Course Coordination Patterns in University Precalculus and Calculus Courses

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In this report we present findings from a preliminary investigation aimed at describing models of course coordination systems currently in place within university precalculus and single variable calculus courses. Hierarchical cluster analysis was used on national survey data to identify homogeneous clusters of courses based on the intended use of uniform course elements across sections. The analysis revealed eleven clusters of courses, nested within five larger groups. We briefly describe each of the eleven clusters in terms of the uniform course elements and the five larger groups in terms of the clusters nested within them. We then characterize these groups with respect to department type (Masters- versus PhD-granting), course level (Precalculus, Calculus 1, and Calculus 2), regularity of instructor meetings, and type of course coordinator.

Keywords: Course coordination; precalculus/calculus; course structure

With enrollments increasing in courses across the Precalculus to Calculus 2 (P2C2) sequence (Blair, Kirkman, & Maxwell, 2018), many mathematics departments have begun implementing course coordination systems in an attempt to create consistency in student learning opportunities within and across these courses (Apkarian & Kirin, 2017; Rasmussen et al., in press). While creating consistency in student learning opportunities broadly underlines such efforts, the exact reasons for implementing such a system vary across institutions. Rasmussen and Ellis (2015) point out that course coordination systems can be used to ensure uniformity in certain course elements (e.g., textbook, exams) for all sections of a particular course, and they suggest that building and implementing coordination systems can engender a sense of community among regular instructors of a coordinated course. Other studies suggest that these systems are important program components for enacting and sustaining change in P2C2 courses (Apkarian, Bowers, O'Sullivan, & Rasmussen, 2018; Pilgrim & Gehrtz, 2018) and that such change has the potential to positively impact student learning (Rasmussen, Ellis, Zazkis, & Bressoud, 2014). Taken together these studies provide some evidence that developing and implementing such a structured system for P2C2 courses has the potential to positively impact both instructor and student experiences within this sequence. Yet surprisingly little is known about how P2C2 coordination systems are currently organized and structured within mathematics departments across the United States. Thus, we ask: *What, if any, patterns of usage exist among different aspects and components of course coordination systems in undergraduate P2C2 courses*?

Background

Theory regarding course coordination systems, or how the coordination of particular course elements affects student experiences in introductory STEM courses is scarce in mathematics education literature. However, it is something many schools are interested in, it is important to begin that conversation (Apkarian, Kirin, Vroom, & Gehrtz, under review). One resource that stands out for thinking about course coordination of introductory mathematics courses is Rasmussen and Ellis's (2015) work, which suggests that in addition to controlling some of the variation in student experiences by controlling course elements, coordination systems may have a social component which engenders a sense of community among instructors. This sense of community affects the development of norms, including norms for teaching, which has an effect on student experiences in the classroom. A related body of work is that regarding *curriculum*. The constructs of written, intended, enacted, assessed, and learned curricula are particularly useful as different dimensions of course structure (Porter & Smithson, 2001; Stein, Remillard, & Smith, 2007). While postsecondary education is not governed by national standards and/or assessments as are the lower grades, a coordination system should, in principle, affect the curriculum. For example, a common course syllabus in Calculus 1, which includes a common textbook and a common set of topics to be covered, can be thought of as a common written curriculum; common exams relate to commonality in terms of the assessed curriculum. The transformation of written curriculum into intended and enacted curricula is affected by instructors' identity and situational context, which are likely impacted by regular course meetings and conversations about instruction.

Methods

Data for this analysis comes from a national survey aimed at investigating P2C2 programs across the country. The survey was completed by 223 of the 330 university departments offering an MA, MS, and/or PhD in mathematics in the United States of America. The survey covered many aspects of these programs and departments, as well as details about the P2C2 courses themselves. From this survey, we gathered detailed information about course delivery and management for 889 courses, 261 of which were categorized as Precalculus, 327 categorized as Calculus 1, and 301 categorized as Calculus 2. These details include what, if any, course elements are uniform across sections; regularity of instructor meetings; primary instructional approach for regular course meetings and recitations; role of coordinator; and the regularity with which the course is taught by differently ranked members of the university (e.g., research faculty, teaching faculty, graduate students). Our knowledge of the literature related to course coordination led us to select particular items from the survey to investigate as part of modeling course coordination systems. In particular, Rasmussen and Ellis (2015) point out that *systems* of coordination include both superficial aspects, such as coordinating course elements across specific course sections, and departmental features, such as the presence and role of course coordinators. In the analysis presented here, we consider the items related to the presence of uniform elements across course sections, regularity of instructor meetings, and the presence and role of a course coordinator for each of the 889 courses.

We began our analyses by grouping courses based on their response to the question about uniform course elements, specifically whether or not each of eleven[1] course elements were indicated as uniform across different sections of the course. Our intention with this analysis was twofold. First, to explore conjectures about what course elements are coordinated together or separately. Second, to reduce the uniform course element data for further analyses. Grouping was done using agglomerative hierarchical cluster analysis. Due to the binary nature of the data (e.g., coordinated/not coordinated) we used complete-linkage (or *farthest neighbor*) clustering and the Jaccard distance measure (Choi, Cha, & Tappert, 2010; Hastie, Tibshirani, & Friedman, 2009). Agglomerative hierarchical clustering begins by assigning each observations (here, 889 11-tuple course responses) to its own cluster, then sequentially combining the two closest clusters. The result of this process is a sequence of cluster fusion indicating at what step the clusters joined, and from what distance. This sequence preserves nested relationships between clusters and

[1] The survey contained 15 such items. Three were dropped to avoid confounding results based on the rarity of those elements; a fourth was dropped due to improper wording which make responses uninterpretable.

indicates relationships between the groupings. From the clustering sequence, decisions must be made to determine the appropriate number of clusters at which to "cut" the results.

An ideal cluster sequence cutoff is one which minimizes the distance between observations in each cluster and maximizes the distance between clusters, thereby creating distinct clusters of similar observations. With this in mind, we employed the elbow and average silhouette methods to inform the number of clusters selected (Hastie et al., 2009). The elbow method considers within-cluster sum of squared error, and recommends cutting the clustering sequence at a point where increasing the number of clusters corresponds to relatively small decrease in that error. The silhouette method considers the quality of clusters by comparing the relative similarity of each cluster element to others in its cluster as compared to observations outside the cluster. Using this method, high silhouette values indicate an appropriate clustering configuration. Using these two methods and taking advantage of the hierarchical structure of the clusters, we identified eleven clusters of courses nested within five larger groups.

Once the cutoffs were determined, we considered the responses which made up each cluster and group. Table 1 shows the proportion of observations within each cluster which selected each item. This allowed us to characterize the clusters based on the uniform course elements. Having satisfied ourselves that the clusters and groups were sensibly coherent, rather than just noise, we assessed the aggregated responses of courses in each cluster and group to the other course-coordination related items. While future work will expand on describing existing patterns within and across the eleven clusters, for the purpose of this report we focus on describing and comparing the five larger groups.

Preliminary Results

The clusters (1-11) and groups (A-E) from our analysis are shown in Table 1, alongside the proportion of observations in each cluster coordinating each element.

Table 1. Groups and clusters from hierarchical agglomerative clustering methods along with proportion of courses within each cluster that coordinate a particular course element. Reported values are those over 0.5 and true zeros, so that a blank entry carries a value between 0 and 0.50.

Group	A	B	C			D		E			
Cluster #	A4	B7	C3	C9	C11	D2	D5	E1	E6	E8	E10
Number of Courses	99	13	255	8	1	82	64	243	65	49	10
Textbook	0	1	1	0.88	1	0.99	1	1	1	1	0
Topics	0	0	1	1	0	1	1	1	1	1	1
Pacing	0	0	0.87	0	0	1	1	0			0
Midterms	0	0	0.90	0	1	0		0			0
Final Exam	0		1	1	1	0	0.78		1	0	0
HW (Online)	0	0	0.78	0	1	0	0.56	0		1	0
HW (Written)	0	0	0.56	0	0	0		0	0		0
Quizzes	0	0		0	1	0			0	0	0
Grading (Course)	0	0	0.92	1	1	0			0		0
Grading (Exam)	0	0	1		0	0		0		0	0
Approach	0	0		0	0						0

Group A consists of only a single cluster, A4, which consists of 99 courses for which no elements were identified as intended to be uniform across multiple sections. Group B is also a single cluster, B7, which consists of 13 courses which all coordinate their textbook, 2 of which also coordinate final exams. Group C consists of three clusters, C3, C9, and C11, which coordinate a lot of elements. Group D consists of two clusters, D2 and D5, which primarily coordinate textbook, topics, and pacing. Group E is the largest group, consisting of four clusters, E1, E6, E8, and E10. These courses coordinate topics, most also coordinate textbooks, and are then delineated by the few other items which are coordinated. In the following section, we discuss other aspects of course coordination systems as they interact with these groups. For brevity, in this paper we omit a discussion of Group B.

Group A (Cluster A4) is the set of 99 courses with no reported uniform elements across sections, 11% of the total courses reported. This cluster includes 71 (13%) of the 554 courses from PhD-granting universities and 28 (9%) of the 314 courses from MA/MS-granting universities. This cluster includes 22 (8%) of the Precalculus, 43 (13%) of the Calculus 1, and 34 (11%) of the Calculus 2 courses reported. Of the courses in this cluster, only five indicated that there are regular instructor meetings at least once per term of instruction. There were 60 courses with no response to this item (80% of the blanks) and 34 which reported never, 13% of that set. Additionally, Group A accounts for 86% (25) of the courses which skipped this item and 56% (53) of those who responded with "N/A." These findings corroborate the expectations one might have for courses with no uniform elements.

Group C includes a total of 264 courses (30% of all), 255 of which are in cluster C3. Group C includes 249 (45%) of the courses reported by PhD-granting departments, and only 15 (5%) of those from MA/MS-granting departments. This group also accounts for 33% of the Precalculus, 32% of the Calculus 1, and 24% of the Calculus 2 courses reported in the survey. The courses in this group have the most instructor meetings, accounting for 72% (94) of the courses which report weekly instructor meetings, 62% (29) of the biweekly meetings, and 45% (72) of the courses which report meeting 2-4 times per term. Additionally, Group C accounts for 24% (46) of the courses which meet only once per term, 6% (15) of those who report never meetings, and 11% (8) of the courses which left this item blank. 69% (181) of the courses in Group C indicated that the person responsible for maintaining the uniform efforts was someone who took on this role for multiple years, and these 181 courses are 49% of all courses with a similar role for their coordinators. Though smaller in their respective proportion of Group C, courses in this group account for 34% (32) of courses with a one-year rotating coordinator, 37% (42) of those where the coordinator is one of the instructors on a term-by-term basis, and only 2% (4) of the courses coordinated by committees. Of the remaining courses, four were marked "other" and one "N/A;" there were no blanks.

Group D consists of 146 courses, which is 16% of all those reported. This group includes 19% of the courses reported by PhD departments and 13% of those reported by MA/MS departments. These 146 courses include 16% of the PC courses, 15% of the C1 courses, and 18% of the C2 courses. The majority of these courses have lower frequency of instructor meetings. The largest pool is a set of 52 courses for which instructors meet once per term, and this accounts for 27% of all such courses and 34 courses in which instructors meet 2-4 times per term, which is 21% of that set. Group D also contains 15% (38) of the courses which never meet, 14% (18) of those that meet weekly, and 4% (2) of those which meet biweekly. Group D includes 22% (40) of the courses for which a committee is responsible for uniform course elements; 20% (73) of those for which there is an individual coordinator who oversees the course for multiple years; 18% (17) of the courses with a rotating coordinator structure; and 10% of those where one of the instructors in the term manages those elements. Of the remaining, there was one blank, one "N/A", and 3 "other" responses. The clusters within this group, D2 and D5, are similar in size, differentiated primarily by the high rate at which courses in D5 have common exams, which are wholly absent in D2. Each accounts for a similar proportion of courses, with the following exception(s). While D2 includes 13% of the reported C2 courses, D5 includes only 5% of them.

Group E is the largest group, including 346 (39%) of the reported courses. This group includes 25% of the courses from PhD departments and a whopping 72% of the courses reported by MA/MS departments. By course level, this group includes 41% of all reported PC courses, 39% of C1, and 44% of C2. Instructor meetings are fairly rare in this group as well, including 64% (164) of all courses which never have instructor meetings and 45% (87) of those with one meeting per term. This large group also includes 33% (160) of those courses which meet 2-4 times per term; 34% (16) of those with biweekly instructor meetings; and 11% (15) of those which meet weekly. There were also five courses in Group E which did not provide an answer to the instructor meeting question. Group E accounts for 71% (127) of the courses which have a committee overseeing the uniform course elements, which is a large overrepresentation. This group also has 46% (52) of those courses overseen by one of the instructors and 45% (42) of those overseen by a rotating coordinator, 39% (37) of the courses which indicated "N/A"; and 28% (102) of those which are overseen by a multiyear coordinator. There were three blanks and four "other" responses.

Discussion & Questions for the Audience

Our initial analysis of data related to coordination systems in university-level P2C2 courses reveals some structure. Recall that the clustering and grouping was done *only* using the course elements, not responses to other coordination items. Thus, over- or underrepresentation of other components is not due to the delineation of groups and clusters. There appear to be associations, as one might expect, between the number of coordinated course elements and the frequency of instructor meetings. There are also some suggestive patterns in the type of coordinator and groupings. In particular, coordination by committee is overrepresented in Group E, Calculus 2 courses are underrepresented in cluster D5, courses from PhD-granting departments are overrepresented in Group C, and courses from MA/MS-granting departments are overrepresented in Group E. These patterns are suggestive of associations, but as yet we do not have evidence of the strength of these associations and only very preliminary conjectures about potential causes. We continue to analyze this data set and review related literature to strengthen these conjectures (e.g., taking into account school size, class size, number of sections). To further our work, we present the following questions for our audience:

1. Are there other relevant bodies of work that we should be leveraging in order to understand/situate our results more appropriately?
2. What questions do you have about our results, which further exploration might reveal?
3. What conjectures (with what basis) exist about the nature of course coordination systems which are testable with our data and analysis?

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