested instructors have difficulty addressing both senses of meaning simultaneously while other studies have indicated factors that affect graduate teaching assistants' (GTA) instruction. Using APOS theory as a theoretical lense, this study examines how these factors affect GTA instruction of the derivative and in turn, how GTAs navigate differing senses of meaning. Through interviews, the researcher found many parallels between GTA instruction and proposed decompositions of the derivative. Regarding meaning, the researcher found when GTAs experience tension between the two senses of meaning, instructional decisions may be taken that anticipate GTA instructional concerns.

An Examination of Preservice Mathematics Teachers Using Ratios and Proportions in a Social Justice Context

Gregory Downing, North Carolina State University; Brittney Black, North Carolina State University

T4

This study examines ways in which preservice teachers use mathematics in a social justice context. Using a mathematical task and social justice activity adapted from Gutstein and Peterson (2005), participants were asked to respond to questions surrounding their experience with using mathematics topics such as ratios and proportions in a social justice context. Using the preservice teachers' responses from pre- and post-surveys, researchers compared participants initial conceptions of teaching mathematics using a social justice lens to their views after completing a mathematical task involving social justice topics of world wealth and population disparity using ratios and proportions.

Benefits to Students of Team-Based Learning in Large Lecture Calculus

Travis Peters, Iowa State University; Elgin Johnston, Iowa State University; Heather Bolles, Iowa State University ; Craig Ogilvie, Iowa State University; Alexis Knaub, Western Michigan University; Thomas Holme, Iowa State University

T11

Team-Based Learning (TBL) is a specific form of active learning that utilizes the flipped classroom model. We implemented TBL in Calculus I in both large and small classes and investigated the impact of this form of instruction over two semesters. In the second semester, we observed many positive benefits to students, including exceptionally high class attendance, higher midterm and final exam scores, significantly lower DFW rates, and larger gains on the Calculus Concept Inventory when compared to students enrolled in non-TBL sections.

Calculus I Instructors' Desires to Improve Their Teaching

Kevin Watson, Virginia Tech; Sarah Kerrigan, Virginia Tech; Rachel Rupnow, Virginia Tech

T45

As calculus is a course required for many undergraduate programs, several studies over the past decade have examined aspects that create successful calculus programs in the United States. While many of these studies have looked at the teaching practices and beliefs of calculus instructors, none have focused on instructors' desires to improve their teaching. The goal of this research is to examine how desires to improve teaching vary among different types of instructors (GTAs, instructors, tenured faculty, etc.), and how institutional or departmental expectations might influence those desires.

Capture of Virtual Environments for Analysis of Immersive Experiences

Camden Bock, University of Maine

T29

This poster addresses emerging technologies for the capture and reconstruction of participants experiences in immersive virtual environments. These methods might improve communication of participants experiences when solving mathematical problems in three-dimensional contexts.

Children's Topological Thinking

Steven Greenstein, Montclair State University; Adam Anderson, Montclair State University

T24

This poster presents an ongoing investigation into children's topological thinking. Prior research identified and advanced children's informal ideas about topological equivalence – and equivalence and invariance more broadly. This investigation extends that research into children's thinking about related ideas such as order, separation, and coverings. Newly identified forms of geometric thinking have implications for the teaching and learning of geometry and for research into students' mathematical thinking.

Comparing Students' and Teachers' Descriptions of First Year STEM Instruction

Kristen Vroom, Portland State University; Sean Larsen, Portland State University

T33

The precalculus through single-variable calculus (P2C2) sequence is often viewed as a barrier for STEM intending students. Additionally, many students point to poor instruction as a primary reason for leaving STEM. This leads to many questions about student experiences in the P2C2 sequence. This study is part of a larger national project and draws on student and instructor survey data from three universities. We aim to lay groundwork for understanding student experiences in the P2C2 sequence by answering: (1) How do P2C2 students and instructors describe their class, (2) Do students and instructors describe them differently?

Computational Thinking in Mathematics: Undergraduate Student Perspectives

Chantal Buteau, Brock University; Ami Mamolo, University of Ontario Institute of Technology; Eric Muller, Brock University; Meghan Monaghan, Brock University

T18

Computational thinking (CT) is understood as the thinking, strategies, and approaches for solving complex problems with algorithmic considerations and in ways that can be executed by a computer. This survey study (n=104) reports on the conceptions of undergraduate mathematics majors and future mathematics teachers enrolled in a sequence of programming-based mathematics courses. Results suggest that students' emerging conceptions of CT became relatively well-aligned with expert views and that their characterization of CT included many computational practices (e.g. from modeling and simulation) and related affordances (e.g., creativity, agency) and outcomes (e.g., benefits learning, deeper mathematics understanding).

Connecting Physics Students' Conceptual Understanding to Symbolic Forms Using a Conceptual Blending Framework

Benjamin Schermerhorn, University of Maine; John Thompson, University of Maine

T23

In an effort to understand physics students' construction of equations in terms of mathematical structures, previous work has employed a symbolic forms framework. To account for students' contextual physics understanding related to these structures for vector differentials, we mapped symbolic forms into the framework of conceptual blending to model students' construction of equations. This allows us to shed further light on recent literature in this area.

Development of Students' Mathematical Discourse through Individual and Group Work with Nonstandard Problems on Existence and Uniqueness Theorems

Svitlana Rogovchenko, University of Agder; Yuriy Rogovchenko, University of Agder; Stephanie Treffert-Thomas, Loughborough University

T25

Research shows that students' learning is affected by the types of tasks. We explore how the use of nonstandard problems influences understanding of the Existence and Uniqueness Theorems (EUTs) by a group of engineering students. The focus is on the development of students' mathematical discourse during the individual and group work with nonstandard problems. We present the evidence indicating that students developed new mathematical routines gaining a deeper understanding of EUTs and appreciated the experience.

DISA – Digital Self-Assessment for Large University Courses

Jokke Häsä, University of Helsinki; Johanna Rämö, University of Helsinki; Juuso Nieminen, University of Helsinki

T35

We introduce a model for replacing the course exam with self-assessment on a large undergraduate mathematics course. In our course model, the self-assessment method is seen as a part of new learning environment that enhances the students' reflection skills and encourages them to foster their ownership of their own learning. Self-assessment skills are trained throughout the course and the students receive feedback from multiple sources, including teachers and peers. The DISA model is an important initiative to develop large course pedagogy in the university mathematics setting.

Dynamic Textbooks and their Use in Teaching Linear and Abstract Algebra

Angeliki Mali, University of Michigan; Vilma Mesa, University of Michigan

T10

In this poster, we present two analyses of two dynamic textbooks. One analysis attends to their dynamic features, the mathematical practices embedded, and the scope of contents. The second analysis uses the documentational approach (Gueudet & Trouche, 2009) to investigate the ways in which these textbooks are used by instructors and their students. Data collection involves seven instructors and nearly 150 students across four states (New York, Texas, California, and Michigan; 50% female, 30% non-Caucasian or Asian) and surveys, logs, student tests, classroom observations, and clinical interviews. In both textbooks the interactive features are prominent via links and interactive computational cells (with Sage). They both include deduction, symbolization, and representation as mathematical practices. There are differences in the scope of contents. Regarding use we found that instructors took advantage of the features only when those could be integrated into their usual practices.

Essential Aspects of Mathematics as a Practice in Research and Undergraduate Instruction

Eryn Stehr, Georgia Southern University; Tuyin An, Georgia Southern University

T13

A gap between mathematics as used by mathematicians and mathematics as experienced by undergraduate mathematics students has persistently been identified as problematic; A commonly proposed solution is to provide opportunities for students to do mathematics and be mathematicians (e. g., Whitehead, 1911; Harel, 2008). Conceptions or beliefs about what this means may vary depending on a mathematician's research and experience. The authors explore mathematicians' expressed conceptions of mathematics in their research and in their teaching.

Evaluating Perceptions of Proof with Comparative Judgement

Ben Davies, Loughborough University; Lara Alcock, Loughborough University; Ian Jones, Loughborough University

T17

This project offers a quantitative analysis of individuals' perceptions of proof. We used a comparative judgement-based approach to assign numerical value to individuals' written explanations of 'what mathematicians mean by proof'. This poster introduces comparative judgement as an assessment tool, then reports on two studies evaluating the relative quality of participants' explanations. Our research provides insight into perceptions of proof held by undergraduates and mathematicians, the perceptions rewarded by mathematicians in a comparative judgement setting, the reliability of mathematicians' judgements, and finally the differences in scores between mathematicians and undergraduates.

Exploring Neural Correlates for Levels of Cognitive Load During Justifying Tasks

Shiv Karunakaran, Michigan State University; Abigail Higgins, California State University Maritime Academy; James Whitbread, Jr., Washington State University

T42

A recent special issue of ZDM (June 2016) made the case for increasing the interdisciplinary collaboration between researchers in the fields of mathematics education and cognitive neuroscience. Specifically, Ansari and Lyons (2016) argued for increasing the "ecological validity of the testing situations and specific [neurocognitive] tests used to measure mathematical processing" (pp. 379-380). The study reported on in this poster serves as a "proof-of-concept" for the use of Functional Near-Infrared Spectroscopy (fNIRS) to measure the level of cognitive load of the brain under mathematical justifying. The poster will address the pros and cons of using neurocognitive measures, such as the fNIRS, to measure and examine the physiological stresses of the brain under the complex mathematical process of proving.

Faculty Collaboration and its Impact on Instructional Practice in Undergraduate Mathematics

Nicholas Fortune, North Carolina State University; Karen Keene, NC State University

T12

To reform instruction by moving towards student-centered approaches, research has shown that faculty need and could benefit from support and collaboration (Henderson, Beach, & Finkelstein, 2011; Speer & Wagner, 2009). In this qualitative instrumental case study I examine the ways in which a mathematician's instruction developed during his participation in a faculty collaboration geared towards reforming instruction and aligning it with inquiry oriented instruction (Kuster, Johnson, Keene, & Andrews-Larson, 2017; Rasmussen & Kwon, 2007). Preliminary results indicate ways in which student thinking was used as a discussion point in the faculty collaboration connected to the ways in which student thinking was used in the classroom to advance the mathematical agenda. Further, results indicate that the mathematical beliefs of the mathematician sometimes took precedence over the use of student work in the classroom.

Geometric Reasoning in an Undergraduate on the Autism Spectrum: A Magic Carpet Case

Jeffrey Truman, Virginia Tech

T38

In this report, I examine the unusually precise geometric reasoning of a student in linear algebra given the beginning of the Magic Carpet sequence outside of their normal curriculum. Analysis of possible reasons for taking this approach and implications for teaching are presented.

Graduate Teaching Assistants' Evolving Conceptualizations of Active Learning

Elijah Meyer, Montana State University; Elizabeth Arnold, Montana State University; Jennifer Green, Montana State University

T44

Graduate teaching assistants (GTAs) play a critical role in undergraduate mathematics education, but most have no experience using active learning to promote higher-order thinking. This research investigates how beginning GTAs conceptualize active learning and how these understandings evolve as they engage in a teaching program. This poster describes the program, as well as the evolution of and variation in GTAs' conceptualizations and uses of active learning.

Growth Mindset Assessments in Mathematics Classrooms

Hannah Lewis, Utah State University; Kady Schneiter, Utah State University

T30

Recent scientific evidence shows the incredible potential of the brain to grow and change. Equally important are the observations of the positive impact that having growth mindset has upon students' achievement. Students with a growth mindset view errors and obstacles as opportunities for growth. These students welcome challenges and the opportunity to learn from their mistakes. Although some university instructors are incorporating growth mindset into their lectures and attitudes, unfortunately, the traditional exam method used in undergraduate mathematics classrooms is a fixed mindset model. This poster shows the implementation of a growth mindset structured exam in a multivariable calculus class. The implementation incudes structured opportunities to rework exam problems, give presentations, and papers. All of these focus on assessing the student's achievement of the objectives in the class.

Is Mathematics Important for Accounting Learning? – An study on Students' Attitudes and Beliefs

Ruixue Du, University of Wisconsin Stevens Point; Senfeng Liang, University of Wisconsin Stevens Point; Christine Schalow, University of Wisconsin Stevens Point

T47

This study examined students' attitudes toward mathematics, and beliefs in mathematics' influence on accounting learning. It also explored how these two factors correlate to students' scores in accounting courses. This study found that students believed that being good at mathematics is a necessary, but not a sufficient, condition for performing well in accounting. Students who performed well in accounting usually showed a relatively positive attitude towards mathematics.

Learning Assistants in Undergraduate Mathematics Courses: Impacts on Retention Rates

Natasha Speer, University of Maine; Laura Millay, University of Maine; Erin Vinson, University of Maine

T20

Research findings indicate that active, engaged student learning approaches to instruction improve outcomes for science, technology, engineering and mathematics (STEM) undergraduate students. We report findings about a program that supports faculty as they implement active learning utilizing undergraduate learning assistant. In particular, we found institutional retention from freshman to sophomore year was improved for students who completed two or more mathematics courses where learning assistants were utilized as part of an active approach to instruction. Implications and further research into these findings are discussed.

Mathematical Reasoning and Proving for Prospective Secondary Teachers

Orly Buchbinder, UNIVERSITY OF NEW HAMPSHIRE; Sharon McCrone, University of New Hampshire

T15

The design-based research approach was used to develop and study a novel capstone course: Mathematical Reasoning and Proving for Secondary Teachers. The course aimed to enhance prospective secondary teachers' (PSTs) content and pedagogical knowledge by emphasizing reasoning and proving as an overarching approach for teaching mathematics at all levels. The course focused on four proof-themes: quantified statements, conditional statements, direct proof and indirect reasoning. The PSTs strengthened their own knowledge of these themes, and then developed and taught in local schools a lesson incorporating the proof-theme within an ongoing mathematical topic. Analysis of the first-year data shows enhancements of PSTs' content and pedagogical knowledge specific to proving.

Mathematics Tutors' Perceptions of Their Role

Christopher McDonald, Oklahoma State University; Melissa Mills, Oklahoma State University

T41

Undergraduate students who work as mathematics tutors completed surveys and interviews to assess their attitudes towards mathematics and their beliefs about the roles of a tutor and instructor. Tutors in this study viewed their role as supplementary to that of a teacher, and emphasized tutors' ability to tailor their mathematical content to the individual.

MPWR-ing Women in RUME: Continuing Support

Stacy Musgrave, Cal Poly Pomona; Jessica Ellis, Colorado State University; Kathleen Melhuish, Texas State University; Eva Thanheiser, Portland State University; Megan Wawro, Virginia Tech

T1

For the past four years, we have run the MPWR Seminar, a daylong mentoring and networking event for women in RUME. Each year, we have hosted 60 – 90 women at various career stages (graduate students, postdoctoral fellows, faculty and professionals outside of academic positions). The seminar takes place the day before the annual RUME conference, allowing most of the participants to continue engaging with each other throughout the subsequent three days during RUME. In this poster, we address the motivation for the seminar, the structure and topics from the 2017 seminar, modifications in the structure for sustainability purposes, and research related to the efficacy and transferability of MPWR. Our aim in sharing this poster is to disseminate our experiences and gather input from the community on what more could be done.

Multivariable Calculus Textbook Analysis Highlights a Lack of Representation for Non-Cartesian Coordinate Systems

Chaelee Dalton, Pomona College; Brian Farlow, North Dakota State University; Warren Christensen, North Dakota State University

T5

Upper-division undergraduate physics coursework necessitates a grasp of mathematical knowledge, including an understanding of non-Cartesian coordinate systems. To fully grasp what upper-division physics' students understanding of non-Cartesian coordinates is, it is worthwhile to study the mathematics course where non-Cartesian coordinate systems are taught most extensively, Multivariable Calculus. Seven Multivariable Calculus textbooks were examined using content analysis techniques. Additionally, textbook items in four textbooks were qualitatively coded by coordinate system. Results indicate that there were few instances where non-Cartesian coordinate systems were present. These findings suggest that before upper-division physics coursework, students' instruction on non-Cartesian coordinate systems is minimal and that it might be difficult for students to employ mathematical techniques that involve non-Cartesian coordinates in their upper-division courses.

Perceptions of Underrepresented Community College STEM Majors

Daniel Lopez, Rutgers University

T14

Community College STEM majors from underrepresented groups were interviewed about their experiences in math classes and their motivations behind choosing a STEM major. The goal was to uncover events that may occur in math classrooms that serve to marginalize underrepresented students and contribute to the dearth of these demographic groups in these majors. As interview data was gathered, it became clear that these students did not suffer from feelings of marginalization. Results suggest that involvement in co-curricular activities, guidance and encouragement from faculty members, and support from family and peers may serve to mitigate feelings of alienation that can occur in students from these underrepresented groups.

Perspectives in the Use of Primary Sources in Undergraduate Mathematics Education: A Triangulation of Author, Instructor, and Student

Matthew Mauntel, Florida State University; Kathleen M Clark, FSU

T43

We report on a case study of two different university mathematics classes (both Linear Algebra courses) that implemented the same primary source project (PSP) as part of the Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS) project. One class was taught by the author of the PSP and the other took place at a second university. Data were collected in the form of pre- and post-surveys with Likert items and open-ended items designed to assessed the mathematical attitudes and perceived gains of the students. Instructors completed a PSP implementation report and post-course survey. The author provided "instructor notes," which included plans for intended implementation of the PSP. In our poster presentation, we provide a triangulation of the data from among the perspectives of the author as an instructor, a non-author instructor of a same course (Linear Algebra), and the students.

Pre-Service Teachers' Mathematical Understanding of the Area of a Rectangle

Betsy McNeal, Ohio State University; sayonita Ghosh Hajra, Hamline University; Ayse Ozturk, Ohio State University; Wyatt Ehlke, Hamline University; Michael Battista, Ohio State University

T8

This poster will share contrasting responses of two pre-service teachers (PTs) to problems that were part of an ongoing study of PTs' conceptions of area of a rectangle. They were asked to a) find the area of a rectangle in terms of a non-square rectangular unit and relate that to multiplication, and b) interpret a fictional child's attempt to connect the area formula with counting square units. These cases showed that an ability to explain a systematic covering of a 2D space with an area unit does not imply an ability to respond to a student who might think "the corner square gets counted twice". Further, the ability to describe this structure, L rows with W area units per row, does not imply readiness to understand the area formula for a rectangle.

Preservice Secondary Mathematics Teachers' Conceptions of the Nature of Theorems in Geometry

Tuyin An, Georgia Southern University

T39

Proof plays an important role in school mathematics curriculum across grade levels and content areas. Being able to understand and apply the axiomatic system, such as with theorems, is considered as a high level of proof and reasoning ability in geometry. By adopting a collective case study design, I investigated preservice secondary mathematics teachers' (PSMTs) conceptions of theorems in geometry, in order to develop knowledge about PSMTs' current conceptions and provide mathematics educators and researchers with a possible means to unpack PSMTs' conceptions. This proposal focuses on one dimension of PSMTs' conceptions, the nature of theorems (NoT) in geometry. The Findings include interpretations of PSMTs' conceptions of the NoT, in terms of the ways they claimed the truth of mathematical statements, examined the validity of given proofs, and disproved given statements, as well as the role of task-based interviews in understanding their conceptions.

Queer Students in STEM: The Voices of Amber, Charles, Jenny and Juan

Matthew Voigt, San Diego State University

T26

This report provides a rich narrative documenting the experiences of four queer students in STEM, which showcase both the challenges and power of being queer in STEM. Students viewed the nature of STEM through a paradoxical lens of a discipline that is objective and thus neutral to issues of identity, yet hostile and exclusionary to non-normative identities in STEM spaces. In response, queer students in undergraduate math courses described the difficulties in navigating the amount of personal information they reveal about themselves or be faced with the psychological burden or cognitive stress derived from presenting in non-normative ways.

Red X's and Green Checks: A Preliminary Study of Student Learning from Online Homework

Allison Dorko, Oklahoma State University

T27

Homework is thought to play an important role in students' learning of mathematics, but few studies have addressed what, if anything, students learn from doing online homework. This study is a preliminary attempt to answer the question do students learn what instructors intended they learn from an online homework assignment about sequences?

Reflections on a Peer-Led Mentorship Program for Graduate Teaching Assistants

Laura Broley, Concordia University; Sarah Mathieu-Soucy, Concordia University; Ryan Gibara, Concordia University; Nadia Hardy, Concordia University

T28

This poster will describe a peer-led mentorship program offered to Graduate Teaching Assistants (GTAs) in a Canadian University. We present the creators' rationale for implementing this program, as well as the perspectives of the two graduate student peer mentors who have taken the lead in its development and implementation.

Relationships Between Calculus Students' Ways of Coordinating Units and their Ways of Understanding Integration

Jeffrey Grabhorn, Portland State University; Steven Boyce, Portland State University; Cameron Byerley, Colorado State University

T19

This poster describes results from a paired-student teaching experiment focused on college calculus students' understandings of integration. Our aim was to model relationships between students' covariational reasoning, quantitative reasoning, and numerical reasoning as they were developing meanings for integration, via weekly teaching sessions that were concurrent but independent from students' second-term calculus course. We discuss commonalities between students' ways of reasoning multiplicatively, ways of reasoning about linear rates of change, and ways of understanding integration.

Research on Concept-based Instruction of Calculus

xuefen gao, ZSTU

T40

This study, involving 254 college-level calculus students and 3 teachers, investigated the misunderstanding of concepts in calculus and designed concept-based instruction to help students understand concepts. Multiple achievement measures were used to determine the degree to which students from different instructional environments had mastered the concepts and the procedures. The midterm examination and the final examination results showed that the students enrolled in the concept-based learning environment scored higher than the students enrolled in the traditional learning environment and the investigation at the end of the semester showed that most of students like the concept-based learning environment.

Seminars to Support Learning Assistants in Mathematics

Nancy Kress, University of Colorado, Boulder

T46

This poster reports on the design and implementation of a series of seminars to support undergraduate learning assistants (ULAs) working in university mathematics courses. The ULAs participating in this study work as tutors and classroom assistants in early college mathematics courses through Calculus 2. The seminars support ULAs to more fully understand their roles and to consider ways that they can improve equity and access for all students in their classes. The work is grounded in the perspective that learning is a sociocultural process (Lave, 1991) and that students' learning is significantly impacted by opportunities to participate in actively doing mathematics (Laursen, 2014; Freeman et al, 2014). This project strives to answer research questions related to understanding how ULAs conceptualize teaching for equity and access, how conceptualizations change through participation in seminars, and how these conceptualizations are related to students' experiences in mathematics classes.

Student reasoning with complex numbers in upper-division physics

Michael Loverude, California State University Fullert

T48

Abstract: Students encounter complex numbers in many physics courses. In particular physics uses complex exponentials to describe oscillatory phenomena and requires that students use multiple representations (algebraic, x vs t graphs, complex plane). In this poster we will examine student responses suggesting difficulties with the connection between complex numbers and oscillation, drawn from students in upper-division physics courses in math methods.

Students' Engagement with a Function Vending Machine Applet

Patrick Martin, Middle Tennessee State University; Heather Soled, Bucknell University; Jennifer Lovett, Middle Tennessee State University; Lara Dick, Bucknell University

T7

The purpose of this study is to examine the ways students engage with a Vending Machine applet designed to problematize common misconceptions associated with the function concept. Findings indicated a need to redesign the applet to further disrupt students' misconceptions of the concept of function. Design decisions for the redesigned applet and the new version will be shared.

Students' Experiences in an Undergraduate Mathematics Class: Case Studies from one Student-Centered Precalculus Course

Brooke Outlaw, North Carolina State University; Gregory Downing, North Carolina State University

T3

Using evidenced-based practices in a large undergraduate mathematics classroom demands further investigation as there is still not significant work in this area. Results from this case study show that students perceived that their participation in student-centered instruction in an undergraduate Precalculus course, was helpful to their learning. The results also suggest that students demonstrated positive attitudes in regard to the collaborative efforts active learning components including Team Activities were utilized in this course and that the strategies were considered useful and important by the students interviewed.

Support for Active Learning in Introductory Calculus: Exploring the Relationship between Mathematics Identity and Pedagogical Approaches

Paran Norton, Clemson University

T34

Calculus I is a main gatekeeper course for STEM majors, so increasing student success in this course is imperative to retaining more students in STEM fields. Since students' mathematics identity is a strong predictor of pursing a STEM career, more information is needed about how students are developing these identities in introductory mathematics courses. This study reports a piece of a larger mixed methods approach to gain more insight into how instructional approaches in introductory calculus are related to students' mathematics identity development. Interviews were conducted to explore students' perceptions of the pedagogy used in their introductory calculus class. Students' descriptions of their mathematics identity, which includes the constructs of interest, recognition, and performance/competence beliefs are discussed and compared between an active learning environment and a traditional lecture classroom.

The Development of a Video Coding Instrument for Assessing Instructional Quality in Community College Algebra Classrooms

Dexter Lim, University of Minnesota

T37

This poster presents the evolution of a video coding protocol for mathematics classroom instruction. We highlight challenges encountered while analyzing 18 hours of pilot data from six community college algebra classrooms, entailing calibration of over 150 episodes.

The Distribution of the Mathematical Work during One-on-one Tutor Problem Solving

Linda Burks, Santa Clara University; Carolyn James, University of Portland

T21

Undergraduate math tutoring is an important context for student learning, yet little empirical work has been done to understand tutor-student interactions. Using frameworks for problem-solving and socially mediated metacognition (Carlson and Bloom, 2005; Goos, et. al, 2002), this poster examines who guides the development of mathematical ideas throughout the problem solving process within a drop-in one-on-one tutoring context. We found that the majority of the tutoring interactions closely followed the Orienting-Planning-Executing-Checking phases of problem solving. The "Executing" phase had the highest degree of student contribution, while the "Checking" phase was least represented.

The STEM Service Courses Initiative of Project PROMESAS: Pathways with Regional Outreach and Mathematics Excellence for Student Achievement in STEM

Hortensia Soto, University of Northern Colorado; Cynthia Wyles, California State University Channel Islands; Selenne Banuelos, California State University Channel Islands

T16

In this poster, we present preliminary results of how Project PROMESAS' STEM Service Courses (SSC) initiative assists collegiate instructors (n = 14) in transforming their teaching of Calculus I. These instructors participated in a one-week summer institute focused on integrating student-centered activities via rich tasks and promoting a sense of community in the classroom. During the fall 2017 semester, they adopted and adapted ideas from the summer institute and participated in monthly day-long follow-up meetings. The monthly meetings were an opportunity to continue learning about student-centered activities and to share newly created teaching materials. As part of the evaluation of this project, the instructors journaled during the summer institute and each month during the fall. These journal entries serve as our data: it suggests that although our instructors struggle balancing student-centered activities with teaching the required content, they are committed to transforming their teaching.

Transitional Conceptions of the Orientation of the Cross Product in CalcPlot3D

Monica VanDieren, Robert Morris University; Deborah Moore-Russo, University of Oklahoma; Paul Seeburger, Monroe Community College; Jill Wilsey, Niagara County Community College

T9

Students struggle with computing the direction of the cross product in relation to the two vectors that form it, but very little research has involved a non-contextual geometric cross product activity, especially in an online context. This study uses grounded theory to analyze student work completed for a dynamic, online visualization activity. Our preliminary research aims to develop categories that could outline a conceptual model of student understanding of the cross product.

University Teachers' Meanings for Average Rate of Change: Impacts on Student Feedback

Ian Thackray, University of Maine

T32

Previous research that has used Thompson's mathematical "meanings" framework has focused on secondary teachers' meanings for mathematics. We examine the meanings that graduate students and professors hold for average rate of change. Further, we attempt to connect meanings to the facets of student work that graduate students and professors notice. This work lets us start to extend the meanings framework and has implications for graduate student professional development.

Using Comparative Judgment to Analyze Precalculus Algebra Exam Tasks

Kaitlyn Serbin, Virginia Polytechnic Institute and State University

T36

Mathematics educators hold varying views on the teaching and learning of mathematics. Literature revealed inconsistencies in educators' interpretations of conceptual understanding and procedural fluency. To explore these differences in perspectives, the current study asked mathematics doctoral faculty, college instructors, graduate teaching assistants, and high school teachers to analyze undergraduate Precalculus Algebra final exam items. Participants were asked to compare exam items based on their effectiveness to gauge conceptual understanding, procedural fluency, or differences between students. The method of comparative judgment was used to yield an ordered rank of the items from least effective to most effective, as perceived by the judges. The weak ordinal association between the rankings of the different groups indicated disagreement in educators' judgments. The factors contributing to these differences in perceptions are unknown. Further research includes exploring these discrepancies in item rankings amongst different subgroups of mathematics educators.

What are the Functions of Proof in Introduction to Proof Textbooks

Elizabeth Hewer, Texas State University; Kathleen Melhuish, Texas State University

T2

The role of proof in mathematics is multifaceted. de Villiers (1990) argues students should be exposed to at least five essential roles: verifying, explaining, systematizing, discovering, and communicating. We analyzed six commonly-used Introduction to Proof (ITP) textbooks for their treatment of the function of proof. We used a thematic analysis approach leveraging de Villiers categorizations. We found that verification was heavily emphasized, while discovery was almost excluded. However, the textbooks emphasized verifying, explaining, systematizing, discovering, and communication in ways that de Villiers did not. For example, de Villiers focused on the global view of systematizing, where the books were more likely to emphasize the local structure of proof.

Day	Time	Session Type
Thursday	**19:00-20:00**	***Meal***

Dinner

Room: Point Loma 2&3

Day	Time	Session Type
Thursday	**20:00-21:00**	***Plenary***

Blending Chemical and Mathematical Knowledge: Explorations in Solving Rate Problems in Chemistry

Marcy Towns

Room: Point Loma 2&3

Answering recent calls for discipline-based education research and interdisciplinary work, this study seeks to investigate how students integrate chemistry and mathematics during problem solving in chemical kinetics, a context that has received little attention in the literature. This work investigates the following research question: In what ways do chemistry and mathematics knowledge interact as students engage in chemical kinetics problem solving? Personal constructs, a blend of personal and social constructivism, served as the theoretical framework for this study, which involved semi-structured interviews using a think-aloud protocol with 36 general chemistry students, 5 upper-level physical chemistry students, and 3 chemical engineering students. Audio and written data were collected using a Livescribe pen, which were then combined to create interpreted narratives for each student. Blended processing, a theory from cognitive science that characterizes human knowledge integration, was used as a methodological framework to guide the analysis. Open coding revealed themes relating to common topics that were discussed when blending occurred; variation in the depth of blending was also characterized. Results provide implications for supporting student problem solving and the modeling of

	Day	Time	Session Type
	Friday	**7:00-8:00**	***Meal***

Breakfast

Room: Pt. Loma Terrace

	Day	Time	Session Type
Session 6	**Friday**	**8:00-8:30**	***Preliminary Report***

A Framework for Analyzing Written Curriculum from a Shape-Thinking and (Co)variational Reasoning Perspective

Halil Tasova, University of Georgia; Irma Stevens, University of Georgia; Kevin Moore, University of Georgia

Room: Point Loma 2&3

This preliminary study provides a framework to analyze the extent and nature of (co)variational and quantitative reasoning in written curriculum. In order to test and refine our framework, we examined both the narratives and worked examples in calculus textbooks on lessons dealing with the topic of functions. We present examples from those textbooks to illustrate the categories of our framework. We conclude with questions concerning potential areas to improve our framework.

Building Lasting Relationships: Inquiry-Oriented Instructional Measure Practices

Rachel Rupnow, Virginia Tech; Tiffany LaCroix, Virginia Tech; Brooke Mullins, Virginia Tech

Room: Point Loma 1

This study examines the relationships between instructional practices in the Inquiry-Oriented Instructional Measure (IOIM). The IOIM consists of seven practices developed from four guiding principles of Inquiry-Oriented (IO) instruction: generating student ways of reasoning, building on student contributions, developing a shared understanding, and connecting to standard mathematical language and notation. A 2-tailed correlation test was applied to IOIM scores from 36 instructors and found six of the practices had strong positive correlations to each other and the seventh had a moderate positive correlation. This suggests the IOIM is a valid measure. A regression analysis was conducted to determine if any practice predicted the others and found practice one significantly predicted the others. Finally, Cronbach alpha was calculated indicating the IOIM is a reliable measure. Together, these findings indicate the IOIM is a valid and reliable measure for IO instructional practices.

Exploring the Pedagogical Empathy of Mathematics Graduate Teaching Assistants

Karina Uhing, University of Nebraska - Lincoln

Room: La Jolla

Mathematics graduate teaching assistants (GTAs) are an important part of the mathematics education community. Recently, there has been a concentrated effort to better understand GTAs' pedagogical beliefs and teaching practices. The purpose of this study is to explore how GTAs would respond to student feelings and if their feedback to student questions can be characterized as attending to emotion. Data was collected through interviews of current GTAs in which participants were shown samples of student work and asked to respond to questions about that work. Preliminary analysis has revealed varying abilities of GTAs to express student feelings.

How may Fostering Creativity Impact Student Self-efficacy for Proving?

Paul Regier, University of Oklahoma; Milos Savic, University of Oklahoma

Room: Del Mar

Mathematical creativity has been emphasized by mathematicians as an essential piece of doing mathematics, yet little research has been done to study the effects of fostering creativity in the undergraduate classroom. In this paper, we seek to understand creativity in the classroom using Sriraman's (2005) five principles for fostering mathematical creativity by studying how these principles impact student self-efficacy for proving. Using online student surveys, interviews, and classroom observation, we demonstrate how Sriraman's five principles and Bandura's four sources of self-efficacy may be used to explore the impact of fostering mathematical creativity on student's self-efficacy for proving. Then from the interviews, we highlight how the use of the free market and scholarly principles may influence student self-efficacy via vicarious role-modeling, and explain why the use of these two principles may be of particular importance in fostering student self-efficacy for proving.

Investigating Students' Meta-Level Object-Reflections and Discourse-Reflections: The Provocative Power of Primary Historical Sources

Cihan Can, Florida State University; Janet Barnett, Colorado State University - Pueblo; Kathleen Clark, Florida State University

Room: Bay Room

We discuss research based on Sfard's theory of mathematics as a discourse, which we used to investigate the potential of engagement with primary historical sources for motivating undergraduate students to participate in and accept new mathematical discourses. This preliminary report focuses on characterizing the nature of students' participation in mathematical discourse in their written work on primary source projects (PSPs), as well as the question of what constitutes evidence of students' noticing of meta-level rules in that work. We present our analysis of a brief excerpt from one PSP, and provide an analysis of two student work samples to exhibit students' object- and discourse-reflections at the meta-level.

Mathematical Competencies and E-Learning: A Case Study of Engineering Students' Use of Digital Resources

Shaista Kanwal, Universitetet i Agder

Room: Private Dining Room

This paper explores how an e-learning environment affords the execution of mathematical competencies in an undergraduate engineering context. Considering the students' mathematical practice as action mediated by the digital resources in a sociocultural sense, we employ the competence framework by (Niss & Højgaard, 2011) to make sense of students' learning. Case-study research design has been implemented to thoroughly observe the mathematical practices of a small group of participants. Observing students' group work and following their mathematical discussions elucidated the way this environment afforded the execution of competencies. Closer analysis revealed that the availability of online tools in this environment has the twofold effects on mathematical thinking, mathematical reasoning and problem-tackling competencies.

Prerequisite Knowledge of Mathematics and Success in Calculus I

Lori Ogden, West Virginia University; Jennifer Kearns West Virginia University; Nicholas Bowman, West Virginia University

Room: Coronado

This preliminary report describes how prerequisite content knowledge is related to success in a first semester calculus course. Data collected included adaptive assessments administered in both Pre-Calculus and Calculus I, standardized test scores, prior enrollment in Pre-Calculus, prior enrollment in Calculus I, and final grades in Calculus I. Analysis revealed that (1) standardized metrics such as ACT, SAT, and placement test scores did not reliably predict students' success in Calculus I, (2) passing Pre-Calculus directly impacted students' prerequisite content knowledge which in turn led to a stronger performance in Calculus I, and (3) students lost a significant amount of knowledge between the end of Pre-Calculus and the beginning of Calculus I. Lastly, in an effort to identify how deficits in specific knowledge domains impact student performance in Calculus I, additional analysis revealed that students' ability to graph trigonometric functions was most predictive of their performance in Calculus I.

	Day	Time	Session Type
Session 7	**Friday**	**8:40-9:10**	***Theoretical Report***

Drawing on Three Fields of Education Research to Frame the Development of Digital Games for Inquiry-Oriented Linear Algebra

Michelle Zandieh, Arizona State University; David Plaxco, Clayton State University; Caro Williams-Pierce, University at Albany, SUNY; Ashish Amresh, Arizona State University

Room: Point Loma 2&3

Demands in undergraduate education are shifting to reach larger student populations - especially learners beyond the brick-and-mortar classroom - which has led to more pressing demands to incorporate technologies that afford such learners access to high-quality, research-based, digital instructional materials. In this article, we explore three theoretical perspectives that inform the development of such instructional materials. In our team's efforts to develop a game-based learning applet for an existing inquiry-oriented curriculum, we have sought to theoretically frame our approach so that we can draw on the corpus of researcher knowledge from multiple disciplines. Accordingly, we will discuss three bodies of literature – realistic mathematics education's [RME's] approach to curriculum development, inquiry-oriented instruction and inquiry-based learning [IO/IBL], and game-based learning [GBL] - and draw on parallels across the three in order to form a coherent approach to developing digital games that draw on expertise in each field.

Finite Mathematics Students' Use of Counting Techniques in Probability Applications

Kayla Blyman, USMA - Westpoint; Casey Monday, Northern Kentucky University

Room: Del Mar

In this study we seek to better understand how students are using counting techniques within the context of the probability application. To do so we investigate three semesters of finite mathematics students' use of enumeration, Venn diagrams, and counting formulas on probability free-response exam questions at a large public university in the mid-south. The study found that appropriate use of enumeration techniques and Venn diagrams both statistically significantly increased a student's likelihood of arriving at a correct answer, while there is statistically significant evidence that the use of counting formulas decreased a student's likelihood of arriving at a correct answer. We conclude with a discussion of the implications of this study for the practice.

Future Middle Grades Teachers' Solution Methods on Proportional Relationship Tasks

Merve Kursav, Michigan State University; Sheri Johnson, University of Georgia

Room: Private Dining Room

This study examines the solution methods that future middle grades teachers chose when solving a problem on proportional relationships. The examination of the solution methods was framed by a new perspective on proportional reasoning that connects multiplication, division, and proportional relationships into a coherent framework. This framework places emphasis on multiple batches and variable parts. The data were collected from a sample of 22 future middle grades teachers' exams completed as part of a content course at a large university in the Southeastern United States. Findings revealed that future middle grades teachers utilized strategies involving multiple batches and variable parts after completing a two-semester sequence of mathematics content courses on proportional relationships tasks. Keywords: Proportional relationship, Proportional reasoning, Variable parts perspective, Multiple batches perspective

Generalisation, Assimilation, and Accommodation

Allison Dorko, Oklahoma State University

Room: La Jolla

This paper builds theory by connecting Piaget's assimilation and accommodation constructs to Harel and Tall's (1991) framework for generalisation in advanced mathematics. Based on what they imagined to be the cognitive processes underlying generalisation, Harel and Tall proposed that generalisation might be expansive (occurring when a student expands the applicability range of an existing schema without reconstructing it), reconstructive (occurring when a student reconstructs a schema to widen its range of applicability), or disjunctive (occurring when a student constructs a new, disjoint schema to deal with a new context). I contend that expansive and reconstructive generalisation align with assimilation and accommodation, respectively. I provide 'proof of concept' using data from a study of students' generalisation of graphing from R2 to R3. Further, I show how linking Piagetian constructs to Harel and Tall's work provides a theoretical explanation for other empirical findings about generalisation.

Key Memorable Events During Undergraduate Classroom Learning

Ofer Marmur, Simon Fraser University

Room: Coronado

This paper presents a theoretical construct termed Key Memorable Events (KMEs): classroom events that are perceived by students as memorable and meaningful in support of their learning, and are typically accompanied by strong emotions, either positive or negative. As such, the proposed concept implies a hierarchy between different events and their contribution to the learning by focusing on those moments perceived by students as most significant. The concept is exemplified in context of large-group undergraduate-calculus tutorials. Theoretical and pedagogical implications are discussed in terms of lesson design, data analysis, and conceptualization of learning in the undergraduate classroom.

Partitioning a Proof: An Exploratory Study on Undergraduates Comprehension of Proofs

Eyob Demeke, California State University Los Angeles; David Earls, University of New Hampshire

Room: Point Loma 1

In this paper, we explore eleven undergraduate students' comprehension of two proofs taken from an undergraduate abstract algebra course. Our interpretation of what it means to understand a proof is based on a proof comprehension model developed by Mejia-Ramos, et al. (2012). This study in particular examines the extent to which undergraduate students are able to modularize a proof using the proof's key ideas. Additionally, eleven doctoral students in mathematics, referred in this paper as experts, were asked to provide modular structures for the same proofs that the undergraduate students received. We employed experts' modular structures of the proofs to analyze that of undergraduates'. The main finding of the study is that, contrary to experts' proof modularization, undergraduates partitioned the proofs in a way that failed to highlight how key components of the proofs are logically linked, suggesting an inadequate proof comprehension.

	Day	Time	Session Type
Session 8	**Friday**	**9:20-9:50**	***Contributed Report***

An Undergraduate Mathematics Student's Counterexample Generation Process

Kristen Lew, Texas State University; Dov Zazkis, Arizona State University

Room: Point Loma 1

This paper illustrates the processes and struggles involved in a student's generation of a counterexample. The data involves one student's at-home proving while working on homework for his introduction-to-proof course. In this paper, we present an episode where a student engaged in substantive efforts in order to generate a proof by counterexample. We compare and contrast this episode against results from the literature on example generation to provide insights regarding the similarities and differences between example and counterexample generation as they relate to proof.

Building Models of Students' Use of Sigma Notation

Kristen Vroom, Portland State University; Sean Larsen, Portland State University; Stephen Strand, CSU Chico

Room: Private Dining Room

Summation notation is a widely-used standard that can represent all kinds of sums. Despite its utility, the literature on this topic points to the notation being difficult for students. Our research project gives insight into how students think about summation notation and why it is so challenging. This report builds off of the first phase in our project, which proved the existence of students' uncertainties with elements of the notation. Survey data from 285 undergraduates suggested that uncertainties are common amongst students. We also found that the act of encoding a sum in sigma notation is more cognitively demanding than interpreting a summation notation expression. In this paper we present models of students' ways of thinking about summation notation.

Katlyn's Inverse Dilemma: School Mathematics Versus Quantitative Reasoning

Teo Paoletti, Montclair State University

Room: La Jolla

In this report, I examine the interplay between Katlyn's (an undergraduate student's) inverse relation (and function) meanings developed through her continued school experiences and her reasoning about relationships between quantities. I first summarize the literature on students' inverse function meanings and then provide my theoretical perspective, including a description of a quantitative approach to inverse relations (and functions). I then present Katlyn's activities in a teaching experiment designed to support her in reasoning about a relation and its inverse relation as representing an invariant relationship. Although she engaged in such reasoning, her continued school mathematics experiences constrained her in reorganizing her inverse function meanings. I conclude with a discussion and areas for future research.

Networking Theories to Design Dynamic Covariation Techtivities for College Algebra Students

Heather Johnson, University of Colorado Denver; Evan McClintock, University of Colorado Denver; Jeremiah Kalir, University of Colorado Denver; Bary Olson, University of Colorado Denver

Room: Coronado

Covariational reasoning is a challenging form of reasoning for undergraduate students to develop and employ. Yet, students' lack of opportunities to use covariational reasoning may account, in part, for some of their difficulties. Building from the work of mathematics education researchers (e.g., Kaput, Thompson, Moore), we developed a suite of Techtivities—free, accessible, digital media activities linking dynamic animations and graphs. Using a Cannon Man Techtivity to illustrate, we provide four key design components and three theoretically based design principles underlying the Techtivities. To inform design both within and across the Techtivities, we network theories of different grain sizes: Thompson's theory of quantitative reasoning and Marton's variation theory. Developing Techtivities for students in the gatekeeping course, College Algebra, we intend to expand students' opportunities to employ covariational reasoning. We discuss implications stemming from students' opportunities to use free, accessible digital media activites, such as Techtivities, to promote their covariational reasoning.

Performance and Participation Differences for In-Class and Online Administration of Low-Stakes Research-Based Assessments

Ben Van Dusen, CSU Chico; Jayson Nissen, CSU Chico; Manher Jariwala, Boston University; Xochith Herrera, CSU Chico; Close Eleanor, Texas State San Marcos

Room: Del Mar

Research-based assessments (RBAs), such as the Calculus Concept Inventory, have played central roles in many course transformations from traditional lecture-based instruction to research-based teaching methods. In order to support instructors in assessing their courses, the online Learning About STEM Student Outcomes (LASSO) platform simplifies administering, scoring, and interpreting RBAs. Reducing barriers to using RBAs will support more instructors assessing the efficacy of their courses and transforming their courses to improve student outcomes. The purpose of this study was to investigate the extent to which RBAs administered online and outside of class with the LASSO platform provided equivalent quantity and quality of data to traditional paper and pencil tests administered in class for both student performance and participation. We used an experimental design to investigate the differences between these two test modes. Results indicated that the LASSO platform can provide equivalent quantity and quality of data to paper and pencil tests.

The next time around: Shifts in argumentation in initial and subsequent implementations of inquiry-oriented instructional materials

Christine Andrews-Larson, Florida State University; Shelby McCrackin, Florida State University

Room: Point Loma 2&3

Considerable learning is entailed in adopting an inquiry-oriented approach to teaching a class. In this analysis, we examine classroom video data of three instructors' initial implementation of an inquiry-oriented instructional unit and their implementation of the same unit one year later. We document consistent increases in instances of eliciting and building on student contributions across tasks and instructors, and use Toulmin's argumentation scheme to offer an illustration of how classroom discussions became more mathematically robust and student-centered from initial to subsequent implementations. Implications for instructor learning are discussed.

Day	Time	Session Type
Friday	**9:50-10:15**	***Break***

Coffee Break

Room: Cabo Courtyard (Weather Permitting)

	Day	Time	Session Type
Session 9	**Friday**	**10:20-10:50**	***Contributed Report***

An APOS Study on Undergraduates' Understanding of Direct Variation: Mental Constructions and the Influence of Computer Programming

Cynthia Stenger, University of North Alabama; James Jerkins, University of North Alabama; Jessica Stovall, University of North Alabama; Janet Jenkins, University of North Alabama

Room: Bay Room

This study explores undergraduates' understanding of direct variation before and after instruction using computer programming to teach generalization over the concept. Based on an initial genetic decomposition for direct variation, the four math/CS researchers developed a research design that included lessons featuring computer programming and mathematical proof writing activities. This study shares results from an application of the instructional research design to N=33 undergraduates interested in teaching. Lessons were from a secondary education math methods course. Follow up interviews were conducted with seven participants. The analysis, using APOS as a framework, categorized mathematical behaviors at the Action, Process or Object level. The study identified obstacles that may have prevented progression through deeper levels of understanding such as deficient prerequisite skills and an affinity for routine algebraic manipulation rather than considering underlying relationships. The student data demonstrated how computer programming activities influenced undergraduates' mental images.

Computational Thinking in University Mathematics Education: A Theoretical Framework

Chantal Buteau, Brock University; Eric Muller, Brock University; Joyce Mgombelo, Brock University; Ana Isabel Sacristán, Cinvestav-IPN

Room: Del Mar

In recent years "Computational thinking" has become a trending topic among teachers who have seen their curricula include the term, and researchers who seek to pinpoint both what it means and how it can be implemented in a meaningful way in classrooms. We see a crucial need in mathematics education to understand how students could be empowered to participate in the computational thinking that is now becoming an integral part of the mathematics and broader community. In our research, we are interested in examining how university mathematics students may come to appropriate programming and engage in computational thinking for mathematics, as mathematicians would do. In this paper, we present the theoretical framework that grounds our research.

Developing Proof Comprehension and Proof by Contradiction Through Logical Outlines

Darryl Chamberlain Jr., University of Florida; Draga Vidakovic, Georgia State University

Room: Point Loma 1

Proof is central to the curriculum for undergraduate mathematics majors. Despite transition-to-proof courses designed to facilitate the shift from computation-based mathematics to proof-based mathematics, students continue to struggle with mathematical proof. In particular, there are few tasks beyond writing proofs that are specifically designed to develop students' understanding of the proofs they read and the proof methods they utilize. The purpose of this paper is to introduce and discuss the merits of two such tasks: constructing and comparing logical outlines of presented proofs. Grounded in APOS Theory, this paper will illustrate a case study that suggests students can improve their understanding of the proofs they read as well as a particular proof method - proof by contradiction – through these two tasks.

Implementation and Impact of a Web-based Activity and Testing System in Community College Algebra

Shandy Hauk, WestEd; Bryan Matlen, WestEd

Room: Coronado

Most community college students in the U.S. must complete at least one developmental class, such as elementary algebra, before they can enroll in a college-level mathematics course. Increasingly common in such courses is the use of a web-based activity and testing system (WATS). This report presents initial results of a mixed-methods study of elementary algebra learning among 510 students in the classes of 29 instructors across 18 community colleges. Instructors were randomly assigned to use a particular WATS (treatment condition) or their usual approach (control condition). The focal WATS had adaptive problem sets, hints, and videos. Treatment group instructors had access to online support for implementation. For the study, students completed common pre- and post-tests and instructors regularly provided information about their teaching practices. The early results reported here indicate that greater instructor fidelity to developer intentions regarding frequency of assignments are positively associated with greater student learning.

The Effect of Self-Efficacy on Student Performance in Calculus

Asli Mutlu, North Carolina State University; Heather Ba, North Carolina State University; Karen Keene, North Carolina State University

Room: La Jolla

Self-efficacy is an important variable that has been used to study students' performance at all educational levels and in many content areas. In the report, we discuss the results of a quantitative study considering self-efficacy in college Calculus and its correlation to other variables available in a large scale study. Ultimately, our findings contradict existing findings regarding the effect of self efficacy on class performance. We add to these results an interesting finding regarding the effect of self efficacy on student's study habits: while time spent on course homework does not mediate the effect of self efficacy, more time spent on course preparation by students with high self efficacy tends to decrease their expected final course grade. Results contribute to math instructors' understanding of their teaching and may help with the construction of more effective instruction.

The Emergence of a Prototype of a Contextualized Algorithm in a Graph Theory Task

John Griffith Moala, The University of Auckland; Caroline Yoon, The University of Auckland; Igor' Kontorovich, The University of Auckland

Room: Private Dining Room

Research has described the necessity and dangers of prototypes in mathematical learning, without offering explanations for what makes prototypes appropriate or inappropriate, or indeed how prototypes emerge in the first place. We explore one part of the emergence of a prototype: how a feature of a concept's example becomes predominant in subsequent generated examples. We describe how three students developed what they regarded as four examples and one non-example of an algorithm suitable for a client with a contextualized graph theory problem. The students engaged in a 'patching process' that preserved an inappropriate feature of the initial example in the other examples that were generated. We argue that the development of appropriate prototypes may depend on the types of processes (like the 'patching process') that students use to abstract and preserve features of the concept examples.

Using Machine Learning Algorithms to Categorize Free Responses to Calculus Questions

Matthew Thomas, Ithaca College; Spencer Bagley, University of Northern Colorado; Mark Urban-Lurain, Michigan State University

Room: Point Loma 2&3

Researchers in various science disciplines have begun exploring use of machine learning algorithms to categorize students' answers to constructed-response tasks, achieving inter-rater reliability on par with that between expert raters. We report on a proof-of-concept experiment in which we categorized student responses to conceptually-focused tasks on a calculus final exam. Our results were only modestly successful, but promising. We identify ways in which responses to mathematics tasks are uniquely challenging for these algorithms, and ways in which the algorithms' performance on mathematics tasks can be improved.

	Day	Time	Session Type
Session 10	**Friday**	**11:00-11:30**	***Contributed Report***

Generalizing in Combinatorics Through Categorization

Zackery Reed, Oregon State University; Elise Lockwood, Oregon State University

Room: La Jolla

In this report we discuss students generalizing within a combinatorial setting. To facilitate reflection on prior activity, we prompted students in a teaching and a design experiment to categorize a myriad of problems they had previously engaged in. We will discuss the combinatorial underpinnings behind the students' generalizations according to Lockwood's (2013) model for combinatorial understanding. We saw that the students were able to produce generalizations of various basic combinatorial problems while each maintaining different understandings of the combinatorial structures. We conclude by discussing uniformity in the students' reasoning pertaining to combinations and the productive nature of such discussions.

Physics students' construction of differential length vectors for a spiral path

Benjamin Schermerhorn, University of Maine; John Thompson, University of Maine

Room: Del Mar

As part of an effort to examine student understanding of non-Cartesian coordinate systems and differential elements related to vector calculus, we interviewed students using tasks similar to typical electricity and magnetism problems. In one task, students were asked to calculate the change in electric potential along a spiral path, involving a common line integral. Analysis focused on conceptual understanding and symbolic expression of differential length vectors. Students were heavily drawn to the angular motion of the path through the radial electric field, often only expressing angular component of the length vector. This contrasts with earlier work, suggesting context may distract from correct mathematical expression.

Shape Thinking and the Transfer of Graphical Calculus Images

M. Katie Burden, University of Central Arkansas; Jason Martin, University of Central Arkansas

Room: Coronado

Shape thinking has previously detailed how students may view function graphs. Students using static shape thinking view a function graph as if it were a wire where learned rules, formulas and quantities appear as a consequence of the perceived shape. This study presents a case study that demonstrates how static shape thinking can be extended to other graphs seen within calculus. Results demonstrate how one first-semester calculus student perceived a "triangular" shape within a function graph. Quantities appeared as a consequence of this perceived shape and his reasoning on multiple related tasks was influenced by his transfer of this perceived shape onto subsequent graphs. Even the the student's reasoning led to inaccurate responses to interview tasks, his reasoning was accurate and consistent within his perception.

Stepping Through the Proof Door: Undergraduates' Experience One Year After an Introduction to Proof Course

Younggon Bae, Michigan State University; John Smith, Michigan State University; Mariana Levin, Western Michigan University; V. Rani Satyam, Michigan State University; Kevin Voogt, Michigan State University

Room: Point Loma 2&3

Navigating the transition from computing to proof writing remains a key challenge for mathematics departments and undergraduate students. Numerous departments have developed courses to introduce students to the nature of proof and effective argument (David & Zazkis, 2017), but research assessing the impact of these courses has just begun. This paper reports the experience of four introduction to proof "graduates" after they completed a semester of real analysis. Each had participated in our prior study of students' experience in the introduction to proof course. Results indicate that students' success in real analysis was supported by their work in the introduction to proof course. Two students exploited the structure common to many proofs in real analysis; the other two relied on extensive practice with example problems. For both pairs, we see linkages between students' work in real analysis and their prior procedurally-oriented work in mathematics.

Teaching Linear Algebra: Modeling One Instructor's Decisions to Move between the Worlds of Mathematical Thinking

Sepideh Stewart, University of Oklahoma; Jonathan Troup, University of Oklahoma; David Plaxco, Clayton State University

Room: Point Loma 1

In this article, we report results from a year-long study in which a linear algebra instructor worked with the research team to document his instructional decision-making via journals and interviews as well as to code and analyze the data. This work supports the development of a more general model of the instructor's decision-making and provides a lens with which to make sense of the instructors shifts between representations from each of Tall's Three Worlds. With the introduction of the model, we include an example to show how the various codes interact in the instructor's decision-making. We also provide a detailed description of one incident that provides a second perspective on the instructor's decisions, helping to support a more robust understanding of the data.

Themes in Undergraduate Students' Conceptions of Central Angle and Inscribed Angle

Biyao Liang, University of Georgia; Carlos Castillo-Garsow, Eastern Washington University

Room: Private Dining Room

Researchers have investigated students' multifaceted conceptions of angle and their difficulties with connecting angle measure to arcs or circles. In this study, we investigated three undergraduate students' thinking about angles in the context of circle geometry, specifically their conceptions of central and inscribed angle. Conceptual analysis of the data revealed that students involved in the tasks and interviews had various conceptions of these angles that either supported or constrained their ability to complete the tasks. Particularly, conceiving the dynamic transformation of both central and inscribed angles, or identifying their common subtended arc was productive, while considering angle as area or ray pair constrained their thinking.

	Day	Time	Session Type
Session 11	**Friday**	**11:40-12:10**	***Preliminary Report***

Assessing Visual Literacy Competency in Undergraduate Mathematics

Deniz Kardes Birinci, University of South Florida; Milé Krajcevski, University of South Florida; Gregory McColm, University of South Florida

Room: Bay Room

We investigated how linear algebra students acquired mathematical knowledge from visualization objects, and to what extent these students exhibited visual literacy standards in higher education. Seven linear algebra students were the subjects of this research projects. The data were collected through questions with high visual content and through semi-structured interviews. We analyzed the data by using descriptive and content analysis techniques. Our study found that linear algebra students were not sufficiently competent in using visualization techniques.

First-generation Low-income College Student Perceptions about First Year Calculus

Gaye DiGregorio, Collaborative for Student Achievement; Jessica Ellis, Colorado State University

Room: Point Loma 2&3

The purpose of this study was to explore first-generation low-income students' experiences with first-year calculus, including their self-belief in being successful in math. As part of the Progress though Calculus project, one STEM-focused institution was studied with survey results from students enrolled in first year calculus, and interviews and a focus group of three first-generation low-income students who completed first year calculus. Qualitative findings emphasized the value of creating connections with other students and faculty, and faculty's impact on students' sense of belief in being successful in calculus. Quantitative results illustrated statistically significant higher rates of faculty interaction outside of class and increased confidence in math while taking first year calculus for first-generation low-income students, in comparison to their continued generation, higher income peers. Promoting non-cognitive factors such as student support and self-belief in math success may influence math completion of first-generation low-income students.

Productive Failures: From Class Requirement to Fostering a Support Group

Milos Savic, University of Oklahoma; Devon Gunter, University of Oklahoma; Emily Curtis, University of Oklahoma; Ariana Paz Pirela, University of Oklahoma

Room: Point Loma 1

Mistakes occur frequently in mathematics. In two classes (Abstract Algebra and Calculus II), mistakes were brought to the forefront in the form of a "productive failure." Through five interviews with students, we initially looked for affectual responses to the pedagogical allowance and student-led demonstration. Many of the responses, both benefits and drawbacks of the productive failure, were interpreted by the research group to resemble peer-led support groups such as Alcoholics Anonymous. Descriptions of both productive failure and support groups, as well as quotes from the students, aim to shed light on psychological benefits of valuing mistakes.

Self-Regulation in Calculus I

Carolyn Johns, The Ohio State University

Room: Del Mar

Improving STEM retention is a major focus of universities and studies have shown calculus to be a barrier for STEM intending students. Prior to this study, local data indicated students did not pursue STEM fields because they were not passing calculus. In this work, I report on the results of a study on factors that seemingly impacted student success in Calculus I. In particular, I examined the relationship between final grades and self-reported self-regulatory aptitudes after accounting for incoming math aptitude. Results indicate self-regulatory aptitudes predict final grades above and beyond math aptitude. In addition, measures of self-regulation differed amongst high and under achievers as well as low and over achievers. This indicates self-regulation plays a role in student success. Furthermore, gender differences were present in measures of self-regulation which may be of importance for improving retention of women in STEM.

Student Understanding of Linear Combinations of Eigenvectors

Megan Wawro, Virginia Tech; Kevin Watson, Virginia Tech; Michelle Zandieh, Arizona State University

Room: La Jolla

Student understanding of eigenspace seems to be a particularly understudied aspect of research on eigentheory. In this report, we present preliminary results regarding students' reasoning on problems involving linear combinations of eigenvectors in which the resultant vector is or is not an eigenvector of the matrix. We detail three preliminary themes gleaned from our analysis: (a) using "is a linear combination of" to support both correct and incorrect answers; (b) conflating scalars in a linear combination with eigenvalues, and (c) reasoning about the dimension of eigenspaces versus a number of eigenvectors.

The Ways Graduate Teaching Assistants Learn to Teach Through Various Professional Development Interactions

Hayley Milbourne, San Diego State University; Susan Nickerson, San Diego State University

Room: Private Dining Room

Across the nation, there is increased national interest in improving the way mathematics departments prepare their GTAs. However, without an understanding of how GTAs interpret and make sense of various teaching practices, we are working without all of the information. I report preliminary results on the ways in which the understandings of GTAs of various teaching practices changed over a term. With this analysis, we will be able to better understand how to better support GTAs with their teaching in the future. The research presented here represents the start of an increased understanding of how GTAs form their own teaching practices.

Validating Proofs in Parallel Mathematical and Pedagogical Tasks

Erin Baldinger, University of Minnesota; Yvonne Lai, University of Nebraska-Lincoln

Room: Coronado

Educators often use tasks that situate teachers in pedagogical contexts, under the assumptions that such tasks activate knowledge authentic to teaching; and, furthermore, purely mathematical contexts may not activate such knowledge. These assumptions are based on analyses that contrast actual engagement with pedagogical context to hypothetical engagement without pedagogical context. We propose that it is important to conduct a direct comparison of responses, and we report on such a study using a set of tasks with and without pedagogical contexts – featuring the same underlying mathematics. The results revealed differences in how secondary teachers validated proof based on context. Context also influenced the importance participants placed on algebraic notation in validating a proof. This study has implications for how and when secondary teachers attend to validity and the role of algebraic notation, and the messages they may convey to their students about validity and notation.

	Day	Time	Session Type
	Friday	**12:10-13:10**	***Meal***

Lunch

Room: (Boxed Lunch)

	Day	Time	Session Type
	Friday	**12:10-13:10**	***Special***

Business Meeting

Room: Point Loma 2&3

	Day	Time	Session Type
Session 12	**Friday**	**13:20-13:50**	***Contributed Report***

An Initial Exploration of Students' Reasoning about Combinatorial Proof

Elise Lockwood, Oregon State University; Zackery Reed, Oregon State University

Room: Private Dining Room

Combinatorial proof involves proving relationships among expressions by arguing that the two expressions count sets with the same cardinality. It is an important topic because it is a kind of proof that has not been studied extensively, yet it represents an aspect of combinatorial reasoning that students should develop. In this paper, we report on data from two students who participated in a paired teaching experiment during which they solved tasks involving combinatorial proof. We highlight some productive aspects of their conceptions of combinatorial proof, and we also report on some pedagogical interventions that seemed to help students progress with successful combinatorial proving. We also argue that combinatorial proofs may naturally tend to be semantic rather than syntactic proof constructions (Weber & Alcock, 2004).

Conceptualizing Students' Struggle with Familiar Concepts in a New Mathematical Domain

Igor' Kontorovich, The University of Auckland

Room: Del Mar

This article is concerned with cognitive aspects of students' struggles in situations in which familiar concepts are reconsidered in a new mathematical domain. Examples of such cross-curricular concepts are divisibility in the domain of integers and in the domain of polynomials, multiplication in the domain of numbers and in the domain of vectors. The article introduces a polysemous approach for structuring students' concept images in these situations. Post-exchanges from an online forum were analyzed for illustrating the potential of the approach for indicating possible sources of students' misconceptions and meta-ways of thinking that might make students aware of their mistakes.

Conventions or Constraints? Pre-service and In-service Teachers' Understandings

Teo Paoletti, Montclair State University; Kevin Moore, University of Georgia; Jason Silverman, Drexel University; David Liss, University of Wisconsin-La Crosse; Stacy Musgrave, Cal Poly Pomona; Madhavi Vishnubhotla, Montclair State University; Zareen

Room: Point Loma 2&3

Several researchers have noted that it is important for students and teachers to be able to differentiate between what is mathematically critical to a concept or representation and what is a convention maintained for the purposes of communication. In this report, we describe two studies examining the extent to which pre-service and in-service teachers (PSTs and ISTs) understand graphing conventions either as conventions or as rules that must be unquestionably maintained. We highlight the extent to which conventions are pervasive in both PSTs' and ISTs' meanings for graphs and related ideas (i.e., function and rate of change) and describe why such meanings are problematic.

How Does Problem Context Shape Students' Mathematical Reasoning on Calculus Accumulation Tasks?

William Hall, Washington State University

Room: Coronado

Calculus serves many students from myriad fields of study. Investigations into the ways students from these fields of study reason about calculus concepts are vital, yet lacking (Rasmussen, Marrongelle, & Borba, 2014). The biological and life sciences make up 30% of traditional Calculus I students (Bressoud, 2015) and yet we know very little about how these students utilize context as they reason about calculus ideas like the definite integral. In this study, task-based interviews were conducted with 12 undergraduate students majoring in the biological and life sciences. Data were analyzed via open coding from a constructivist grounded theory approach (Charmaz, 2000) and a new analytic tool, local theory diagrams was developed. Results indicate problem context influenced students' assessment of the viability of their solution strategies as well as enabled them to reason through apparent contradictions in their work.

Individual and Situational Factors Related to Lecturing in Abstract Algebra

Estrella Johnson, Virginia Tech; Rachel Keller, Virginia Tech; Tim Fukawa-Connelly, Temple University; Valerie Peterson, University of Portland

Room: Point Loma 1

In this study, we report the results of a national survey of 219 abstract algebra instructors concerning their instructional practices and pedagogical decision-making. Organizing our respondents into groups (Alternative, Mixed, Traditional) based on proportion of class time lecturing, we investigated differences in the prevalence of specific pedagogical practices and the individual/situational factors influential therein. We used the reported teaching practices to generate profiles of the salient features of each instructional type and attempted to explain these differences through a combination of individual/situational factors. Results indicate that while significant differences in teaching practices exist, these are primarily explained by individual factors such as personal beliefs, level of experience, and interest in various scholarly activities. Situational characteristics, apart from institution type as identified by terminal degree, such as perceived departmental support and situation of abstract algebra in the broader mathematics curriculum did not appear to be related to instructional differences.

Insights into Students' Images of a Geometric Object and its Formula from a Covariational Reasoning Perspective

Irma Stevens, University of Georgia

Room: La Jolla

In covariational reasoning, when a student conceives of a situation as composed of measurable attributes that vary in tandem, discussing the relationship between quantities represented in a formula requires an interplay between a students' image of the situation and their conception of a formula. In this study, I categorize four pre-service teachers' images of both the situation and the formula as they describe the relationship between a given triangle's height and area. The results indicate how students' images of the situation and conceptions of a formula influence reasoning about the relationship between two quantities, specifically the role of numerical values and the development of a sophisticated dynamic image of the situation from which the student is able to draw conclusions.

	Day	Time	Session Type
Session 13	**Friday**	**14:00-14:30**	***Contributed Report***

A Preservice Mathematics Teacher's Covariational Reasoning as Mediator for Understanding of Global Warming

Dario Gonzalez, University of Georgia

Room: Del Mar

I examine one preservice mathematics teacher's (PST's) covariational reasoning in relation to two functions involved in modeling global warming. I also discuss how her covariational reasoning mediates her understanding of important concepts related to global warming. Jodi, the PST, completed a mathematical task I created for the study during an individual, task-based interview. The analysis of Jodi's responses revealed that: (a) the level of covariational reasoning and conceptions regarding quantities can constrain/facilitate the understanding of concepts related to global warming, (b) overreliance on discrete variation can led to conflicting notions regarding global warming, and (c) reasoning about rate of change is necessary to make sense of mathematical models for global warming based on energy balance.

An Activity Theory Approach to Mediating the Development of Metacognitive Norms During Problem Solving

Emilie Hancock, University of Northern Colorado

Room: Point Loma 2&3

Metacognition has long been identified as an essential component of the problem-solving process. Research on metacognition and metacognitive training has historically adopted an acquisitionist view. This study takes a participaionist lens by considering metacognition as a habit of mind or dispositional tendency. Problem-solving habits of mind can be viewed as normative ways of thinking to which students become attuned by participating in authentic problem-solving situations. This study explored one such situation, in which portfolio problem-solving sessions and write-ups were used to mediate metacognitive thinking. Periodically, students worked together on non-routine problems and submitted individual write-ups documenting their judgement and decision-making processes. Analysis utilized Activity Theory, which operationalizes the participation structure of a classroom, to document the nonlinear development of classroom metacognitive norms during problem solving. Micro-analysis revealed a shift from product- to process-oriented metacognitive norms. Macro-analysis situated these results, highlighting social mediators of activity and contradictions as catalysts for change.

Conceptual Analysis in Cognitive Research: Purpose, Uses, and the Need for Clarity

Alan O'Bryan, Arizona State University

Room: La Jolla

This theoretical paper discusses conceptual analysis of mathematical ideas relative to its place within cognitive learning theories and research studies. In particular, I highlight specific ways mathematics education research uses conceptual analysis and discuss the implications of these uses for interpreting and leveraging results to produce empirically tested learning trajectories.

Conceptual Blending: The Case of the Sierpinski Triangle Area and Perimeter

Naneh Apkarian, San Diego State University; Chris Rasmussen, San Diego State University; Michal Tabach, Tel Aviv University; Tommy Dreyfus, Tel Aviv University

Room: Point Loma 1

In this report, we present an analysis of 10 individual interviews with graduate mathematics education students about the area and perimeter of the Sierpinski triangle (ST). We use conceptual blending as a theoretical and methodological tool for analyzing students' reasoning to investigate how students encounter and cope with the ST having zero area and infinite perimeter. Our analysis documents the diverse ways in which the students reasoned about the situation. Results suggest that conceptualizing an infinite perimeter is more accessible to these students than is zero area, that encountering the paradox is dependent on how blends are composed, and that resolution of the paradox comes through completion and elaboration. The analysis furthers the theoretical/methodological framing of conceptual blending as a useful tool for revealing the structure and process of student reasoning.

Examining the Relationship Between Students' Covariational Reasoning When Constructing and When Interpreting Graphs

Kristin Frank, Towson University

Room: Coronado

Graphing tasks require students to engage in at least one of two activities: construct a graph and/or interpret a graph. Ideally, the meanings a student re-presents when constructing a graph are consistent with the meanings the student constructs from his/her sketched graph. However, this coherence is nontrivial. In this paper I present results from clinical interviews with university precalculus students to illustrate how students' graphing actions can be governed by different images of covarying quantities. More specifically, I present two students' mathematical activity to illustrate how these students' imagined quantities to covary in different ways depending on whether they were reasoning about a situation, constructing a graph, or reasoning about that sketched graph. I conclude by hypothesizing that the way a student coordinates two quantities' measures (e.g., asynchronous coordination of varying quantities or static coordination of measures) can inhibit him/her from imagining the same covariational relationship when constructing and interpreting graphs.

Gestures as Evidence of Assimilation When Learning Optimization

Keith Gallagher, West Virginia University; Nicole Infante, West Virginia University

Room: Private Dining Room

Teachers and students often produce gestures during communication about mathematical concepts and processes. Our goal in this study was to determine whether students would produce gestures similar to those used by the teacher. Each of five students in a first semester calculus course was asked to solve two optimization problems based on a video lesson in which the teacher used primarily pointing, primarily depictive gestures, or no gestures at all. Though our data do not show the students' gestures directly imitating the teacher's, they provide support for the claim that frequent gesture use during communication may indicate assimilation of new concepts and that assimilation improves student performance on optimization tasks.

	Day	Time	Session Type
Session 14	**Friday**	**14:40-15:10**	***Contributed Report***

Examining a Mathematician's Goals and Beliefs about Course Handouts

Sepideh Stewart, University of Oklahoma; Clarissa Thompson , Kent State; Noel Brady, University of oklohma

Room: Coronado

In this qualitative narrative study, we employed Schoenfeld's theory of Resources, Orientations and Goals (ROGs) to analyze a mathematician's beliefs and goals in creating handouts for his students. Some of the instructor's primary goals in creating the handouts were: (1) to help students gain an intuition about Algebraic Topology, (2) to provide a resource for students to revisit the difficult material outside of class, and (3) to prompt students to complete exercises so that they could monitor their own mastery of the course content. As part of this study, one of the students in class took daily journals. These journal entries revealed that he appreciated the time and careful preparation that was necessary to create the handouts, particularly the pictures that the instructor drew in the margins to help students gain an intuition. However, one obstacle that the student faced was struggling to appreciate the instructor's goal of expecting students to monitor their mastery of content outside of class time through completion of ungraded exercises in the handouts.

Figurative Thought and a Student's Reasoning About "Amounts" of Change

Biyao Liang, University of Georgia; Kevin Moore, University of Georgia

Room: Point Loma 1

This paper discusses a student coordinating changes in covarying quantities. We adapt Piaget's constructs of figurative and operative thought to describe her partitioning activity in terms of the extent that it is constrained to carrying out particular sensorimotor actions on perceptually available material, and we relate such descriptions to her thinking about quantitative amounts of change. We conclude the paper by discussing how characterizing these nuances of student thinking in terms of figurative and operative thought contributes to current literature on covariational reasoning and conceptualization of concept construction.

Mathematical Knowledge for Teaching Examples in Precalculus: A Collective Case Study

Erica Miller, University of Nebraska-Lincoln

Room: La Jolla

The purpose of this collective case study is to examine mathematical knowledge for teaching examples in precalculus. The instructors involved in the study were experienced graduate teaching assistants who were teaching their course for the third time and were identified as good teachers. Utilizing a social constructivist and cognitive theory approach, I analyzed video recordings of enacted examples. The central question that guided this analysis was: What is the mathematical knowledge for teaching examples in precalculus? The goal of this study is to examine undergraduate mathematical knowledge for teaching from the perspective of practice, instead of relying on existing frameworks. As a result of this study, the author developed a model of mathematical knowledge for teaching examples in precalculus that includes knowledge of representations, students, instruction, specialized content, and connections when enacting high cognitive demand examples.

Mathematics cognition reconsidered: on ascribing meaning

Thorsten Scheiner, The University of Auckland

Room: Private Dining Room

In contrast to the common assumption that mathematics cognition involves the attempt to recognize a previously unnoticed meaning of a concept, here mathematics cognition is reconsidered as a process of ascribing meaning to the objects of one's thinking. In this paper, the attention is focused on three processes that are convoluted in the complex dynamics involved when individuals ascribe meaning to higher mathematical objects: contextualizing, complementizing, and complexifying. The aim is to discuss emerging perspectives of these three processes in more detail that speak to the complex dynamics in mathematics cognition.

Revisiting Reducing Abstraction in Abstract Algebra

Kathleen Melhuish, Texas State University; Annie Bergman, Portland State University; Jennifer Czocher, Texas State University

Room: Del Mar

In this paper, we revisit Hazzan's (1999) fundamental work on reducing abstraction in abstract algebra tasks. As we analyzed hundreds of students' activity related to abstract algebra tasks, we identified many ways students reduced abstraction that did not align with the original framework. We leverage additional theories of abstraction to expand and refine Hazzan's framework to reflect new aspects of familiarity, contextualization, complexity and connectedness, and formality. For each of the new categorizations, we provide illustrations of students engaged in the relevant reduction of abstraction. We conclude with consideration to how the expanded framework may highlight productive types of abstraction reduction.

The Generation and Use of Examples in Calculus Classrooms

Vicki Sealey, West Virginia University; Johnna Bolyard, West Virginia Unversity; Matthew Campbell, West Virginia University; Nicole Engelke Infante, West Virginia University

Room: Point Loma 2&3

In this paper, we analyze video data of five instructors teaching the Mean Value Theorem in a first-semester calculus course. Throughout the lessons, graphical examples were provided by the instructors and/or the students of functions that satisfied or did not satisfy the conclusion of the Mean Value Theorem. Through the use of thematic analysis, we identified four themes related to emergence and use of examples: who generated the example, who evaluated the example, for what purpose the example was used, and the richness of the example. We emphasize that instruction that leverages student generated examples can provide a great deal of richness in a mathematics lesson and create opportunities to engage students in authentic mathematical activity. This work contributes to an evolving notion of what is entailed in students' active learning of mathematics and the role of the instructor.

Day	Time	Session Type
Friday	**15:10-15:35**	***Break***

Coffee Break

Room: Cabo Courtyard (Weather Permitting)

	Day	Time	Session Type
Session 15	**Friday**	**15:40-16:10**	***Preliminary Report***

Assessing Group Learning Opportunities in a First Semester Calculus Course

Jennifer Kearns, West Virginia Unversity; Johnna Bolyard, West Virginia Unversity

Room: La Jolla

The purpose of this study was to examine how undergraduate calculus students positioned themselves within group work and how that positionality influenced their own and others' learning opportunities. Using qualitative methods, this study examines the specific group social interactions and positionalities that led to productive and unproductive group problem solving. The study used a sociocultural lens to identify productive group work and learning. The findings of this paper suggest the roles students assume are very fluid throughout the problem solving process. In addition, the roles that the students assume influenced the learning opportunities. Furthermore, groups that utilized individualistic group practices were not able to build opportunities for conceptual understanding nor have productive group learning.

Engaged Learning through Creativity in Mathematics

Mika Munakata, Montclair State University; Ashwin Vaidya, Montclair State University; Ceire Monahan, Montclair State University; Erin Krupa, Montclair State University

Room: Del Mar

This study examined the impact of a deliberate attempt to present creativity as a mathematical endeavor on students' conceptions about mathematics, attitudes toward learning mathematics, and approaches to mathematics. Course modules were developed as part of a grant-funded project on creativity in STEM education and implemented in a course for non-mathematics majors. Throughout the course, students engaged in hands-on, active learning of mathematics through discovery. The mathematical topics for the course were chosen based on their relevance to students' everyday lives and their suitability for encouraging creativity. Data were collected through surveys, focus group interviews, and written artifacts. In this paper, we describe the preliminary results of our study and offer implications for both research and teaching.

Examining Students' Problem Posing Through a Creativity Framework

Steven Silber, University of Delaware

Room: Private Dining Room

Understanding how students pose problems can inform the development of posing activities to further enhance students' understanding of mathematics. Analyzing students' problem posing through the lens of mathematical creativity provides insight into the creative process of posing problems; namely, the cognitive tools students use to formulate questions. Three undergraduate students, enrolled in a developmental mathematics course, participated in a problem-posing intervention to examine the cognitive resources students used as the foundation for their mathematical problem posing. Session transcripts were analyzed using an analytical framework derived from an investment perspective on creativity, and identified resources were organized into two categories: mathematical knowledge and skills, and social interactions and experiences. Preliminary findings from the fifth session suggest that students associated the mathematical content of a posing task with previously encountered problems, as well as appealed to their familiarity with the situational context of the posing task.

Leveraging the Perceptual Ambiguity of Proof Scripts to Witness Students' Identities

Stacy Brown, California State Polytechnic University, Pomona

Room: Point Loma 2&3

Recognizing identity not only as an important educational outcome but also being inter-related to students' knowledge and practice, this paper explores an affordance of proof scripts; the witnessing of students' identities. Drawing on proof scripts from teaching experiments and the construct of perceptual ambiguity, this paper argues that proof scripts afford access not only to students' understandings, problematics, and ways of reasoning but also students' identities.

Modeling the Spread of Ideas in an Inquiry-Oriented Classroom

Rachel Rupnow, Virginia Tech; Sarah Kerrigan, Virginia Tech

Room: Point Loma 1

In this study, we model the spread of student understanding of linear combinations in an Inquiry-Oriented Linear Algebra (IOLA) class based on video analysis. Methods adapted from modeling biological systems were used to estimate the rate of spread of Process-level understanding of linear combinations, measured according to Action-Process-Object-Schema (APOS) theory. The amount of time required for all students to achieve Process-level understanding was also estimated.

Student's Semantic Understanding of Surjective Functions

Kelly Bubp, Ohio University

Room: Bay Room

Reasoning and proof are essential to mathematics, and surjective functions play important roles in every mathematical domain. In this study, students in a transition to proof course completed tasks involving composition and surjective functions. This paper explores students' semantic understandings of surjective functions, both individually and in the context of composition of functions. Most students demonstrated productive semantic understandings of surjective functions that allowed them to produce counterexamples and arguments for the truth of statements. Furthermore, in the struggle of using the syntactic definition of surjective in a proof, some students used their semantic understanding to try to make sense of the definition. This demonstrates the potential of students' ability to reason semantically to build understanding of the syntactic definition and structure of proofs of surjective functions.

Testing the Stability of Items in a Survey to Measure Relative Instructional Priorities Among Graduate Teaching Assistants

Eliza Gallagher, Clemson University; Aubrie Pfirman, Clemson University; Tony Nguyen, Clemson University; Khushikumari Patel, Clemson University

Room: Coronado

The results presented in this paper are part of a larger mixed-methods study examining relative instructional priorities among mathematics graduate teaching assistants (MGTAs). In this paper we share some early results and observations from a limited test-retest analysis of a pilot survey administered to MGTAs in two large public institutions in the Southeast United States. This is not intended to be an exhaustive statistical analysis of the pilot survey results or test-retest analysis. Instead, we focus on specific items to serve as a lens for better understanding the complexity of the choices MGTAs make in instructional settings.

	Day	Time	Session Type
Session 16	**Friday**	**16:20-16:50**	***Contributed Report***

Developing Preservice Teachers' Mathematical Knowledge for Teaching in Content Courses

Jeremy Strayer, Middle Tennessee State University; Alyson Lischka, Middle Tennessee State University; Candice Quinn, Middle Tennessee State University; Lucy Watson, Middle Tennessee State University

Room: La Jolla

In this paper we present evidence that a) providing opportunities for PSMTs to engage with simulations of practice and b) making connections between advanced perspectives on geometry and 7-12 mathematics allows PSMTs to develop MKT in university mathematics content courses.

Factors Supporting (or Constraining) the Implementation of DNR-based Instruction in Mathematics

Osvaldo Soto, University of California San Diego; Guershon Harel, University of California San Diego

Room: Coronado

DNR-based instruction in mathematics (Harel, 2008a, 2008b, 2008c) is a theoretical framework for the learning and teaching of mathematics. DNR-based professional development is a long-running program spanning seven years with multiple cohorts of in-service secondary mathematics teacher participants. This report investigates teacher change among five key variables: facilitating public debate, using holistic problems, attending to students' intellectual need, attending to the meaning of quantities and use of students' contributions. Commitment to change and perseverance, the nature of available curricular materials and teachers' view of the role of curriculum, a view of students as partners in knowledge construction, institutional context, collaboration and content knowledge were identified as factors that afford or constrain DNR implementation. This work has implications for the design of future professional development efforts and the development of a more robust theory of teacher change.

Peter's Evoked Concept Images for Absolute Value Inequalities in Calculus Contexts

Erika David, Arizona State University

Room: Point Loma 2&3

Statements involving absolute value inequalities, such as the definition of continuity at a point, abound in Advanced Calculus. In textbooks, such statements are frequently illustrated with graphical representations. Despite their abundance, how students think about absolute value inequalities and their representations in these contexts is not widely known. This study examines one undergraduate mathematics student's evoked concept images (Tall & Vinner, 1981) for absolute value inequalities in various contexts, including those from Advanced Calculus. The student's evoked concept image differed based on the context of the statements involving absolute value inequalities. Notably, the student's evoked concept image did not support his understanding of the visual representation of the formal definition of continuity. The results of this study suggest that some students may not conceive of absolute value inequalities in ways that are productive for understanding the formal definitions of Advanced Calculus concepts.

Schema Development in an Introductory Topology Proof

Ashley Berger, University of Oklahoma; Sepideh Stewart, University of Oklahoma

Room: Private Dining Room

This is an exploratory study into schema development of introductory topology students. We discuss Skemp (1987) and Dubinsky and McDonald's (2001) definitions of schema and how they fit with Piaget and Garcia's (1989) triad framework. We employed these theoretical instances on the idea of schema to analyze students' responses to a final exam problem about a basis for the product topology on a product space. Our analysis indicates that the majority of the students were still in the beginning stages of schema development by the end of the semester in a topology course.

Student Status in Peer Conferences

Daniel Reinholz, San Diego State University

Room: Point Loma 1

This paper provides an analysis of students' peer assessment conversations in introductory college calculus. In particular, it explores gender differences in the types of feedback and word choices used by students. Using computer-aided textual analysis, it draws connections between the types of words that students use and their relative status in the class. Surprisingly, the use of pronouns based on gender did not follow what one would predict based on prior studies. Possible explanations and implications for future research are discussed.

Toward a Functional Grammar of Physics Equations

Kirk Williams, CSU Chico; David Brookes, CSU Chico

Room: Del Mar

An area of difficulty for students in introductory physics courses is how they use and reason with equations. We propose that part of this difficulty is due to meaning that is embedded in the structure of equations. As equations are manipulated, their structure and concomitant meanings change. As mathematics is considered the "language of physics," our starting point will be to propose that it has a grammar. As equations change form and meaning, they are doing so within a certain grammatical system. We will show how physics equations can be categorized and mapped to ideational clause types as devised by Halliday (1985). This mapping could be useful in relating the mathematical "language" used in physics to "natural language," benefiting physics instructors who are trying to understand the struggles of their students, and helping students to understand the rich meanings embedded in physics equations.

	Day	Time	Session Type
Session 17	**Friday**	**17:00-17:30**	***Theoretical Report***

Building on Covariation: Making Explicit Four Types of "Multivariation"

Steven Jones, Brigham Young University

Room: La Jolla

Covariation and covariational reasoning have become key themes in mathematics education research. In this theoretical paper, I build on the construct of covariation by considering cases where more than two variables relate to each other, in what can be called "multivariation." I share the results of a conceptual analysis that led to the identification of four distinct types of multivariation: independent, dependent, nested, and vector. I also describe a second conceptual analysis in which I took the mental actions of relationship, increase/decrease, and amount from the covariational reasoning framework, and imagined what analogous mental actions might be for each of these types of multivariation. These conceptual analyses are useful in order to scaffold future empirical work in creating a complete multivariational reasoning framework.

Challenging the stigma of a small N: Experiences of students of color in Calculus I

Jessica Hagman, Colorado State University; Vincent Basile, Colorado State University; Daniel Birmingham, Colorado State University; Bailey Fosdick, Colorado State University

Room: Point Loma 1

Because students of color are underrepresented in undergraduate mathematics classes, their experiences are often ignored in studies drawing on large data sets or are inferred based on the experiences of other underrepresented populations, specifically women. This exclusion and misrepresentation of students of color is often attributed to methodological limitations. In this study, we reexamine the data studied for a previous analysis attending to student race and ethnicity rather than to gender. Due to the smaller numbers of non-white students, we utilize different analytic tools, and draw on students' open-ended responses to a survey question asking about their experiences in Calculus I. In addition to adding to the literature on students from marginalized populations in undergraduate mathematics, this paper argues for a reframing of how we value papers with a small N, and what this value indicates about our value of the students making up the small samples.

Future Middle Grades Teachers' Coordination of Knowledge Within the Multiplicative Conceptual Field

Andrew Izsak, University of Georgia; Sybilla Beckmann, University of Georgia

Room: Del Mar

We report theoretical and empirical results generated through studying several cycles of a number and operations content course offered to future middle grades mathematics teachers. A main feature of the course is using an explicit, quantitative definition for multiplication to connect a range of topics in the multiplicative conceptual field (Vergnaud, 1983, 1988). Course topics include multiplication and division with both whole numbers and fractions, proportional relationships, and linear functions. The theoretical results include a mathematical analysis of multiplication as coordinated measurement and a (still emerging) psychological framework that emphasizes coordinating diverse cognitive resources. Empirical data come from clinical interviews conducted with 6 future teachers enrolled in the content course in Fall 2016. One empirical result is the importance of connecting partitioning quantities, dividing measurements by whole numbers, and multiplying measurements by unit fractions when expressing relationships between quantities through multiplication expressions and equations.

Observable Manifestations of A Teacher's Actions to Understand and Act on Student Thinking

Sinem Bas Ader, Istanbul Aydin University; Marilyn Carlson, Arizona State University

Room: Coronado

This study produced a framework that describes different levels of teacher-student interactions during teaching. The framework characterizes observable teacher behaviors that are associated with each of the four levels of decentering that emerged from analyzing the teacher-student interactions of three teachers when teaching.

Reasoning About One Population Hypothesis Testing: The Case of Steve

Annie Childers, University of AR at Little Rock; Draga Vidakovic, Georgia State University; Harrison Stalvey, University of Colorado, Boulder; Aubrey Kemp, Georgia State University; Leslie Meadows, Georgia State University; Darryl Chamberlain Jr.,

Room: Private Dining Room

Hypothesis testing is a key concept included in many introductory statistics courses. Yet, due to common misunderstandings of both scientists and students, the use of hypothesis testing to interpret experimental data has received criticism. With statistics education on the rise, as well as an increasing number of students enrolling in introductory statistics courses each year, there is a need for research that investigates students' understanding of hypothesis testing. This paper describes results obtained from a larger study designed to investigate introductory statistics students' understanding of one population hypothesis testing. In particular, we present on one student's understanding of the concepts involved in hypothesis testing, Steve, who provided us the best spectrum of different levels of knowledge according to APOS Theory, our guiding theoretical framework. Based on this data, we suggest implications for teaching.

Sensemaking in Statewide College Mathematics Curriculum Reform

Matthew Wilson, Oklahoma State University; Michael Oehrtman, Oklahoma State

Room: Point Loma 2&3

We draw on the theory of sensemaking and sensegiving to characterize the social cognitive aspects of transformative organizational change in the context of statewide college mathematics curriculum reform efforts with the goal to understand the barriers to implementing the forms. In order to further understand the change process and the challenges these changes present, we conducted interviews with institutional, state, and national leaders of these efforts.

Day	Time	Session Type
Friday	**17:40-18:40**	***Plenary***

Re-imagining Online Videos for Mathematics Learning

Joanne Lobato

Room: Point Loma 2&3

Despite the tremendous growth in online mathematics videos, there is surprising uniformity in the use of talking heads or hands to demonstrate step-by-step procedures. In an effort to contribute to the discussion about what's possible in video-based online mathematics learning, we developed dialogically-intensive, conceptually-oriented videos (available at www.mathtalk.org). Each video features the unscripted dialogue of a pair of secondary school students as they convey sources of confusion and resolve their own dilemmas by arguing for and against particular ways of reasoning. Other students who learn by viewing and engaging with the videos are called vicarious learners (because they are participating in the original dialogue indirectly). This talk highlights several of our studies on vicarious learning, by both high school students and prospective secondary mathematics teachers. By also drawing upon vicarious learning studies conducted with undergraduates in several different disciplines, I discuss theory regarding the foundations, benefits, and constraints of vicarious learning.

	Day	Time	Session Type
	Saturday	**7:00-8:30**	***Meal***

Breakfast

Room: Pt. Loma Terrace

	Day	Time	Session Type
Session 18	**Saturday**	**8:30-9:00**	***Preliminary Report***

Curricular Presentation of Static and Process-Oriented Views of Proof to Pre-service Elementary Teachers

Taren Going, Michigan State University

Room: Bay Room

Engaging students in proof-related reasoning is an important but often challenging task for pre-service elementary teachers. Given that limited mathematics content courses and their associated textbooks offer some of the only opportunities for preservice elementary teachers to engage with proof, it is vital to understand what opportunities they offer to understand proof. I conducted an analysis of two textbooks used for elementary mathematics content courses to investigate the view(s) of proof promoted within and the opportunities to learn about proof-related reasoning. My findings suggest a mixed emphasis on static and dynamic views of proof and proving, but also many opportunities for instructors of mathematics content courses to promote an explanatory, process-oriented view of proof.

Gauging College Mathematics Instructors' Knowledge of Student Thinking About Limits

Natasha Speer, University of Maine; Jessica Gehrtz, Colorado State University; Jessica Ellis, Colorado State University

Room: Point Loma 1

A theme in the literature surrounding instructional practices and knowledge for teaching is that knowledge of how students think about mathematical ideas plays important roles in supporting effective instruction. However, the undergraduate mathematics education community lacks tools for assessing this kind of knowledge. As an initial step toward the development of such assessments, we documented instructors as they examined students' work on calculus tasks during individual interviews. Transcripts were coded as exhibiting robust, limited, or no evidence of knowledge of student thinking using Jacobs, Lamb, and Philipp's (2010) framework. The coding process highlighted the varying depth and breadth of instructors' knowledge. Once refined, this coding process can be used to develop instruments for gauging knowledge of student thinking through means other than interviews. Such instruments will be of use to researchers, to those who design professional development for experienced and novice instructors, and for evaluation of professional development efforts.

Investigating Student Success in Team-Based Learning Calculus I and in Subsequent Courses

Heather Bolles, Iowa State University ; Travis Peters, Iowa State University; Elgin Johnston, Iowa State University

Room: Del Mar

With recommendations for active-learning strategies and challenging courses, we applied mixed methods to examine students' success in Calculus I and subsequent courses following instruction using Team-Based Learning (TBL). Overall, TBL students performed better on midterm and final calculus exams, gave more explanations, and completed Calculus I at a higher rate than their peers. These results remained true when students' incoming competencies for calculus were considered. TBL students performed comparably to their peers in Calculus II and Physics.

Modus Tollens in Modeling

Jennifer Czocher, Texas State University; Jenna Tague, Fresno State University

Room: Private Dining Room

The purpose of this paper is to present a case study of a mathematics major exhibiting logical reasoning to validate her mathematical model. The case study demonstrates how constructing a mathematical model can be construed as making an argument for its validity.

Surveying Professors' Perceptions of Incorporating History into Calculus I Instruction

Aaron Trocki, Elon University; Madison Jaudon, Elon University

Room: La Jolla

The goal of this study is to document undergraduate mathematics professors' perceptions of incorporating the history of mathematics into their Calculus I instruction. Although research has been documented on benefits of incorporating history into mathematics teaching and learning, little has been documented on undergraduate professors' beliefs and how they may incorporate history into Calculus I. To address this question, we created a survey based on Schoenfeld's (1999) theoretical framework of knowledge, goals, and orientations to capture perceptions about instructional decisions related to history incorporation. Calculus I professors in a southeastern state were surveyed to gain an understanding of perceptions on the importance of history and how they incorporate history. The majority of professors (80%) view history as important for Calculus I learning for a variety of reasons and incorporate it in different ways. Implications for supporting undergraduate Calculus I teaching and learning are shared along with questions for further research.

The Emergence of a Video Coding Protocol to Assess the Quality of Community College Algebra Instruction

April Strom, Scottsdale Community College; AI@CC Research Group, AI@CC Research Group

Room: Point Loma 2&3

The Evaluating the Quality of Instruction in Post-secondary Mathematics (EQIPM) is a video coding instrument that provides indicators of the quality of instruction in community college algebra lessons. The instrument is based on two existing instruments that assess the quality of instruction in K-12 settings —the Mathematical Quality of Instruction (MQI) instrument (Hill, 2014) and the Quality of Instructional Practices in Algebra (QIPA) instrument (Litke, 2015). EQIPM addresses three dimensions focused on quality of instruction via 17 codes. In this paper, we describe two codes: Instructors Making Sense of Procedures from the Quality of Instructor-Content Interaction dimension, and the Mathematical Errors and Imprecisions in Content or Language, a code spanning all three dimensions. The purpose of the paper is to illustrate what we have learned from these codes and the new instrument to advance our understanding of post-secondary mathematics instruction.

The Instructor's Role in Promoting Student Argumentation in an Inquiry-Oriented Classroom

Chris Rasmussen, San Diego State University; Karen Marrongelle, Portland State University; Oh Nam Kwon, Seoul National Univerisity

Room: Coronado

Four class sessions in inquiry-oriented differential equations were analyzed to understand the role of the instructor in supporting student argumentation. Three coding schemes were developed to identify arguments, characterize instructor utterances, and connect instructor talk to argumentation goals in inquiry-oriented instruction. Results show that students generated the majority of arguments tendered in the four class sessions. The instructor used questions to generate student arguments more than other types of instructional utterances (e.g., revoicing, telling). Nearly half of the instructor's utterances were aligned with argumentation goals. More detailed examples of student-generated arguments in the class sessions are being constructed to illustrate the flow and function of different goal alignment routines to understand what it is that the instructor did during class to promote student argumentation.

	Day	Time	Session Type
Session 19	**Saturday**	**9:10-9:40**	***Contributed Report***

Adapting an Exam Classification Framework Beyond Calculus

Brian Katz, Augustana College; Sandra Laursen, Ethnography & Evaluation Research

Room: Point Loma 2&3

This paper reports a methods-building project that seeks to make inferences about mathematics instructors' teaching practices from their exams. We adapt and revise a framework by Tallman et al. (2016) and expand its applicability across the undergraduate curriculum, beginning with a sample of seven exams from early-career mathematics instructors. We describe the rationale for the adaptation process and patterns of differences between exam sets. Future work includes coordinating this analysis with results from other data sets from the same instructors.

Convergent and Divergent Student Experiences in a Problem-Based Developmental Mathematics Class

Martha Makowski, University of Alabama

Room: La Jolla

In recent years low success rates in traditionally taught pre-college mathematics classes has led to new courses that use group work and problem solving to teach the required content. Early results examining student outcomes are promising, but say little about students' classroom experiences. This study uses interviews from six students and one instructor in a single class to explore differences between student experiences and the intentions of the instructor. Although several students expressed positive perceptions of the class, tensions arose between students who wanted to learn efficiently versus the classroom expectation that students stay together in their groups. Practices such as copying and dictation arose, at least partially, as coping mechanisms for students caught between these conflicting values. Future work should examine alternative grouping methods and ways of using early indicators of need to provide additional support.

Informal Content and Student Note-Taking in Undergraduate Calculus Classes

Alex Kopp, Temple University; Tim Fukawa-Connelly, Temple University

Room: Coronado

This study investigates four hypotheses about calculus instruction: (i) that lectures include informal content (ways of thinking and reasoning that are not captured by the formal symbolic statements), (ii) that informal content is usually presented orally but not written on the blackboard, and (iii) that students do not record the informal content that is only stated orally but do if it is written on the blackboard, and (iv) that professors often most want students to learn the content they state informally. Via interviews, we also explored why professors chose to write on the board, or not, content. We recorded 5 calculus mathematics lectures and photographed the notes of 78 students. We found that informal content was common, although most informal content was presented in a written form. Typically students recorded formal content while not recording informal content in their notes.

Learning Our Way into Effective Professional Development: Networked Improvement Science in Community College Developmental Mathematics

Haley McNamara, The Carnegie Foundation for the Advancement of Teaching; Ann Edwards, The Carnegie Foundation for the Advancement of Teaching; Carlos Sandoval, University of California Irvine

Room: Del Mar

Every year, hundreds of thousands of college students are placed into, and do not complete, developmental math courses. The Carnegie Math Pathways, a nationwide initiative aimed at addressing this problem, is comprised of a student-centered instructional system that forefronts mathematical sense-making and conceptual understanding; structural changes to course offerings; and a system of faculty professional development. This paper reports on the use of Improvement Science, an approach grounded in methods and tools of quality improvement, to design, improve, and scale a professional development program for first-time Pathways instructors. We also report on insights derived from the improvement approach about effective professional development in the Pathways and findings related to common challenges faced when teaching the Pathways. We conclude with implications for professional development in higher education and the use of improvement science to scale effective professional development.

The Counter-storytelling of Latinx Men's Co-Constructions of Masculinities and Undergraduate Mathematical Success

Luis Leyva, Vanderbilt University

Room: Point Loma 1

While Latinxs complete undergraduate engineering degrees at lower rates than Whites and Asians, Latinx men trail behind Latinx women who recently earned over half of engineering and science degrees conferred to Latinxs. With multiple semesters of mathematics required in engineering majors, qualitative analyses of undergraduate Latinx men's strategies of persistence and success in engineering can illuminate ways to inform more socially-affirming postsecondary educational opportunities and thus increase retention in STEM (science, technology, engineering, and mathematics). This report presents findings from a phenomenological study that characterized variation in two undergraduate Latinx men's negotiations of their masculinities with pursuits of mathematical success as engineering majors at a large, predominantly White four-year university. Findings illuminate the Latinx men's strategies of managing risks of mathematics classroom participation, building academically and socially supportive relationships with faculty members, and negotiating pursuits of STEM higher education with their gendered sense of commitment to family.

Theorizing Silence

Matthew Petersen, Portland State University

Room: Private Dining Room

Different norms govern the use of silence in mathematical collaboration and in every-day Anglo-conversation. Research is therefore needed into the ways students are enculturated into the distinctive uses of silences employed in mathematics collaboration. This project will require a new theoretical perspective that facilitates the study of silence. Drawing off studies of silence and embodiment from multiple disciplines, this paper advances a view of silence and the body, and so lays the groundwork for a rigorous study of silence in mathematics education.

	Day	Time	Session Type
	Saturday	**9:40-10:05**	***Break***

Coffee Break

Room: Cabo Courtyard (Weather Permitting)

	Day	Time	Session Type
Session 20	**Saturday**	**10:10-10:40**	***Preliminary Report***

A Course in mathematical modeling for pre-service teachers: Designs and challenges

Joshua Chesler, California State University, Long Beach; Jen-Mei Chang, California State University, Long Beach

Room: Del Mar

The increased status of mathematical modeling in the K-12 curriculum requires teacher preparation programs to adapt. This design experiment examines a course in mathematical modeling for pre-service secondary mathematics instructors that was co-developed and co-taught by a mathematics educator and an applied mathematician. The students in the course, all mathematics majors, experienced growth as well as challenges, some rooted in quantitative reasoning.

A Student's Use of Definitions in the Derivation of the Taxicab Circle Equation

Aubrey Kemp, Georgia State University; Draga Vidakovic, Georgia State University

Room: Coronado

Research shows that by observing properties and making conjectures in other geometries, students can better develop their understanding of concepts in Euclidean geometry. It is also known that definitions in mathematics are an integral part of understanding concepts, and are often not used correctly in proof or logic courses by students. APOS Theory is used as the framework in this preliminary data analysis to determine one students' understanding of certain definitions in Euclidean and Taxicab geometry, and her use of these definitions in deriving an equation for a circle in Taxicab geometry.

Classroom Experiences of Students in a Community College Intermediate Algebra Course

Anne Cawley, University of Michigan

Room: Bay Room

There is little understanding of the ways in which students experience developmental mathematics courses at community colleges (Crisp, Reyes, & Doran, 2015). This study investigates the instructional experiences of students in an Intermediate Algebra course using qualitative methods that rely on interviews, surveys, classroom observations and classroom artifacts. I aim to understand (1) what are the experiences of students in a developmental mathematics class at a community college and (2) how students make sense of particular experiences. The findings from this study will support college mathematics departments by providing evidence of the classroom instructional experiences of students.

Impacts of Peer Mentorship in a Calculus Workshop on the Mentors' Identities and Academic Experiences in Undergraduate STEM

Aditya Adiredja, The University of Arizona; Luis Leyva, Vanderbilt University; Jorge Mendoza, The University of Arizona

Room: Point Loma 2&3

Research has shown the positive impact of peer mentorship on the educational experiences of mentored students from underrepresented backgrounds. National survey data of peer leaders indicate that peer mentors also benefit from the mentoring experience. This report unpacks this survey finding related to peer mentors' increase in a sense of belonging in college and academic persistence as a result of participating in the mentorship. Our data draws from interviews with six historically marginalized students of color after their participation as mentors for a group of first-year calculus students during a summer bridge calculus workshop. The mentors' main responsibility was facilitating critical conversations about racial and gender in Science, Technology, Engineering, and Mathematics (STEM). Preliminary analyses found that mentoring contributed to their confidence in succeeding in a STEM field and their ability to make sense of gendered and racialized educational experiences in STEM.

Integrals, Volumes, and Visualizations

Krista Bresock, West Virginia University; Vicki Sealey, West Virginia University

Room: Point Loma 1

Many studies have been done on student understanding of integration and this research aims to add to that knowledge base with the study of student understanding of integration when applied to volume problems and how visualizations and sketches are used in the problem-solving process. Participants were recruited from a large, public, research university and interviews consisted of students working through routine and novel volume problems while discussing their thought processes aloud. Preliminary results show that students rely heavily on memorized formulas and have difficulties explaining the concepts behind the formulas. The idea of the integral as a sum of small pieces is present in most students studied, but they have trouble relating this idea to the formulas in their volume integrals. All students drew sketches of the geometric situation for all the problems, but the extent to which they could use their sketch meaningfully varied greatly.

Teachers' Use of Informal Conceptions of Variability to Make Sense of Representativeness of Samples

Gabriel Tarr, Arizona State University; April Strom, Scottsdale Community College

Room: Private Dining Room

A key factor in statistical thinking is reasoning about variability. This paper contains data on how in-service middle school teachers and a community college faculty member reasoned through two statistical tasks. The researcher presents his analysis of the data through the lens of how teachers reasoned about variability as they worked through the two statistical tasks.

Validation of an Assessment for Introductory Linear Algebra Courses

Muhammad Haidar, Florida State University

Room: La Jolla

Research-based and validated open-ended assessments are useful tools to explore students' reasoning and understanding of a subject. The primary goal of this study is to validate an assessment which can accurately measure students' conceptual understanding of four focal topics, typically covered in an introductory linear algebra course; span and linear independence, systems of linear equations, linear transformations, and eigenvalues and eigenvectors. I used the assessment data of 255 students, from nine linear algebra classes at eight different institutes across the country to validate the assessment. By administering the assessment in their classes, linear algebra instructors can gauge their students' conceptual understanding of linear algebra concepts and can identify the concepts which are generally vexatious for students.

	Day	Time	Session Type
Session 21	**Saturday**	**10:50-11:20**	***Contributed Report***

A Department-Level Protocol for Assessing Students' Developing Competence with Proof Construction and Validation

Tabitha Mingus, Western Michigan University; Mariana Levin, Western Michigan University

Room: Private Dining Room

This methodological paper describes a protocol for assessing the development of students' competence with proof, created by the assessment committee within the Department of Mathematics at Midwestern University. The assessment protocol we describe evolved over a period of 20 years and aims to collect information that is meaningful and actionable for improving mathematics instruction within the department. While there are several unique features of Midwestern University that have created a context in which such work can be undertaken at the level of the department, we believe that this case will be of interest to mathematics departments seeking to find ways to measure their students' developing competence with proof.

A Model of Task-Based Learning for Research on Instructor Professional Development

Billy Jackson, State University of New York, Geneseo; Jenq-Jong Tsay, U. Texas Rio Grande Valley; Shandy Hauk, WestEd

Room: Del Mar

We present a theoretical framework that synthesizes and increases the descriptive power of existing models of task-based learning. Grounded in social constructivism and activity theory, the framework supports collegiate mathematics education researchers in identifying, investigating, and reporting on task-based learning in instructor professional development contexts. Relevant definitions and connections to the larger realm of inquiry-based, problem-based, and other general inquiry-oriented instruction are addressed. We conclude with a discussion and illustration of how the framework may be used in design, materials development, and evaluation research related to instructor professional learning.

Could Algebra be the Root of Problems in Calculus Courses?

Sepideh Stewart, University of Oklahoma; Stacy Reeder, University of Oklahoma; Kate Raymond, University of oklohma ; Jonathan Troup, University of Oklahoma

Room: La Jolla

Calculus serves as the gateway for most STEM degrees. Due to students' challenges successfully completing calculus, more than half of students are deterred from a career in STEM. Our preliminary investigation indicates that students' difficulties with algebra cause significant problems in many first-year math courses. The aim of this paper is to investigate in what ways the difficulties with algebra impact students' success in calculus.

E-IBL, Proof Scripts, and Identities: An Exploration of Theoretical Relationships

Stacy Brown, California State Polytechnic University, Pomona

Room: Point Loma 1

The purpose of this theoretical report is to further current discussions on the relationships between Equity-Oriented Instruction (EOI) and Inquiry Based Learning (IBL) pedagogies. Specifically, it proposes a framing of Equity-Oriented Inquiry Based Learning (E-IBL) that foregrounds equitable practice, as oppose to viewing equitable practice as a gratuitous outcome of IBL pedagogies. Drawing on data from teaching experiments conducted in IBL-Introduction to Proof courses, the inter-relationships between knowledge, identity and practice (Boaler, 2002), Pickering's 'dance of agency,' Gutiérrez's dimensions of equity, and Bourdieu's notion of habitus, this paper explores why intentional attention towards the critical axis of equity – that which links identity and power - is necessary, if IBL pedagogies are to promote equity.

Exploring the secondary teaching of functions in relation to the learning of abstract algebra

Nicholas Wasserman, Teachers College, Columbia University

Room: Coronado

Secondary mathematics teachers regularly take advanced mathematics courses, but many regard them as unrelated to their work as teachers. In accord with a novel instructional approach (Author(s), 2017), we designed materials for an abstract algebra course that connect to the teaching of functions in secondary schools. In this paper, we describe findings from a small-scale teaching experiment employing design research, which provides evidence that particular tasks were productive for accomplishing some of the mathematical and pedagogical aims.

First Results From a Validation Study of TAMI: Toolkit for Assessing Mathematics Instruction

Charles Hayward, Ethnography & Evaluation Research; Timothy Weston, University of Colorado Boulder; Sandra Laursen, Ethnography & Evaluation Research

Room: Point Loma 2&3

Many researchers consider observation to be a 'gold standard' for measuring classroom practices since self-report surveys may be prone to bias. In this paper, we explore how the design of survey instruments and observation protocols affects the trustworthiness of the data collected. We describe our process of developing well-aligned observation and survey instruments in order to reduce sources of measurement error. We present results from a large-scale test of these instruments in 176 observations of 17 different math courses. Our results indicate that when survey instruments are designed to describe what happens in a course, rather than evaluate the quality of the instruction, and when those survey results are compared to observation protocols measuring teaching in the same way, self-report surveys are largely trustworthy.

	Day	Time	Session Type
Session 22	**Saturday**	**11:30-12:00**	***Preliminary Report***

Cognitive Resources in Student Reasoning about Mean Tendency

Kelly Findley, Florida State University; Jennifer Kaplan, University of Georgia

Room: Bay Room

The ability to conceptualize the sample mean as having a distribution is essential to the development of statistical reasoning. Considerable research on student thinking exists on this topic, but this literature largely assumes a misconception model. This study takes a grounded theory approach to investigate the cognitive resources incoming students possess to reason about sampling distributions and mean tendency. This preliminary report includes data from a pilot study with one student enrolled in an introductory statistics course. She completed both a pre- and post-instruction interview that involved prompts about the distribution of the ages of pennies in circulation and related questions about average ages of groups of pennies. We identify several cognitive resources elicited by the pre- and post-interviews, consider the influence of instruction on the activation of these resources, and briefly discuss implications to statistics teaching. Finally, we outline next steps for data collection with 8-10 students.

Emerging Instructional Leadership in a New Course Coordination System

Naneh Apkarian, San Diego State University

Room: Coronado

This paper reports on the instantiation of a coordination system in a university mathematics department, and in particular the transition of three faculty members into their new roles as course coordinators. Course coordination, characterized by uniform course elements and instructor meetings, is a programmatic feature that supports student success in introductory mathematics courses. When courses are coordinated, the person or people responsible for the coordination play a critical and complex role in ensuring that all students experience comparable, well-designed classes – but building such a system is complex and has not been studied in situ. In this report, I explore one coordinator's transition from a peripheral participant in discussions of teaching to a highly central figure with significant influence on instructors and colleagues in the department. Surveys and interviews with involved parties reveal nuance of this shift in leadership and shed some light on the process.

Guiding whose reinventions? A gendered analysis of discussions in inquiry-oriented mathematics

Christine Andrews-Larson, Florida State University; Cihan Can, Florida State University; Alexis Angstadt, Florida State University

Room: Point Loma 1

The under-representation of women in STEM fields is well-documented and undisputed. Evidence suggests that students' experiences in undergraduate mathematics courses contributes to this disparity, and that student-centered approaches to instruction may be more equitable than lecture-based approaches. However, the generalizability of this finding has not been established. In this study, we explore how female students are positioned in whole class discussions in two inquiry-oriented mathematics classes selected to reflect differences in how female students reported experiencing whole class discussions.

Identifying Subtleties in Preservice Secondary Mathematics Teachers' Distinctions Between Functions and Equations

James Alvarez, The University of Texas at Arlington; Theresa Jorgensen, The University of Texas at Arlington; Kathryn Rhoads, The University of Texas at Arlington

Room: La Jolla

For more than thirty years, the secondary school mathematics curriculum has seen a shift to functions-based approaches to algebra. Advancing comprehension of the equals sign as an equivalence relation is critical for beginning algebra students studying equations, and developing understanding of functions is foundational as a gateway to courses required of science, technology, engineering, and mathematics majors. This study explores the ways in which mathematics majors seeking secondary mathematics teaching certification distinguish between the concepts of function and equation. Participants (n=24) completed a ten-item pre- and post-assessment on functions and equations. Open coding techniques were used to identify emerging categories that describe participants' distinctions between the concepts. After a mathematics course experience with an eight-week unit on functions, the participants' concept image for functions focused primarily on input and output whereas their concept image for equations centered broadly on the equivalence of two quantities.

Measuring Self-Regulated Learning: A Tool for Understanding Disengagement in Calculus I

Benjamin Sencindiver, Colorado State University; Mary Pilgrim, Colorado State University; James Folkestad, Colorado State University

Room: Point Loma 2&3

Calculus 1 has been and continues to be a key gateway course to STEM majors, which contributes to a loss of students in the STEM pipeline. Student-learning behaviors impact performance and, in turn, the student experience. By analyzing early online homework activity and help seeking, rich descriptions of students can be used for early prediction for at-risk students, but can be misrepresentative for students who have not yet engaged with these resources. This preliminary report presents self-regulated learning (SRL) theory as a way to understand student behaviors. Using this framework, online tools were designed to collect behavioral data which was used to create a SRL score based on in-course student activity. This preliminary report presents findings on the relationship between student behaviors in Calculus I, a behavioral SRL score, and failure rates, particularly with students disengaged with course content.

Shape Thinking: Covariational Reasoning in Chemical Kinetics

Jon-Marc Rodriguez, Purdue University ; Marcy Towns, Purdue University; Kinsey Bain, Purdue University

Room: Private Dining Room

This work addresses the following research question: In what ways do students use mathematics in combination with their knowledge of chemistry and chemical kinetics to interpret concentration versus time graphs? The study was designed and implemented using a resource-based model of cognition as the theoretical framework. Data was collected through the use of an assessment involving short-answer test items administered to 109 students in a first-year, non-majors chemistry course at a Swedish university. The student responses were translated from Swedish to English and subsequently coded. Data analysis involved using the shape thinking perspective of graphical reasoning as a methodological framework, which was adapted to analyze the covariational reasoning used by students in the context of chemical kinetics. Open-coding and considerations of shape reasoning have provided insight into student understanding of mathematical models of chemical processes.

Teacher Learning About Mathematical Reasoning: An Instructional Model

Robert Sigley, Texas State

Room: Del Mar

First, I describe an instructional model for Teacher Learning about Mathematical Reasoning (TLMR), designed for pre-service (PSTs) and in-service teachers (ISTs) to: (a) build knowledge of the various forms of mathematical reasoning that students naturally make use of in their justifying solutions to problems, (b) attend to the development of students' mathematical reasoning from studying videos and student written work, and (c) learn about the conditions and teacher moves that facilitate student justifications of problem solutions. Second, I provide a detailed description of activities from a representative cycle of the TLMR model. Finally, I report briefly on preliminary results indicating teacher growth in identifying and recognizing student reasoning for PSTs and ISTs who underwent the TLMR model compared to a comparison group.

	Day	Time	Session Type
	Saturday	**12:00-13:00**	***Meal***

Lunch

Room: (Boxed Lunch)

Day	Time	Session Type
Saturday	**12:00-13:00**	***Special***

Mentoring Tables

Room: Point Loma 2&3

Mentors from the RUME community will lead roundtable discussions of the following topics:
1. The job search: Identifying fit and preparing your materials ;
2. The job search: navigating online and on-campus interviews;
3. Publishing in RUME;
4. Seeking NSF funding (in the Coronado Room)
5. Seeing fellowships to support graduate study;
6. Being a parent and academic;
7. Fostering new research collaborations

	Day	Time	Session Type
Session 23	**Saturday**	**13:10-13:40**	***Contributed Report***

Examining the Effectiveness of a Support Model for Introductory Statistics

Seth Chart, Towson University; Melike Kara, Towson University; Felice Shore, Towson University; Sandy Spitzer, Towson University

Room: Private Dining Room

This paper describes a pilot program, aimed at improving outcomes in Introductory Statistics, in which undergraduate peer coaches led teams of students in activities designed to address common misconceptions about statistics during weekly sessions. Preliminary analysis suggests that introducing these sessions may reduce the percentage of students that finish Introductory Statistics with a grade of D, F or W, although the small number of students in the pilot program did not provide sufficient power to detect statistical significance. We also observe that the population of students who attend most of the optional sessions seems to be a mixture of high performing students and lower performing students. Participants in the program reported mainly positive perceptions of the program's usefulness. We intend to continue investigating these observations in future iterations of the program where we hope to improve participation and refine the session activities.

Learning Progressions in Mathematics and Physics: An Example for Partial Derivatives

Paul Emigh, Oregon State University

Room: La Jolla

We describe the creation of a learning progression about partial derivatives that extends from lower-division multivariable calculus through upper-division physics courses for majors. This work necessitated three modifications to the definition of a learning progression as described in the literature. The first modification is the need to replace the concept of an upper anchor with concept images specific to different (sub)disciplines. The second modification is that rich interconnections between ideas is the hallmark of an expert-like concept image. The final modification is using representations in several ways to support the development of translational fluency in emerging experts. These theoretical changes are supported by examples of research and curriculum in the use of differentials in thermodynamics.

Pedagogical Considerations in the Selection of Examples for Definitions in Real Analysis

Brian Katz, Augustana College; Tim Fukawa-Connelly, Temple University; Keith Weber, Rutgers University; Juan Pablo Mejia-Ramos, Rutgers University

Room: Coronado

This study investigates mathematicians' pedagogical practices and associated beliefs about the use of examples to instantiate definitions in a real analysis textbook. We used task-based interviews, asking participants to revise the introductory presentation of a concept, including definitions and examples, to be of higher pedagogical quality. All mathematicians believed that examples and counter-examples are important in learning about a concept. In this report, we concentrate on how mathematicians take the collection of examples and student thinking into account when deciding on which examples to use and the types of criteria they use to determine an appropriate collection of examples for a definition.

Peer Mentoring Mathematics Graduate Student Instructors: Discussion Topics and Concerns

Kimberly Rogers, Bowling Green State University; Sean Yee, University of South Carolina

Room: Del Mar

We developed and implemented a peer-mentoring program at two US universities whereby nine experienced mathematics graduate student instructors (GSIs) each mentored three or four first- and second-year GSIs (novices). Mentors facilitated bi-weekly small group meetings with context-specific support to help novices use active-learning techniques and augment productive discourse (Smith & Stein, 2011). Meeting discussion topics were informed by novices' interests, concerns raised by both mentors and novices, and ideas from other small groups. We examined what topics from small-group peer-mentoring meetings novices valued and timing of the topics that mentors suggested for future cycles. We qualitatively coded meeting topics and analyzed novices' ratings of topics discussed. Results indicate specific topics novices valued and the importance of timing some topics appropriately, informing future professional development for GSIs. These results offer insight and synergy between educating GSIs and improving undergraduate mathematics teacher pedagogy.

Scaling-Continuous Variation: A Productive Foundation for Calculus Reasoning

Robert Ely, University of Idaho; Amy Ellis, University of Georgia

Room: Point Loma 2&3

This paper introduces a new mode of variational and covariational reasoning, called scaling-continuous reasoning. Scaling-continuous reasoning builds on Leibniz' ideas of increments and infinitesimals and does not rely on images of motion. Instead, it entails (a) imagining a variable taking on all values on the continuum at any scale, (b) understanding that there is no scale at which the continuum becomes discrete, and (c) re-scaling to any arbitrarily small increment for x and coordinating that scaling with associated values for y. We present one clarifying example of this type of reasoning and argue that scaling-continuous reasoning can support a robust understanding of foundational ideas for calculus, including rates of change, differentiation, and the definite integral.

	Day	Time	Session Type
Session 24	**Saturday**	**13:50-14:20**	***Contributed Report***

Evaluation of Impact of Calculus Center on Student Achievement

Cameron Byerley, Colorado State University; Travis Campbell, Colorado State University; Brian Rickard, University of Arkansas

Room: Private Dining Room

Many universities are spending resources to establish math tutoring centers. Sharing information about the effectiveness of such centers is crucial to determine how to allocate resources. We illustrate methods of evaluating tutoring centers. We investigate the question, "what is the association between students' attendance at the Colorado State University Calculus Center and their grade in Calculus II?" We found a statistically significant positive correlation between students' tutoring center participation and their grades.

Graphing as a Tool for Exploring Students' Affective Experience as Mathematics Learners

V. Rani Satyam, Michigan State University; Mariana Levin, Western Michigan University; John Smith, Michigan State University; Theresa Grant, Western Michigan University; Kevin Voogt, Michigan State University; Younggon Bae, Michigan State University

Room: Point Loma 1

Researching affective issues can be difficult in education; methods like interviews and surveys can place artificial categories on participants' experience and exert biased influence. This lack of tools to study affect calls for better methods. We explore graphing as a potential tool with affordances for studying affect, by reporting results of three separate studies at different timescales where undergraduates graphed affective phenomena like confidence or emotion: two in an introduction to proof course and one in a pre-service teacher content course. By systematically describing each study and looking across the three, we argue that graphing can be a useful technique for representing experience. Its utility lies in aligning research goals with the structure imposed by the temporal axis. More structure along the temporal axis allows researchers access to what a student experiences at predetermined temporal points and less structure allows access to what students themselves find to be salient events.

How Do We Teach Thee? Let Me Count the Ways. A Syllabus Rubric with Practical Promise for Characterizing Mathematics Teaching

Sandra Laursen, Ethnography & Evaluation Research; Tim Archie, IDEA Center

Room: Point Loma 2&3

Good methods to characterize teaching are needed to describe both current status and changes in teaching practice, and to link student outcomes to particular instructional practices. Such methods are understudied and thus the relative merits of different methods are not well understood. As part of a study examining multiple methods for characterizing teaching in college mathematics, we analyzed syllabi using three rubrics. Syllabi are authentic course artifacts that reflect course design and instructor's intentions; they are readily available from instructors. One of these rubrics, an evaluative rubric known as Measuring the Promise (MtP), proved useful in distinguishing courses taught by a sample of seven early-career instructors and a comparison sample of experienced active learning practitioners. Good correlation of MtP scores with observation scores using the well-established Reformed Teaching Observation Protocol suggest that the MtP may be a useful alternative to costly and time-consuming observations.

Students' Strategies for Setting up Differential Equations in Engineering Contexts

Omar Naranjo, Brigham Young University; Steven Jones, Brigham Young University

Room: La Jolla

Ordinary differential equations (ODEs) comprise an important tool for mathematical modelling in science and engineering. This study focuses on how students in an engineering system dynamics course organized the act of setting up ODEs for complex engineering contexts. Through the lens of ODEs as a "coordination class" concept, we examined the strategies that seemed to guide the students' interpretations of problem tasks and their activation of knowledge elements during the tasks, as the students worked to produce ODEs for those tasks. This led to our uncovering of three main strategies guiding the students' work, and the finding that being able to flexibly draw on all of these strategies may be beneficial for student success.

What are Conveyed Meanings from a Teacher to Students?

Hyunkyoung Yoon, Arizona State University

Room: Coronado

In this paper we provide a new lens to explain conceptual connections between what a teacher knows, what a teacher does in the classrooms, and what his or her students learn. We observed three teachers' lessons and interviewed the three teachers and their students. By examining our data, we see that teachers' meanings and their assumptions about what students already understand have an impact on the ways they expressed their meanings during instruction. Then, students developed their meanings in trying to understand what the teacher said and did. Our analyses suggest that teachers need to think about how students might understand their instructional actions so that they can convey what they intend to their students.

	Day	Time	Session Type
Session 25	**Saturday**	**14:30-15:00**	***Contributed Report***

Collective Argumentation Regarding Integration of Complex Functions Within Three Worlds of Mathematics

Brent Hancock, University of Northern Colorado

Room: Private Dining Room

Although undergraduate complex variables courses often do not emphasize formal proofs, many widely-used integration theorems contain nuanced hypotheses. Accordingly, students invoking such theorems must verify and attend to these hypotheses via a blend of symbolic, embodied, and formal reasoning. This report explicates a study exploring student pairs' collective argumentation about integration of complex functions, with emphasis placed on students' attention to hypotheses of integration theorems. Data consisted of task-based, semistructured interviews with pairs of undergraduates, as well as classroom observations. Findings indicate that participants' explicit qualifiers and challenges to each other's assertions catalyzed new arguments allowing students to reach consensus or verify conjectures. Although participants occasionally conflated certain formal hypotheses, their arguments married traditional integral symbolism with dynamic gestures and clever embodied diagrams. Participants also took care to avoid invoking attributes of real numbers that no longer apply to the complex setting. Teaching and research implications are discussed as well.

Generalizations of Convergence from R to R^2

Zackery Reed, Oregon State University

Room: Point Loma 1

Sequential convergence is a powerful tool in the field of real analysis. Though its structure persists throughout various metric spaces, students initially understand sequential convergence as it manifests on the real line. Students often do not encounter more generalized forms until advanced analysis courses. As part of multiple teaching experiments, students were given the opportunity to generalize sequential convergence from R into the R2. This report will demonstrate various generalizations rooted in reflective abstraction of convergence in R. We will also discuss students generalizing by reduction, reflecting on the utility of distance as a map between spaces.

Planning to Succeed in a Computer-Centered Mathematics Classroom

Geillan Aly, University of Hartford

Room: Del Mar

Most developmental mathematics students in community colleges, particularly those of color, are unsuccessful and fail to reap the benefits of higher education. In-class computer-centered (ICCC) classes are a possible solution to this issue because students work independently at computers during class time while instructors facilitate learning by answering students' questions. This case study focuses on one student's ICCC classroom experience by focusing on how the student's plan to pass the course were validated by the classroom environment. Ultimately, that plan was insufficient to address the needs of the student.

Relationships between Precalculus Students' Engagement and Shape Thinking

Derek Williams, NC State University

Room: Coronado

This study examines relationships between community college precalculus students' understanding and engagement to link mathematical success to a malleable construct, and offer new insights for addressing consistently poor success rates in community college precalculus (Barnes, Cerrito, & Levi, 2004). Two-part interviews, consisting of a task and debriefing, were conducted with 8 students to investigate their shape thinking (Moore & Thompson, 2015), and engagement, conceptualized through flow theory (Csikszentmihalyi, 1975). Results suggest that students can be highly engaged in mathematics tasks regardless of understanding and that students exhibiting different ways of thinking about graph construction tended to experience different forms of engagement.

Sparky the Saguaro: A Teaching Experiment Examining a Student's Development of the Concept of Logarithms

Emily Kuper, ASU; Marilyn Carlson, Arizona State University

Room: La Jolla

A number of studies have examined students' difficulties in understanding the idea of logarithm and the effectiveness of non-traditional interventions. However, few studies have examined the understandings students develop when completing conceptually oriented exponential and logarithmic lessons that build off prior research and understandings. This study explores one undergraduate precalculus student's understandings of concepts foundational to the idea of logarithm as she works through an exploratory lesson on exponential and logarithmic functions. Over the course of a few weeks, the student participated in a teaching experiment that focused on Sparky – a mystical saguaro that doubled in height every week. The lesson was centered on growth factors and tupling periods in an effort to support the student in developing the understandings necessary to discuss logarithms and logarithmic properties meaningfully. This paper discusses an essential component that students must conceptualize in order to hold a productive meaning for logarithms and logarithmic properties.

The Authority of Numbers: Fostering Opportunities for Rational Dependence in a Mathematics Classroom

Ander Erickson, University of Washington Tacoma

Room: Point Loma 2&3

This cross-case analysis of quantitative literacy instruction at the undergraduate level compares three different settings where activities were introduced that required students to seek out and make use of information outside of the classroom. These activities provided students with opportunities to engage with quantitative claims made by experts and by comparing these cases I was able to identify several axes of variability that affect the extent to which the problems supported the practice of rational dependence, or the reasoned dependence on the knowledge of others. These variables include the extent to which students are held accountable for their choices of information sources, the way in which the teacher frames what it means to critically appraise a quantitative claim, and the role that mathematics plays in the activity.

		Day	Time	Session Type
Session	**Poster Session 2**	**Saturday**	**15:00-16:00**	***Poster***

A Hypothetical Learning Trajectory (HLT) for Preservice Secondary Teachers' Construction of Congruence Proofs

Rachel Zigterman, University of Nebraska-Lincoln; Yvonne Lai, University of Nebraska-Lincoln

S35

With the advent of the Common Core State Standards, there has been renewed interest in teaching geometry from a transformation perspective; however, most geometry teachers are unfamiliar with this approach as they learned geometry from a perspective based on Euclid's Elements. Consequently, there is little knowledge of how teachers who come from this traditional perspective learn geometry from a transformation approach. One major difference that teachers must reconcile is in the construction of congruence and similarity proofs. As such, there is a need to understand how teachers learn these proofs from a transformation perspective. We propose to present a poster reporting a hypothetical learning trajectory (HLT) for preservice teachers' construction of such congruence proofs, based on the coursework of 15 preservice secondary teachers and cognitive interview responses to geometry tasks.

Adaption of Sherin's Symbolic Forms for the Analysis of Students' Graphical Understanding

Jon-Marc Rodriguez, Purdue University ; Kinsey Bain, Purdue University; Marcy Towns, Purdue University

S15

We describe a methodological presentation of Sherin's (2001) symbolic forms, discussing adaptions made to the framework to analyze graphical reasoning. Symbolic forms characterize the ideas students associate with patterns in an expression. To expand symbolic forms beyond equations, we supplement it with another framework that considers modeling as discussing mathematical narratives. This affords the language to describe how students think about the process or "story" that could have given rise to a graph. By considering registrations in general terms as structural features students attend to (parts of the "story"), when students assign ideas to registrations (parts of an equation or regions of a graph), they are using symbolic forms.

An Instructional Resource for Improving Students' Conceptual Understanding of Functions through "Reflective Abstraction"

Jessica Lajos, University of Oklahoma; Sepideh Stewart, University of Oklahoma

S19

It has been widely documented that undergraduate-level students' understanding of functions is rigid and indicative of an action view which constrains conceptual understanding (Carlson, Jacobs, Coe, & Hsu, 2002). Duval affirms that, "to understand the difficulties that many students have with comprehension of mathematics, we must determine the cognitive functioning underlying the diversity of mathematical processes" (2006, p.103). What are the underlying cognitive skills students need to gain a better conceptual understanding of functions? How should instruction of function content and training in these cognitive skills be combined? We propose the theoretical model "Structural-Schema Development for a Function", to address these questions. This model defines developmental stages students pass through to form a global view for the function concept, identifies underlying cognitive mechanisms involved in each stage, and develops instructional exercises that combine content with cognitive skills training for these cognitive mechanisms.

Assessing the Development of Students' Mathematical Modeling Competencies: An Information Entropy Approach

Yannis Liakos, University of Agder; Yuriy Rogovchenko, University of Agder

S10

We suggest a new scaling tool for converting big amounts of qualitative data into quantitative data based on the recent developments in the information theory. We believe that it can be used with reasonable efficiency for the monitoring of the development of students' mathematical competencies, and not only. Discussing advantages and shortcomings of a new tool along with the possibilities for further development, we invite to discussion of the new approach.

Bridge Programs for Engineering Calculus Success

Sandra Nite, Texas A&M University; Michael Sallean, Texas A&M University

S34

Mathematics is often the gatekeeper for students aspiring for a college degree in any field. Engineering is no different. A precalculus bridge program to support engineering students for success in their engineering calculus sequence was initiative at a southern university in Texas in summer of 2010. Students who did not meet the cut score on the Mathematics Placement Exam for Engineering Calculus I were offered the program. The program was changed in length, and bridges to Engineering Calculus II and II were added. In addition to MPE scores and course grades, surveys were administered to examine student beliefs about college level mathematics expectations and requirements. This poster will illustrate information from various papers about the bridge programs and their results. Keywords - Calculus, Bridge program

Characterizing Self Explanations for Undergraduate Proof Comprehension

Kristen Amman, Rutgers University

S27

A study was conducted with 11 undergraduate students in a real analysis course to further investigate important results reported by Hodds, Alcock, and Inglis (2014) on self explanation and undergraduate proof comprehension, and by Ainsworth, S., & Burcham, S. (2007) on self explanation and textual coherence. The main product of the current study is a framework of self explanations in proof comprehension that takes into account students' questions as they self explain both high and low coherence proofs.

Connecting Advanced Undergraduate Mathematics to School Mathematics

Elizabeth Arnold, Montana State University; James Alvarez, The University of Texas at Arlington

S30

META Math is a project to create, pilot, and field-test modules for use in undergraduate mathematics and statistics courses taken by pre-service teachers. Materials in calculus, discrete mathematics, algebra, and statistics showcase vital connections between college mathematics and the mathematics taught in high school. Drawing on recommendations in the Mathematical Education of Teachers II and the Statistical Education of Teachers, the project puts attending to the needs of pre-service teachers on par with attending to the needs of other undergraduate students by focusing on applications related to high school mathematics teaching.

Constant Rate of Change: The Reasoning of a Former Teacher and Current Doctoral Student

Natalie Hobson, Sonoma State University

S17

In this work, I provide brief illustrations of multiple ways of reasoning about constant rate of change that I observed in a mathematics education doctoral student's activity when tasked to draw graphs relating two varying quantities. These ways of reasoning suggest that textbook authors and instructors critically examine those illustrations and experiences provided to students in order for students to come away from mathematics courses with consistent and productive reasonings about rate of change.

Content Analysis of Introductory Textbooks in Point-Set Topology

Daniel Cheshire, Texas State University; Joshua Fagan, Texas State University

S32

This study compared twelve point-set topology textbooks at the introductory level. The goal was to differentiate each textbook according to its overall conceptual approach to the field, as well as its mathematical approach to four fundamental topological ideas. The analysis indicated significant differences in the conceptual and mathematical presentation of those topics among the twelve textbooks. These findings highlight the need for researchers to distinguish between the conceptual and mathematical approaches found in textbooks for proof-intensive courses in undergraduate mathematics education.

Cooperative Learning and its Impact in Developmental Mathematics Courses: A Case Study in a Minority-Serving Institution

Eyob Demeke, California State University Los Angeles

S47

In this poster, we report on the evolution of developmental students' mathematics background knowledge after a four-week long course that emphasized active learning. The research took place at a large Hispanic serving institution in the state of California. Students' progress or lack thereof was measured using a diagnostic test developed by the Mathematics Diagnostic Testing Project (MDTP). These students were initially considered not ready for college level mathematics course work and were subsequently enrolled, in a four-week summer course which is designed to prepare them for a college level math course. During each class, students would spend at least 30 minutes engaging in cooperative learning that utilizes active learning strategies such as think-pair share, peer lesson, and wait time. A pre/posttest analysis of SYART showed that these students showed a statistically significant growth, leading us to conclude that the four-week intervention in math remediation had a considerable impact.

Design Research in German Mathematics Tertiary Education Focusing on Profession-Specificity

Wessel Lena, University of Education Freiburg

S16

The poster proposal presents design research projects in the context of German tertiary education for preservice secondary teachers and service mathematics courses. The approach of design research for university students with a content-specific focus on profession-specificity is exemplified by two concrete design research projects.

Development of reasoning about rate of change, based on quantitative and qualitative analysis

inyoung lee, Arizona State University

S45

Pre-calculus and Calculus are two big compartments as we consider their developmental and complemental attribute. I analyzed data quantitatively from a series of pre-calculus assessments conducted in a large public university 2017 fall, then investigated the result and its impacts qualitatively in calculus context focused on rate of change. The two-part analysis consists of discerning intrinsic factors in the assessment items that have a large effect on overall performance followed by clinical interviews about meaning of the Fundamental Theorem of Calculus and its applications. The results support my claim that the ability to conceptualize constant rate of change has a considerably positive effect on students' reasoning about rate of change and the Fundamental Theorem of Calculus as well.

Examining Exams, Evaluating Evaluations: An Alternate Approach Assessed

Kayla Blyman, USMA - West Point Kristin Arney, USMA - West Point David del Cuadro-Zimmerman, USMA - West Point Lisa Bromberg, USMA - West Point

S29

In an attempt to bring a more realistic environment into the classroom during assessments, an alternative form of assessment was piloted during a mathematical modeling course at (Redacted for Blinding) in the fall of 2017. The "alternate" assessments are primarily conceptual in nature and consist of three parts: a night before read-ahead introducing a new application, an in class individual portion, and an in-class group portion. Through the evaluation of this assessment technique, we hope to determine if it should be expanded to a larger audience in the future. Preliminary finding of this evaluation are presented.

Exploring Pre-service Elementary Teacher's Relationships with Mathematics via Creative Writing and Survey

Taekyoung Kim, Oklahoma State University

S25

Thirty-two pre-service elementary teachers completed a survey regarding their beliefs and attitudes regarding learning and teaching mathematics and two creative writing tasks. In the writing tasks, participants described their relationship with personified mathematics and introduced personified mathematics to their future students. By interpreting the survey and writings, different aspect of attitudes towards mathematics were discovered.

Exploring Students' Reports of Exclusion in Calculus II

Jessica Hagman, Colorado State University; Matthew Voigt, San Diego State University; Nancy Kress, University of Colorado, Boulder; Jessica Gehrtz, Colorado State University

S12

We seek to understand experiences of students who report exclusion in the Calculus II class at one technical institution. From the perspective that learning is a sociocultural process we understand students' sense of inclusion or exclusion to play an important role in how they experience mathematics learning opportunities. Our paper explicates the methods used for our analysis as well as our findings related to describing the population of students who report high levels of exclusion and their experiences in Calculus II.

Exploring the Efficacy of a Game-Based Learning Application in Undergraduate Mathematics: Functions of the Machine

Jessica Lajos, University of Oklahoma; Yutian Thompson, University of Oklahoma; Sarah Klankey, Unversity of Oklahoma; Javier Elizondo, University of Oklahoma; Will Thompson, University of Oklahoma; Melanie Lewis,

S20

To explore the efficacy of a game-based learning application to facilitate conceptual understanding of function concepts through covariational reasoning, the Virtual Learning Experience Team affiliated with the Universities Education Research and Outreach Center developed a digital game titled Functions of the Machine. In the game the student plays the role of a scientist tasked with making a complex machine run. The student explores, tests and fixes the machines moving parts which consists of gears, fluid tanks, and conveyor belt contraptions. Through a series of interactive scaffolded problems, students' transition from proportional reasoning to complex covariational reasoning simulations. It is hypothesized that the visually dynamic game-based learning environment may be better equipped at helping students develop covariational reasoning compared to visually static homework exercises.

Exploring the Role of Active Learning in a Large-Scale Precalculus Class

Gregory Downing, North Carolina State University; Brooke Outlaw, North Carolina State University

S5

In a large undergraduate mathematics classroom, introducing evidence based learning practices can be challenging. Due to persisting outdated methods of teaching, results of recent research call for more investigation of active-learning in all STEM classrooms, including large scale ones. Using Fraser's (1989) lens on perception, results from this study indicate that students who participated in Team Activities and other learner-centered activities in a large scale precalculus undergraduate class reported good experiences and are more positive in their attitudes towards mathematics.

Features of Tasks and Instructor Actions That Promote Preservice Secondary Mathematics Teachers' Understanding of Functions

Janessa Beach, The University of Texas at Arlington; James Alvarez, The University of Texas at Arlington; Theresa Jorgensen, The University of Texas at Arlington; Kathryn Rhoads, The University of Texas at Arlington

S22

The Enhancing Explorations in Functions for Preservice Secondary Mathematics Teachers Project is developing research-based tasks and explorations as well as instructor materials to be used in mathematics courses for preservice secondary mathematics teachers. The project, now in year two, continues to develop and refine these items based on data collected in year one and the advice of an expert panel and advisory board. The goal of this poster presentation is to provide information on lesson development and methods used in determining key characteristics of instructor moves for building student understanding of functions as well as gather feedback and suggestions on further design and development.

Gender-based Analysis of Learning Outcomes in Inquiry-Oriented Linear Algebra

Brooke Athey, Florida State University

S28

In order to better understand gender-based differences in learning experiences and outcomes in inquiry-oriented instructional settings, we analyze data from a common end-of-term assessment administered across 7 sections of linear algebra. This analysis focuses on data from 58 students, 22 of whom identified as female and 36 of whom identified as male. Distribution of the 58 students' scores was negatively skewed, similar to that of broader sample of 153 assessment scores (many of which did not have gender information available as that was collected separately from the assessment). A two-tailed t-test with independent samples was administered, revealing that the difference between the scores of the two groups is not statistically significant.

Goals, Resources, and Orientations for Equity in Collegiate Mathematics Education Research

Shandy Hauk, WestEd; Katie D'Silva, WestEd

S37

Though the terms equity, diversity, inclusion, and social justice have entered the research lexicon, we face significant challenges in gaining a nuanced understanding of the various ideas associated with these words and how those ideas are consequential for collegiate mathematics education research. This interactive poster presents a theoretical framework for making sense of (and making sense with) "equity" as an essential component of research. The poster offers tools for thinking and talking about equity and research design, implementation, and reporting. Poster visitors will have an opportunity to contribute questions and observations about the definitions of equity and proposed connections among approaches to courageous conversations about equity in research, self- and other-awareness, and aspects of equity in the mathematics content, curricula, and instruction at the heart of the research.

Here's What You Do: Personalization and Ritual in College Students' Algebraic Discourse

Cody Patterson, University of Texas at San Antonio; Luke Farmer, University of Texas at San Antonio

S14

We present results of a discourse analysis focused on college algebra students' uses of personal and impersonal language, references to endorsed mathematical routines, and inferences about mathematical objects in responses to a small-group problem-posing activity. We analyze students' responses with respect to selected dimensions of the arithmetical discourse profile of Ben-Yehuda et al., and provide evidence of a positive association between impersonal language and the presence of object-level mathematical statements and precise uses of algebraic terminology.

Historical analysis on predictive practices: the case of chaotic dynamics

JESUS ENRIQUE HERNÁNDEZ-ZAVALETA, CINVESTAV; Ricardo Cantoral, CINVESTAV

S13

This poster focuses on the historical analysis of three main characters of the history of chaos: the Poincare's error in his memoire about the three-body problem, the ideas of Edward Lorenz about the deterministic non-periodic flow, and the work of Robert May in the analysis of the logistic map. This into the framework of Variational Thinking and Language research program from the Socioepistemological Theory. The results show a predictive practice characterized for four main actions: to search periodicities, to recognize the uncertain, to compare temporal states, and to classify kinds of behaviors. We assume that the promotion of landscapes and activities out of school are a way for construct specialized mathematical knowledge, and the incorporation of the ability to wait for the unexpected is necessary for teachers and students living this century.

How Diagrams are Leveraged in Introduction to Proof Textbooks

Michael Abili, Texas State University; Elizabeth Hewer, Texas State University; Kristen Lew, Texas State University; Kathleen Melhuish, Texas State University; Robert Sigley, Texas State

S1

According to research, diagrams can play a vital role in the constructing and understanding of proofs (Samkoff, Lai, & Weber, 2012). Introduction to Proof (ITP) courses are usually a student's first exposure to proofs. Therefore, the ITP curriculum reflects important opportunities for students to develop proof construction and proof understanding skills. We analyzed how diagrams were presented in the top four market share ITP textbooks across a set of standard topics. Through this process, we categorized the role and nature of diagrams in the curricula. We found that a majority of diagrams were used to illustrate statements and definitions. Other important roles such as supporting proof construction, building conjectures, or finding counterexamples were infrequent.

How Experts Conceptualize Differentials: The Results of Two Studies

Tim McCarty, West Virginia University; Vicki Sealey, West Virginia University

S7

The mathematical symbol "dx" is a symbol for which there can exist different views about its characteristics, purposes, and roles. We conducted two studies to see how experts viewed the dx in a variety of contexts. For our first study, we interviewed four mathematicians in order to understand their various concept images of the dx, and for our second study, we interviewed two mathematicians and one physicist about both their own concept images and the concept images they would like for their students to have. Overall, we found little agreement among all of the experts' responses, and we believe that that further study of experts' concept images of the differential is warranted.

If f(2)=8 then f'(2)=0: A Common Misconception, Part 2

Alison Mirin, Arizona State University; Stephen Shaffer, Arizona State University

S46

This study reports calculus students' failure to differentiate the cubing function when represented piecewise as f(x)=x^3 if x/=2, f(x)=8 if x=2. The data reported here suggest that students did not fail simply due to inattention to the function definition; when reminded that 2 cubed is 8 and prompted to compare the graph of f to that of the cubing function, student performance increased, but was still poor, suggesting the presence of deep-seated misunderstandings.

Investigating Student Learning and Sense-Making from Instructional Calculus Videos

Aaron Weinberg, Ithaca College; Matthew Thomas, Ithaca College; Jason Martin, University of Central Arkansas; Michael Tallman, Oklahoma State University

S18

Growing interest in "flipped" classrooms has made video lessons an increasingly prominent component of post-secondary mathematics curricula. This format, where students watch videos outside of class, can be leveraged to create a more active learning environment during class. Thus, for very challenging but essential classes in STEM, like calculus, the use of video lessons can have a positive impact on student success. However, relatively little is known about how students watch and learn from calculus instructional videos. This research generates knowledge about how students engage with, make sense of, and learn from calculus instructional videos.

Math Help Centers: Factors that Impact Student Perceptions and Attendance

Christine Tinlsey, University of Oklahoma; Beth Rawlins, University of Oklahoma; Deborah Moore-Russo, University of Oklahoma; Milos Savic, University of Oklahoma

S40

Mathematics help centers have become more common in post-secondary education, but there is scant research on them. In this study we use data from 1088 students over six academic semesters and grounded theory analysis techniques to study and draw initial conclusions on student perceptions of and reasons for attending a math center.

Mathematics Through the Lens of Service-learning

sayonita Ghosh Hajra, Hamline University; Jen England, Hamline University; Chloe Mcelmury, Hamline University; Hani Abukar, Hamline University

S9

In this poster, we report on a study conducted at a midwestern private liberal arts university where researchers incorporated service-learning into a non-major mathematics course. Data reveal students felt more confident learning mathematical concepts because of its real-world application in the community. Additionally, students reported an increase in the value of mathematics and its importance in societal reforms.

Quantum Physics Students' Reasoning about Eigenvectors and Eigenvalues

Kevin Watson, Virginia Tech; Megan Wawro, Virginia Tech; Warren Christensen, North Dakota State University

S3

Eigentheory is an important mathematical tool for modeling quantum mechanical systems, but little is known about how physics students reason about eigenvectors and eigenvalues as they transition from linear algebra courses into quantum mechanics. In this poster, we share examples of the resources (elements of students' knowledge) we have identified in physics students' reasoning about the eigenvectors and eigenvalues of real 2x2 matrices, as well as connections among these resources within and across students.

Student Intuition Behind the Chain Rule and How Function Notation Interferes

Justin Dunmyre, Frostburg State University; Nicholas Fortune, North Carolina State University

S8

Recently, Speer and Kung (2016) informed the RUME community on what was missing from our research. In an effort to begin to fill these gaps in the literature, we explored students conceptual understanding of the chain rule in Calculus I classrooms taught by the first author. In this teaching experiment (Steffe & Thompson, 2000), our preliminary results indicate that if students are afforded opportunities to engage in experientially real tasks (Freudenthal, 1991; Rasmussen & King, 2000) on the chain rule, they understand the purpose it serves and can extend that understanding to varied contexts. However, the largest interference to this understanding was function notation, particularly nested function notation. Implications indicate that the instruction of chain rule could be enhanced by preempting a chain rule unit with nested function notation, while still maintaining tasks centered around a conceptual understanding of the chain rule.

Student Resources for Unit and Position Vectors in Cartesian and Non-Cartesian Coordinate Systems

Warren Christensen, North Dakota State University; Brian Farlow, North Dakota State University; Marlene Vega, California State University - Fullerton; Michael Loverude, California State University Fullert

S4

As part of a broader study into students understanding of students' use of mathematics in upper-division physics courses, this study investigates how students conceptualize unit and position vectors in Non-Cartesian Coordinate Systems using a theoretical framework of resources. We present a case study of Mark, a Senior physics major, and identify the resources that Mark activates while answering conceptual questions without a direct physics context during a one-on-one interview protocol. This analysis identifies specific resources that Mark brings to bear when reasoning about vectors. The results of this case study provide a guide for analyzing additional interviews and allow us to pursue the long-term goal of curriculum development that can be used to improve students use and understanding of non-Cartesian coordinate systems.

Student's Attention to the Conclusion During Proofs

Sindura Subanemy Kandasamy, Texas State University; Kathleen Melhuish, Texas State University

S2

This study investigates students' use of conclusions to structure their proofs for a standard statement in introductory Group Theory. We surveyed 65 students across three classes asking them to evaluate the truth of a statement and provide a proof. We found students tend to use hypothesis-driven second level proof framework (rather than conclusion-driven). These students were then less likely to produce a deductive argument that aligned with the original statement. We conclude with implications for the treatment of proof analysis and proof frameworks to support students' proving activity.

Students' Understanding of Quadratic Equations

Jonathan López Torres, North Carolina State University

S38

Action–Process–Object–Schema theory (APOS) was applied to study student understanding of quadratic equations with one variable. This requires proposing a detailed conjecture (called a genetic decomposition) of mental constructions students may use to understand quadratic equations. The genetic decomposition, which was proposed, can contribute to help students achieve an understanding of quadratic equations with improved interrelation of ideas and more flexible application of solution methods. Semi-structured interviews with eight beginning undergraduate students explored which of the mental constructions conjectured in the genetic decomposition students could do, and which they had difficulty doing. Two of the mental constructions that form part of the genetic decomposition are highlighted and corresponding further data was obtained from the written work of 121 undergraduate science and engineering students taking a multivariable calculus course. The results suggest the importance of explicitly considering these two highlighted mental constructions.

Supporting Prospective Teachers' Understanding of Triangle Congruence Criteria

Steven Boyce, Portland State University; Priya Prasad, University of Texas at San Antonio

S6

This poster describes an instructional sequence for supporting college geometry students' justifying Euclidean triangle congruence criteria using properties of isometries. We hypothesized that investigating transformations in the taxi-cab metric would perturb students' understandings of the relationships between triangle congruence criteria and isometries, so they would more explicitly identify the properties of transformations as a necessary part of their justifications of triangle congruence criteria. We report on the results of pre-post written assessments of our college geometry students' justifying SAS to a hypothetical 10th grade student.

Teachers' Knowledge of Fraction Arithmetic with Measured Quantities

Sheri Johnson, University of Georgia; Merve Kursav, Michigan State University

S44

Abstract: A national sample of 990 middle grades teachers consisting of a variety of undergraduate majors completed a knowledge assessment aimed at measuring teachers' knowledge of fraction arithmetic using measured quantities. Based on this sample, approximately 24% of middle grades teachers have an undergraduate degree in Early Childhood or Elementary Education. Using a simple measurement of the percentage of items answered correctly, these middle grades teachers with an undergraduate degree in Early Childhood or Elementary Education scored significantly lower than those who had a Business or STEM degree. The same cannot be said for students with an undergraduate degree in Mathematics Education. These findings reveal the importance of developing instruction focused on multiplicative reasoning with measured quantities for undergraduates majoring in education.

Teachers' reasoning with frames of reference in US and Korea

Surani Joshua, Arizona State University

S42

We gave approximately 180 US and 380 Korean teachers frame of reference tasks, and coded the open responses with rubrics intended to rank responses by the extent to which their responses demonstrated conceptualized and coordinated frames of reference. In both countries less than half of teachers scored at the highest level on almost every task, showing that teachers frequently struggle to keep track of quantities within a frame of reference in a meaningful way. Our US-Korean comparison also shows that US teachers struggle on most of these tasks significantly more than Korean teachers

Transformers! More than Meets the Eye!

Courtney Simmons, Oklahoma State University; Michael Oehrtman, Oklahoma State

S26

In this study, we characterize a conceptual model some students draw upon in their problem-solving activity when engaged in definite integral tasks. We call this model an Integral as a Transformer conception as it is invoked by students as a means to transform a quantitative relationship suitable for constant values into a structure appropriate for co-varying quantities.

Using Catan as a Vehicle for Engaging Students in Mathematical Sense-Making

Miller Susanna Molitoris, Kennesaw State University; Amy Hillen, Kennesaw State University

S43

Catan is an increasingly popular board game which is rich with opportunities for mathematical applications. The research presented in this poster demonstrates how Catan served as an effective vehicle for engaging students enrolled in a freshman learning community.Students engaged with various mathematical concepts involving probability, combinatorics, and game theory.

Using Everyday Examples to Understand the Concept of Basis

Jessica Knapp, Arizona State University; Michelle Zandieh, Arizona State University; Aditya Adiredja, The University of Arizona

S39

The purpose of this paper is to explore everyday examples given by students to explain the notion of basis. By exploring key aspects of the examples generated by the students we can see what roles and characteristics of basis the students attend to.

Video Case Analysis of Students' Mathematical Thinking to Support Preservice Teacher Candidates' Functional Reasoning and Professional Noticing

Tatia Totorica, Boise State University; Laurie Cavey, Boise State University; Michele Carney, Boise State University; Patrick Lowenthal, Boise State University; Jason Libberton, Idaho State University

S24

Using a design-based research approach, we are developing a series of online video-based instructional modules to engage secondary mathematics teacher candidates in case analyses of students' functional reasoning and to improve their own mathematical and pedagogical understandings. We present our project framework for module development, implementation, and revision, with an end goal of identifying preliminary hypothetical learning trajectories for candidates' functional reasoning and professional noticing.

What Would You Say You Do Here? Metaphor as a Tool to Characterize Mathematical Practice

Joseph Olsen, Rutgers University

S21

In the cognitive science literature, multiple researchers have pointed out the importance of metaphor as a cognitive mechanism for sense-making. In mathematics in particular, metaphor has been shown to be a valuable tool in making sense of and reasoning with mathematics. To our knowledge, there has been no research on the metaphors that professors use when communicating the nature of mathematical practice to students in advanced mathematics lectures. In this poster, we describe the metaphors the research team identified across 11 undergraduate mathematics lectures at the advanced level. We found metaphors used by many lecturers that convey ideas about the nature of mathematical practice. We identified the affordances of these metaphors to better understand the way that mathematicians describe mathematics as a practice to undergraduate students studying advanced mathematics at the undergraduate level.

What? How? Why? Resources Used to Teach the Fundamental Theorem of Calculus

Linda Leckrone, University of Michigan

S31

This poster presents findings from a larger study about what, how and why community college calculus teachers use a variety of resources when they teach and plan to teach the fundamental theorem of calculus (FTC). Interviews with 14 teachers revealed that most teachers used very similar resources in very similar ways, yet their reasons for using those resources varied.

Day	Time	Session Type
Saturday	**16:10-16:40**	***Special***

Awards Session

Room: Point Loma 2&3

Day	Time	Session Type
Saturday	**16:40-17:40**	***Plenary***

The Assessment and Improvement of Reading Comprehension in Proof-based Mathematics

Juan Pablo Mejia-Ramos

Room: Point Loma 2&3

Proof is central to mathematical practice: mathematics researchers and students spend a substantial amount of time constructing, presenting, and reading mathematical proofs. In this talk I will discuss the notion of reading comprehension in mathematics, and focus on the concept of proof comprehension, i.e. an individual's ability to read, process, and understand mathematical proofs. I will talk about the development of a model for assessing proof comprehension, and about different ways of using this model in the classroom and in mathematics education research. In particular, I will describe a method for producing and validating reliable proof comprehension tests, and report findings from the distribution of three of these tests to a large population of students in multiple universities. Finally, I will describe a number of instructional innovations to help improve students' proof comprehension, and discuss their effectiveness as measured by instruments designed using our proof comprehension model. I will end by discussing avenues for future research in the area.