Construction and Application Perspective: A Review of Research on Teacher Knowledge Relevant to Student-Teacher Interaction

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This paper is a review of research that either explicitly or implicitly examines the interplay between teacher knowledge and teaching practices sensitive to students' mathematical thinking. I use radical constructivism as a lens to analyze how the researchers conceptualize the role of teacher knowledge in student-teacher interaction. My analysis reveals that some researchers attribute teachers' observable actions to what knowledge teachers possess (i.e., application perspective) while some others focus on what knowledge teachers construct in-the-moment (i.e., construction perspective). I conclude the paper by discussing the potential causes and consequences of these differences as well as the affordances and limitations of each perspective.

Keywords: Teacher Knowledge, Student-Teacher Interaction, Constructivism, Literature Review

Student-teacher interaction is a common and critical activity in teaching practices. However, some researchers have suggested that, in mathematical teaching and learning, there is a discrepancy between the mathematical meanings teachers intend to teach and the mathematical meanings students actually develop (e.g., Bauersfeld, 1980; Lew, Fukawa-Connelly, Mejía-Ramos, & Weber, 2016; Thompson, 2013; Thompson & Thompson, 1994). This miscommunication between teachers and students creates a fundamental dysfunction in mathematics education (P. W. Thompson, 2013), which highlights the need for mathematics teachers to systematically bring forth the mathematics of students and make instructional decisions sensitive to the mathematics of students (e.g., Steffe & Thompson, 2000b; Teuscher, Moore, & Carlson, 2016). In extant literature, researchers have found that teachers' development and enactment of teaching practices sensitive to student thinking is associated with their knowledge of all kinds (e.g., Hill et al., 2008; Johnson & Larsen, 2012; Seymour & Lehrer, 2006; A. G. Thompson & Thompson, 1996). The goal of this review is to identify the researchers' various ways of operationalizing teacher knowledge in these studies.

Teacher Knowledge and Teaching Practices Sensitive to Student Thinking

Mathematics educators continue to highlight the importance of teachers eliciting and using student thinking in teaching practices. For example, National Council of Teachers of Mathematics (2014) announced eight Mathematics Teaching Practices, among which four are relevant to making instructional actions based on student thinking (i.e., facilitate meaningful mathematical discourse, pose purposeful questions, support productive struggle in learning mathematics, and elicit and use evidence of student thinking). Many researchers have studied how teachers learn and implement these practices, which resulted in a growing body of literature on teacher discourse moves, teacher noticing, teacher decentering and instruction in general. Findings of these studies indicate that most teachers are not used to attending to and interpreting student thinking (e.g., Franke, Carpenter, Levi, & Fennema, 2001; Jacobs, Lamb, & Philipp, 2010). Even when teachers do attend to student thinking, it is especially difficult for teachers to interpret what students are thinking and the sources of the students' difficulties (e.g., Johnson & Larsen, 2012; Maher & Davis, 1990; Speer & Wagner, 2009).

This issue motivates researchers to investigate what contributes to teachers' varying abilities to notice, listen to, model, and act on student thinking. Some researchers have suggested that teachers' knowledge is associated with teachers' abilities to notice student thinking (e.g., Lee,

2017) and instruction quality (e.g., Charalambous, Hill, & Mitchell, 2012; Hill et al., 2008). These researchers typically assessed teacher knowledge by using assessment items grounded in existing teacher knowledge frameworks (e.g., Ball, Thames, & Phelps, 2008) and analyze the teachers' actions independent of the knowledge assessments. Some other researchers have attempted to infer teachers' knowledge in the moment of interaction with students and, at times, combined this with retrospective teacher interviews to support their inferences (e.g., Seymour & Lehrer, 2006; Teuscher et al., 2016). Given these different approaches to teacher knowledge relevant to student-teacher interaction, I conjecture that researchers might have different views on teacher knowledge and affect their methodological designs and interpretation of their data. In this paper, I review select literature that examines the interplay between teacher knowledge and teaching practices sensitive to students' mathematical thinking in order to answer the following research questions: What are the researchers' different conceptualizations of teacher knowledge and what are the affordances and limitations of these conceptualizations?

Radical Constructivism, Knowledge, and Social Interaction

The epistemological stance of radical constructivism (von Glasersfeld, 1995) informs my analysis of the literature. I consider knowledge as actively constructed by a knower through interaction with the environment. Knowledge is not a representation of an objective ontological "reality"; rather, it functions and organizes viably within a knower's experience and is idiosyncratic to the knower. We thus have no access to anyone else's knowledge nor an objective environment; the best we can do is to construct hypothetical models of others' knowledge that viably explain our observation of their behaviors (Steffe & Thompson, 2000b). As it relates to mathematical teaching and learning, students' mathematical knowledge consists of their *ways of understandings* (Harel, 2008) of mathematics that are product of their mental actions constructed from their experience *including* interactions with their teachers; accordingly, teachers' *mathematical knowledge for teaching* (MKT; see Silverman & Thompson (2008)) is grounded in their *ways of understandings* of mathematics constructed from their experience *including* their interactions with students. In order to transform these personal understandings so that they have pedagogical power, teachers need to try to model their students' perspectives and consider how to foster their constructing similar understandings (Silverman & Thompson, 2008).

The radical constructivist view of knowledge and knowing is also useful for operationalizing social interaction among students and teachers. As Steffe & Thompson (2000a) stated, "interaction enters radical constructivism at its very core" (p. 192). Student-teacher interaction (or human communication in general) involves each individual engaged in a conversation interpreting others' meanings, anticipating others' responses, and adjusting her models of the others' meanings in order to decide how to act and what to expect in future conversations. As teachers and students communicate, they reciprocally construct (sometimes with intention) knowledge about the other in the moment of interacting through *assimilating* and *accommodating* (von Glasersfeld, 1995) the language and observable actions of the other (Steffe & Thompson, 2000a) (see the four blue arrows in Figure 1a). A teacher's constructed knowledge potentially perturbs and constrains their personal mathematical knowledge and affect their following actions (see the "enact" arrows on the teacher side). A teacher can also refine and construct knowledge of students' mathematical thinking through reflecting on their own ways of interacting along with the mathematical and pedagogical consequences of these ways of interacting in-the-moment and retrospectively (see the "reflect" arrow on the teacher side).

Related to the current review, I aim at examining select literature by focusing on the extent to which researchers capture this assimilation and accommodation aspects of teacher knowledge in

their conceptualization of teacher knowledge and interaction (see my distinction between the *construction* and *application perspective* in a later section).

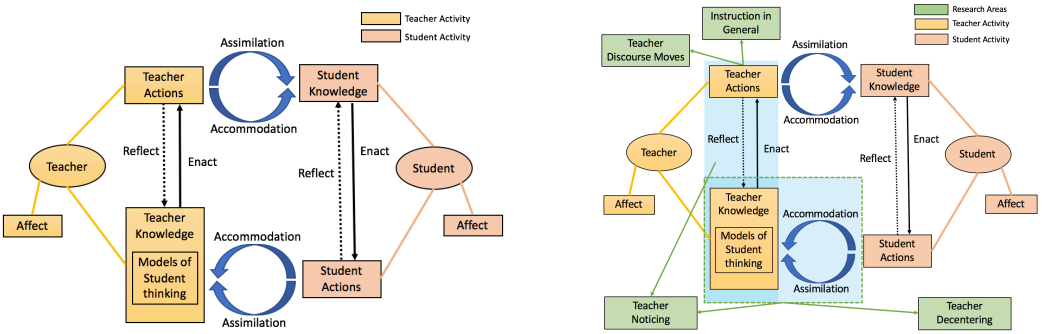

Figure 1. (a) A proposed framework of student-teacher interaction from a radical constructivist perspective with (b) four research areas situated in the framework (the blue area indicates activity relevant to the current review).

Methods

I first situated the literature base in the above framework and located four research areas relevant to teaching practices sensitive to student thinking, which included teacher discourse moves (i.e., teacher talks for eliciting and using evidence of student thinking), teacher noticing (i.e., teachers' ability to observe and recognize student thinking), teacher decentering (i.e., teachers' attempt to set aside her own thinking and model student thinking), and instructional actions in general (see green boxes in Figure 1b).

I selected literature through the consultation of the university library Multi-Search that simultaneously searched more than 130 databases. I elected to include only English articles that had been published in academic journals to guarantee that I included research of a scholarly and authoritative nature. I used keyword combinations of "mathematics", "teacher knowledge", and "decentering" or "discourse" or "listening" or "teacher moves" or "analytic scaffolding" or "teacher noticing" or "student thinking" to search only in abstracts. I then ranked the results of each search by "relevance" and reviewed the abstracts of the first 50 articles to filter 26 articles that satisfied the following four criteria: (1) empirical studies, (2) either explicitly or implicitly touches on teacher knowledge, (3) involves the activity of student-teacher interaction (e.g., research on teacher noticing during analyzing students' written work was excluded), and (4) touches on student thinking. Eighteen of these articles constituted the literature base for the current review, which included two studies on teacher decentering, four on teacher discourse moves, two on teacher noticing, and ten being relevant to teacher knowledge of student thinking.

I followed Galvan and Galvan's (2017) guidelines to conduct this review. I first conducted a vertical analysis (Miles & Huberman, 1994) of each of the 18 articles in my first pass of reading. I used an EXCEL sheet to organize the information of each article in six aspects. In this paper, I only focus on reporting my analysis on "researchers' conceptualization of (teacher) knowledge". I drew on two sources to interpret researchers' perspectives on teacher knowledge. First, I examined the theoretical framework section of the paper to infer the authors' conceptualizations of teacher knowledge or their interpretations and adaptations of the existing teacher knowledge frameworks. In addition, I drew attention to the teacher knowledge claims researchers made for explaining teacher actions to infer what kinds of knowledge they considered to be critical in generating these explanations. I then conducted a horizontal analysis (Miles & Huberman, 1994) to identify similarities and differences within this aspect across the literature. I developed a code for each emerged theme and assigned codes to each article during the second pass of reading. I continually searched for examples that the generated themes could not account for, and I modified my definition of the existing themes or created new themes.

Results

I identify four themes regarding researchers' conceptualizations of teacher knowledge and summarized the literature by themes in Table 1.

Table 1. Summary of literature by emerged themes (literature categorized as more than one theme is underlined).

Theme	Teaching Practices					Number of Literature
	Noticing	Listening	Discourse	Decentering	Instruction in General	
Application Perspective	Kersting (2008)	Johnson and Larsen (2012)	Bray (2011); Speer and Wagner (2009)		Charalambous et al. (2012); Hill et al. (2008); Park and Oliver (2008); Wilkie (2016)	8
Construction Perspective	Lee (2014)	Jenkins (2010); Johnson and Larsen (2012)	Seymour and Lehrer (2006)	Teuscher et al. (2016); Walters (2017)	Franke et al. (2001); Park and Oliver (2008); M. G. Sherin (2002); Wilkie (2016); Wilson et al. (2013)	11 (3 duplicated)
Action/Skill Perspective	Lee (2017)					1
Lack of Perspective			Jacobson and Lehrer (2000)			1

Application Perspective

To take an *application perspective*, a researcher conceives teacher knowledge as possessed knowledge enacted in contexts and being ready for researchers' evaluation. The researcher potentially presumes teachers' observable behaviors as driven by existing knowledge and aims at understanding what knowledge the teachers possess that enables them to act in particular ways. I identified 8 out of 18 studies in which researchers used this perspective. For example, Johnson and Larsen (2012) investigated the role of a teachers' mathematical knowledge for teaching in supporting her ability to listen to students. Although the authors did not elaborate on their conceptualization of teacher knowledge, I inferred their perspective based on their interpretation of the data:

> [The authors demonstrated a transcript in which a researcher was explaining a student's thinking to the teacher, Dr. Bond, in a post-class interview and Dr. Bond responded with a new realization of what the student was thinking] Provided with this extra piece of information, that Adam [the student] was thinking about multiplying symmetries as a left to right sequential procedure, Dr. Bond was able to make sense of his concern…we argue that she was constrained by a limitation in her knowledge of content and students…Dr. Bond was constrained by a lack of knowledge about how her students might have been thinking about the operation of composing symmetries. (p. 122)

Here, the authors claimed that the teacher's lack of knowledge of content and students led to her difficulty with understanding the student's struggles and constrained her from interacting with Adam in ways that were sensitive to his mathematical thinking. They further argued that, if the teacher was armed with the knowledge of students' conceptions of binary operation, she would be able to apply such knowledge in-the-moment to address the student's concern. Because the authors attributed a teacher's thinking and actions to what knowledge she possessed or lacked, I infer that they held a view of knowledge that was consistent with an application perspective.

As another example, Speer & Wagner (2009) provided the following statement about *pedagogical content knowledge* (PCK):

> We use *recognize* in our description of the component practices to denote situations in which teachers are already familiar with the ways that students think about and come to understand

> the mathematics. In other words, their existing PCK may include knowledge of how students think about the specific ideas at hand and/or typical students' difficulties with the topic…At other times, even if teachers are not familiar with the particular ways of reasoning that students offer, they may be able to "figure out" what the students are suggesting and thinking. Therefore, recognizing draws heavily on a teacher's PCK, whereas figuring out requires that a teacher do some mathematical work in the moment. (p. 536-537)

First, the authors claimed that teachers might hold some existing PCK of students' mathematical thinking that enabled them to "recognize" similar students thinking in certain situations. This view of PCK is consistent with the application perspective. I also drew attention to the authors' awareness of situations where teachers' existing knowledge of student thinking might not include all possible ways of thinking they could observe and thus the teachers need to "figure out" student thinking in-the-moment ("do some mathematical work in the moment"). However, it was unclear as to whether the authors considered "figure out" to be a result of a teacher's application of existing knowledge (i.e., an application perspective) or as a process of constructing new knowledge (i.e., a construction perspective). In later sections, I found multiple pieces of evidence consistent with the former case—the authors made claims that the teacher's inability to "figure out" student thinking in-the-moment was due to her lack of knowledge (e.g., "Had Gage's SCK enabled him to figure out the mathematical ideas the students were suggesting…he might have been able to provide different kinds of guidance for the students…" [p. 553].).

Construction Perspective

The above example opens up an alternative interpretation of the "figure out." That is, the presence of novel student thinking may offer teachers an opportunity to *construct* new knowledge as they interpret what the student may be thinking in-the-moment. This interpretation is consistent with my definition of "construction perspective." To take a *construction perspective*, a researcher believes that teacher knowledge is generative, dynamic, evolving, and co-emerging from on-going interaction with students (including their reflection on their own interaction). The researcher explains teachers' observable actions in their teaching or reflection by inferring what knowledge about the students the teachers construct in-the-moment.

The construction perspective applies to 11 out of 18 studies. For example, Seymour and Lehrer (2006) conceived the growth in PCK as "an interactional achievement" in a sense that teachers develop PCK by engaging students in conversations to make sense of their mathematical thinking that is different from their own and by reflecting on students' thinking; meanwhile, students engage in understanding teachers' verbal meanings and actions. The authors also discussed their conception of orchestration as a site for developing PCK, stating "Orchestration and PCK are essentially coconstituted…PCK cannot emerge all at once, but rather evolves during the course of a protracted series of attempts to orchestrate classroom conversations" (ibid, p. 550-553). Transitions in a teacher's PCK can be characterized as "the emergence, stabilization, and adaptation of couplings" between student Discourse and teacher Discourse (ibid, p. 554). I interpret that the authors conceived teacher knowledge as being constructed and developed through sustained negotiation between knowledge of distinct perspectives, which aligns with a construction perspective.

Mixed Perspectives

I should note that making the distinction between an application and a construction perspective does not imply that they contradict each other. A teacher's possessed knowledge can be a result of her construction during *prior* experiences and can inform the construction of knowledge in *future* interactions. I consider it possible for a researcher to take both perspectives

simultaneously in one study since the researcher can conceive teachers' observable actions as results of their application of possessed knowledge and is also aware that they can construct new knowledge or modify existing knowledge through interaction with students. I identified three studies that used a combination of the two perspectives (underlined in Table 1). For example, Park and Oliver (2008) characterized that,

> PCK as knowledge-in-action became salient in situations where a teacher encountered an unexpectedly challenging moment…In order to transform the challenging moment into a teachable moment, the teacher had to integrate all components of PCK accessible at that moment and apply them to students through an appropriate instructional response. In this respect, the development and enactment of PCK is an active and dynamic process. (p. 268)

The authors illustrated with an example that a teacher learned about a student's misconception of a concept in her teaching and integrated this knowledge with her knowledge of subject matter and curriculum to confront student misconceptions through instructional strategies in-the-moment. Characterizing what knowledge the teacher constructed in terms of the student's misconceptions implied that the authors took a construction perspective. Meanwhile, they considered the teacher's instructional decisions as results of her integrating and applying multiple sources of knowledge, which implied an application perspective.

Action/Skill Perspective and Lack of Perspective

In Lee's (2017) conceptualization of teacher knowledge, she conflated teacher knowledge (i.e., what teachers know) and teacher actions or skills (i.e., what teachers do), stating that "PCK for preschool mathematics can be conceptualized as a set of three interrelated skills (p. 233)" that included noticing, interpreting, and enhancing student thinking. I also identified a literature in which the authors did not discuss any theoretical orientations of teacher knowledge and assumed teachers who participated in additional professional development program had more knowledge than those who did not (Jacobson & Lehrer, 2000).

Discussions

My intention of this review is to identify researchers' different approaches to teacher knowledge relevant to student-teacher interaction, which included the different teacher knowledge frameworks they used and their ways of operationalizing these frameworks to make claims about teacher knowledge when explaining teachers' observable actions. First, my analysis unsurprisingly suggests that researchers hold different perspectives on teacher knowledge when conducting research in relation to teacher knowledge and student-teacher interaction. This phenomenon is not necessarily due to the diversity of frameworks prevalent in mathematics education. Researchers who used the same framework might interpret and use the framework differently. For example, researchers of 9 (out of 18) studies used Ball et al.'s (2008) MKT taxonomies but with some of them using it as a construction perspective, some using it as an application perspective, and one using it as an action/skill perspective (see a summary in Table 2). Because Ball et al. (2008) developed the MKT framework with an intention of identifying teacher knowledge demanded by the work teachers do, it made sense that this framework widely applied to the literature on teacher knowledge relevant to student-teacher interaction. However, this conceptualization of teacher knowledge emphasizes specific types of teaching practices that signal teacher knowledge rather than the cognitive content and nature of teacher knowledge itself. As a result, it is not surprising that many researchers used this framework either from an action/skill perspective to conflate knowledge and actions or an application perspective to focus on the function of knowledge in the form of teacher actions. A limitation of these views of teacher knowledge is that they do not allow researchers to gain insights into the cognitive content

and structure of the teachers' knowledge. We are left wondering: what mathematical meanings the teacher construct from their students and how are those meanings organized in the teachers' minds so that they can enact those meanings when interacting with students?

Table 2. Summary of literature using Ball et al.'s (2008) MKT framework.

Theme	Literature	# of literature/Total # of literature in each theme
Application Perspective	Bray (2011); Charalambous et al. (2012); Hill et al. (2008); Johnson & Larsen (2012); Kersting (2008); Speer & Wagner (2009); Wilkie (2016)	7/8
Construction Perspective	Johnson & Larsen (2012); Wilkie (2016); Wilson, Mojica, & Confrey (2013)	3/11
Action/Skill	Lee (2017)	1/1

Second, researchers' views of teacher knowledge reflect what source of knowledge they consider as critical in explaining teachers' observable actions. Researchers who hold a construction perspective attribute teacher actions to what knowledge teachers construct in the moment of interacting with students and make claims about what constitutes the teachers' constructed knowledge. Researchers who hold an application perspective explain teacher actions in terms of what knowledge teachers possess or lack. At times, the researchers make claims about what teachers *cannot* do due to the absence of certain types of knowledge, emphasizing the deficit of teacher knowledge instead of the affordances. While this perspective allows researchers to understand what knowledge enable teachers to notice, listen, and act on student thinking, it may constrain researchers from understanding how noticing, listening, and teaching provides teachers sustained opportunities to *develop* their knowledge of their students.

I am not arguing that a construction perspective is always preferable over an application perspective. I understand that researchers may have different research goals and thus focus on different aspects of teacher knowledge. However, I do believe that researchers who are oriented to radical constructivism should consider taking *both* perspectives to explain teacher actions. A radical constructivist view of social interaction as ongoing assimilation and accommodation of meanings requires researchers to simultaneously take into account how teachers *apply* and *modify* knowledge in the moment of interacting with students. Applying existing knowledge to interpret a current situation implies a mental process of assimilation; namely, the teachers are treating what they perceive about the students' activity as fitting into their existing conceptual structure. In comparison, modifying knowledge from teaching implies that the teachers are experiencing accommodation—the teachers are modifying their existing conceptual structures to account for what they perceive in-the-moment. This echoes some scholars' call that researchers should focus on the knowledge construction process to capture the dynamic and constructive nature of teacher knowledge, as opposed to identifying particular knowledge needed for effective teaching (e.g., Bauersfeld, 1980; Mason & Spence, 1999; Silverman & Thompson, 2008).

A final observation is that some researchers did not provide sufficient descriptions of their views of teacher knowledge in the theoretical framework section (e.g., Bray, 2011; Jacobson & Lehrer, 2000; Lee, 2014)—some of them summarized the existing frameworks without elaborating on how they interpreted and used the framework in their studies. It is important for researchers to carefully consider their uses of teacher knowledge frameworks and to be aware of how they conceive the role of teacher knowledge in student-teacher interaction. As suggested in my analysis, such consideration may not only help situate the research in the literature in a more rigorous way but also impact researchers' ways of interpreting and explaining their data.

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