A Gendered Comparison of Abstract Algebra Instructors' Inquiry-Oriented Instruction

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Inquiry-Oriented Instruction (IOI) holds promise for providing equitable learning opportunities for men and women. We consider two abstract algebra instructors whose women exhibited different learning outcomes. We explore how this disparity in the women's achievement might be related to differences in these instructors' implementation of IOI, specifically regarding how they elicited and evaluated student contributions. We used Reinholz and Shah's (2018) EQUIP observation tool to investigate differences between students' participation opportunities in each class. We found a significant difference in the instructors' number of interactions with men and women during class discussion. We also found significant differences in the instructors' methods for soliciting and responding to student contributions in class discussion. A discussion of these differences in instructional practices, as well as implications for future work, is provided.

Keywords: equity analytics, gender, inquiry-oriented instruction, abstract algebra

The mathematics classroom is a gendered space, as socially constructed differences in achievement and participation exist between genders (Leyva, 2017). These differences might contribute to the undisputed underrepresentation of women in STEM. Lubienski and Ganley (2017) called for researchers to examine how women's choices to leave or persist in mathematics-intensive fields may be constrained by inequitable educational opportunities. Some researchers focus their efforts on identifying teaching practices that give women more access to learning opportunities and yield equitable learning outcomes. Laursen, Hassi, Kogan, and Weston (2014) advocated student-centered teaching approaches, such as inquiry-based learning, suggesting they "level the playing field" (p. 412) for men and women in mathematics. However, there is some dispute on the generalizability of equitable effects of student-centered instruction.

To explore the relationship between Inquiry-Oriented Instruction (IOI) and equitable student learning outcomes, Johnson et al. (under review) compared men's and women's learning outcomes in abstract algebra measured by their performance on the Group Theory Content Assessment (GTCA; Melhuish, 2015). Their participants included students whose instructors had participated in an IOI professional development project and students in the national sample. They found men outperformed women in the IOI sample. However, there was no achievement gap between men and women in the national sample. In recent analysis, a discrepancy was found in gender performance of students of two of the participating instructors from that sample, hereafter referred to as Dr. C and Dr. K. Both men and women in Dr. C's class outperformed the national sample, but men in Dr. K's class outperformed and women underperformed the national sample. These differences in learning outcomes may be attributed to differences in these instructors' practices. Our study seeks to highlight differences between these instructors' implementation of IOI to hypothesize potential instructional practices that may lead to equitable learning outcomes.

Literature Review and Theoretical Perspective

IOI is a student-centered pedagogical approach, which provides opportunities for students to inquire into mathematics and for instructors to inquire into students' mathematical thinking (Rasmusssen & Kwon, 2007). In IOI, students engage in meaningful tasks that allow them to develop informal intuitive understanding of concepts, from which they can develop more formal mathematical reasoning (Wawro, Rasmussen, Zandieh, Sweeney, & Larson, 2012). The four principles of IOI are: "Generating student ways of reasoning, building on student contributions, developing a shared understanding, and connecting to standard mathematical language and notation" (Kuster, Johnson, Keene, & Andrews-Larson, 2017, p. 2). These instructional principles emphasize inquiry into student thinking and formalization of mathematical reasoning. In addition to inquiring into student thinking, facilitating meaningful discourse, and guiding students' progress in the course, IO instructors need to sustain social norms in the classroom that are conducive to students' reinvention of mathematics (Stephan, Underwood-Gregg, & Yackel, 2014). These may include norms of students collaborating, explaining their reasoning, and participating in class discussion. Instructors also need to provide equitable opportunities for participation and position students as competent learners (Reinholz & Shah, 2018).

Equity research focuses on accounting for effects of past marginalization and mitigating systematic differences in ways students experience educational opportunities (Adiredja & Andrews-Larson, 2017; Gutiérrez, 2002; Reinholz & Shah, 2018). In the context of mathematics education, equity has been blurred with equality, in that educators might intend to provide equal access to curricular materials or learning supports (Gutiérrez, 2002). However, to account for differences in student backgrounds, student identities, and social biases, certain students need different learning opportunities to achieve fairness in the classroom. Reinholz and Shah (2018) proposed focusing on equality of participation opportunities as a necessary stepping stone for achieving equity in the classroom. We follow Gutiérrez (2013) and Adiredja and Andrews-Larson (2017) in taking a sociopolitical perspective, recognizing the interplay of knowledge, identity, and power in students' experiences and interactions in social educational contexts. Students associate mathematical success with power (Leyva, 2017), and as they construct their mathematical identities, perceptions of their own mathematical competence might be constrained by opportunities to participate in the classroom. This study considers participatory equity (Reinholz & Shah, 2018), discerning whether there exists a fair distribution of opportunities for participation given in two abstract algebra classrooms. Therefore, we explore the following research questions: What differences exist between two inquiry-oriented abstract algebra instructors' provision of opportunities for men and women to participate in class discussion? What differences exist in these instructors' teaching practices?

Methods

This explanatory case study explores the differences in practices of two abstract algebra instructors whose women students exhibited different achievement outcomes. Both instructors participated in an IOI professional development project in which they received training in implementing IOI in abstract algebra, curriculum materials, and support via online working groups with other instructors. Dr. C is a white man, Dr. K is a white woman, and both instructors teach at large public universities in the western United States. They both had over thirty students in their classes; Dr. C had twice as many men as women in his class, and Dr. K had about the same number of men and women in her class. We explore differences in Dr. C's and Dr. K's use of student contributions to gain insight on gendered experiences in their classes. We investigate which students participated in class discussions and how their contributions were elicited and responded to. We focus on gender to see how men and women's contributions were positioned during class discussion. Students' genders were inferred by the observing researchers rather than self-identified by the students, as this data was unavailable.

Instrument

The Equity QUantified In Participation (EQUIP) was developed by Reinholz and Shah (2018) as an observation tool to evaluate students' participation and instructors' practices of providing opportunities for students to participate in class discussion. The EQUIP rubric has seven dimensions, but we only used three of those dimensions, including solicitation method, teacher solicitation, and teacher evaluation (Figure 1), because they relate to the four principles of IOI (Kuster et al., 2017). Solicitation method refers to the type of strategy the instructor uses to initiate student participation. Teacher solicitation type refers to the type of question or statement the instructor uses to solicit student participation. Teacher evaluation describes how the instructor responds to students' contributions. We adapted the EQUIP tool by adding more specific codes for solicitation method and teacher evaluation.

Dimension	Solicitation Method	Teacher Solicitation	Teacher Evaluation
Levels	1. Not Called on 2. Random Selection 3. Called on Method 4. Called on Group 5. Called on Individual 6. Called on Volunteer	1. N/A 2. Other 3. What 4. How 5. Why	1. N/A 2. Revoice 3. Evaluation 4. Elaborate 5. Follow-Up Question or Task

Figure 1. Dimensions and levels from EQUIP (Reinholz & Shah, 2018) and coding assignments

Data Collection and Analysis

Instructor's classes were video-recorded by project personnel. The first author watched videos of five classes for both Dr. C and Dr. K. She transcribed only the student-teacher talk sequences that occurred during full class discussion. She then used the EQUIP observation tool to record the frequencies of the participatory and instructional practices observed in each student-teacher talk sequence that occurred during class discussion. Student demographics (e.g., gender) were noted for analysis of which students participated in the discussion. The codes were then discussed with the second author to resolve any uncertainties and were transformed to categorical values (Figure 1). A code was assigned to each instructor and gender to allow for comparison within the statistical analysis. We used three omnibus chi-square tests to compare the instructors for each of the three EQUIP dimensions. We then used three omnibus chi-square tests to compare each instructor by gender for each of the three EQUIP dimensions. Post hoc tests were conducted to examine statistically significant differences between instructors and gender. We then calculated the *equity ratio* (Reinholz & Shah, 2018), the "ratio of actual participation to expected participation" (p. 161) for men and women in each EQUIP dimension. Expected participation is the percentage of participation one would expect based on the demographics of the class. For example, if men comprise 40% of the class, they would be expected to participate 40% of the time. If men actually participated 50% of the time, the equity ratio would be 50/40 = 1.25. An equity ratio above 1 demonstrates an over-representation, and an equity ratio below 1 demonstrates an underrepresentation of participation from that group of students. Entrance interviews were also conducted with Dr. K and Dr. C by project personnel at the beginning of the semester they taught IO abstract algebra. We analyzed these interviews for data triangulation.

Results

Equity Analytics Results by Instructor

Dr. C's and Dr. K's instructional practices regarding student participation were compared using the three EQUIP dimensions. There was a total of 175 student-teacher talk sequences for Dr. C's class, and only 66 for Dr. K's class. The frequencies and percentages of each teacher's use of a certain solicitation method and type of evaluation are recorded in Table 1. The difference in instructors' *teacher solicitations* was not statistically significant. The difference in instructors' *solicitation method* was statistically significant $\chi2(4, N = 241) = 21.300$, $p < .001$, Cramer's V = .297. When looking at specific solicitation methods, neither instructor used random selection (e.g., drawing names from a hat). Dr. C's students were not called on more often than Dr. K's students were not called on. Dr. C called on individuals more often than Dr. K did. Dr. K called on volunteers and groups to participate more often than Dr. C did. The difference in *teacher evaluations* was statistically significant $\chi2(4, N = 241) = 28.687$, $p < .01$. Dr. C asked follow-up questions in response to student contributions more often than Dr. K did. They both primarily elaborated on student contributions, but Dr. K elaborated on student contributions much more often than Dr. C did. Both instructors responded by evaluating or revoicing student contributions about the same amount. Dr. C's responses were coded as N/A more often than Dr. K's were. This could be due to students responding to other students' contributions before Dr. C could respond.

Table 1. Code frequency and percentages for solicitation method and teacher evaluation by instructor.

Dimension	Instructor	Not Called on	Called on Method	Called on Group	Called on Individual	Called on Volunteer	Total
Solicitation Method	Dr. C	78 (44.6%)	0 (0%)	4 (2.3%)	41 (23.4%)	52 (29.7 %)	175
	Dr. K	24 (36.4%)	1 (1.5%)	9 (13.6%)*	6 (9.1%)	26 (39.4%)	66
	Instructor	Follow-Up	Elaborate	Evaluation	Revoice	N/A	Total
Teacher Evaluation	Dr. C	41 (23.4%)	54 (30.9%)	17 (9.7%)	14 (8%)	49 (28%)	175
	Dr. K	10 (15.2%)	42 (63.6%)*	6 (9.1%)	6 (9.1%)	2 (3%)*	66

*Represents significant differences in standardized residuals based on post hoc testing

Equity Analytics Results by Instructor and Student Gender

The difference between instructors' *number of interactions with men and women* was statistically significant, $\chi2(1, N = 241) = 5.540$, $p = .019$, Cramer's V=.152. Men in Dr. C's class participated in 104 of the 175 student-teacher talk sequences, while women participated 71 times. Dr. C's class had about twice as many men as women, but women participated proportionally more than men did, considering the equity ratios for the total amount of participation (see Table 2). Women in Dr. C's class had an equity ratio over 1, and men had an equity ratio under 1, implying women were over-represented and men were under-represented in the total amount of participation. Men in Dr. K's class participated in 50 of the 66 student-teacher talk sequences, while women participated only 16 times. Dr. K's class had an equal number of men and women, but women participated less than men did. Women in Dr. K's class had an equity ratio under 1, and men had an equity ratio over 1 for total participation, implying women were under-represented and men were over-represented in their total participation.

Gender differences in both Dr. C's and Dr. K's *teacher solicitation* and *teacher evaluation* were not statistically significant. The observed difference between Dr. C's *solicitation method* for men and women was statistically significant, $\chi2(4, N = 175) = 14.023$, $p = .007$, Cramer's V = .241. However, the observed difference between Dr. K's *solicitation method* for men and women was not statistically significant $\chi2 (4, N=66) = 3.662$, $p =.454$. The frequencies and percentages of each instructor's use of a certain method for soliciting participation from men and women are presented in Table 2. Slightly more men than women participated in Dr. C's class without being called on. However, in Dr. K's class, men participated without being called on much more often than women did. Dr. C rarely called on groups to participate, but when he did, only women shared their group's contribution. In Dr. K's class, men shared their group's contribution more often than women did. In both classes, however, men volunteered to answer more often than women did. Women in Dr. C's class were over-represented in participating in all levels of solicitation method except when Dr. C called on a volunteer. Women in Dr. K's class were under-represented in participating in all levels of solicitation method except when Dr. K called on individuals.

Table 2. Code frequency, percentages, and equity ratio for solicitation method by instructor and student gender

Instructor	Gender	Not Called on	Called on Method	Called on Group	Called on Individual	Called on Volunteer	Total
Dr. C	Men	43 (24.6%) 0.82	0 (0%)	0 (0%) 0	20 (11.4%) 0.73	41 (23.4%) 1.18	104 (59.4%) .88
	Women	35 (20%) 1.36	0 (0%)	4 (2.3%) 3.03	21 (12%) 1.55	11 (6.3%)* 0.64	71 (40.6%) 1.23
Dr. K	Men	20 (30.3%) 1.67	1 (1.5%) 2	6 (9.1%) 1.33	3 (4.5%) 1	20 (30.3%) 1.54	50 (75.8%) 1.52
	Women	4 (6.1%) 0.33	0 (0%) 0	3 (4.5%) 0.67	3 (4.5%) 1	6 (9.1%) 0.46	16 (24.2%) 0.48

*Represents significant differences in standardized residuals based on post hoc testing

Descriptions of Dr. K's and Dr. C's Classes

To contextualize these numbers, we now turn to narratives from both classes. This section describes and compares teaching episodes by Dr. K and Dr. C, in which students developed formal definitions of isomorphic and isomorphism. In the classes prior to these episodes, students determined whether a given mystery Cayley table represented a group that was the same as D_6 (the group of symmetries of a triangle). This task aimed to give students an intuitive understanding of isomorphic groups and isomorphisms (see Larsen, 2013).

Dr. K's class. Dr. K began class by writing an informal definition of isomorphic groups on the board. She gave examples of isomorphic and non-isomorphic groups, and asked students to define isomorphic and isomorphism. Students worked individually, shared their ideas with their groups, and decided upon one idea as a group to contribute in whole class discussion. When Dr. K called on each group (numbered 1 through 8), a student voluntarily shared their contribution. A woman in group 1 claimed an isomorphism maps an element of one group to an element of the same order, and Dr. K elaborated on this. A man in group 2 commented that the isomorphism needs to be one-to-one and onto. Since few students discussed those terms in their groups, Dr. K gave a short lecture to explain the definitions and draw pictures of surjective, injective, and bijective set maps. Then, when Dr. K asked for other traits of isomorphisms or isomorphic groups, she called on a woman to share group 4's contribution. The woman claimed all mappings of corresponding elements had to be the same. Dr. K then asked for other traits, and a man from group 7 asserted the group tables (likely referring to Cayley tables) had to look the same. Dr. K wrote these contributions on the board and called on group 8 to share; a man from that group said isomorphic groups have the same order, but they wondered if they have the same operation. Dr. K elaborated on these contributions, and posed a new task to the class to decide whether isomorphic groups need to have the same operation. After working in groups, Dr. K gathered back the class. She asked if isomorphic groups can have different operations, and a student mumbled "yeah." Dr. K remined students of the example of the rotations of a square and $\mathbb{Z}_4$ having different operations but still being isomorphic groups. She then wrote the beginning of the formal definitions of isomorphic and isomorphism on the board, and she referred to group 4 and 7's informal ideas of homomorphisms. Students were asked to formalize those ideas using function notation, which they worked on for the rest of class.

Dr. C's class. Dr. C reminded the students of their previous task, in which they saw the group in the mystery table was isomorphic to D_6. He told students to write their own definition of isomorphic groups. Students (pseudonymed W# for women and M# for men) worked individually, and wrote definitions on whiteboards (see Figure 2), which were displayed at the front of the room. Dr. C had students talk with their groups about the different definitions. Dr. C initiated whole class discussion to address definition 2a; W1 said she had a similar definition. When Dr. C asked why, W1 explained her reasoning. W2 then stated a concern about the equal sign in the definition. Dr. C asked what was wrong with the equal sign, and W2 explained her reasoning. W3 said she did the same thing (definition 2d), but wrote "corresponds" instead of "equals." W2 added, "which implies there's a map." Dr. C discussed these ideas, and recalled the correspondence between elements of D_6 and the mystery table. Discussing definition 2e, students claimed $a * b$ is not necessarily in G, and they need to "split up the phi." Dr. C discussed these ideas, and acknowledged the idea of correspondence in the definition. Considering definition 2b, M1 said, "There exists a homomorphism G to H, and there also exists a homomorphism H to G." Dr. C mentioned that if M1's definition was true, then something about the definition 2b had to be wrong. Dr. C claimed definition 2c required a bijection, 2b required a homomorphism, and 2f required the existence of both. He then asked students to think of counterexamples to definitions 2b and 2c. After students worked in their groups, Dr. C called on W4 to share her counterexample. W4 described the trivial homomorphism that maps every element of G to the identity of H. When Dr. C asked W4, "Why is it not isomorphic?" W4 said this homomorphism could exist between groups with different numbers of elements. Dr. C asked for other comments, and M3 elaborated on W4's example. Dr. C then asked if groups with the same number of elements were always isomorphic; W5 explained her counterexample. Dr. C then led students to prove or give a counterexample to M1's idea, and M4 clarified that the homomorphism from H to G in M1's definition should be the "inverse that maps H to G." When Dr. C asked why, M4 explained his reasoning. Dr. C then explained this inverse homomorphism is a bijection, and he finalized the definitions of isomorphic groups and isomorphism.

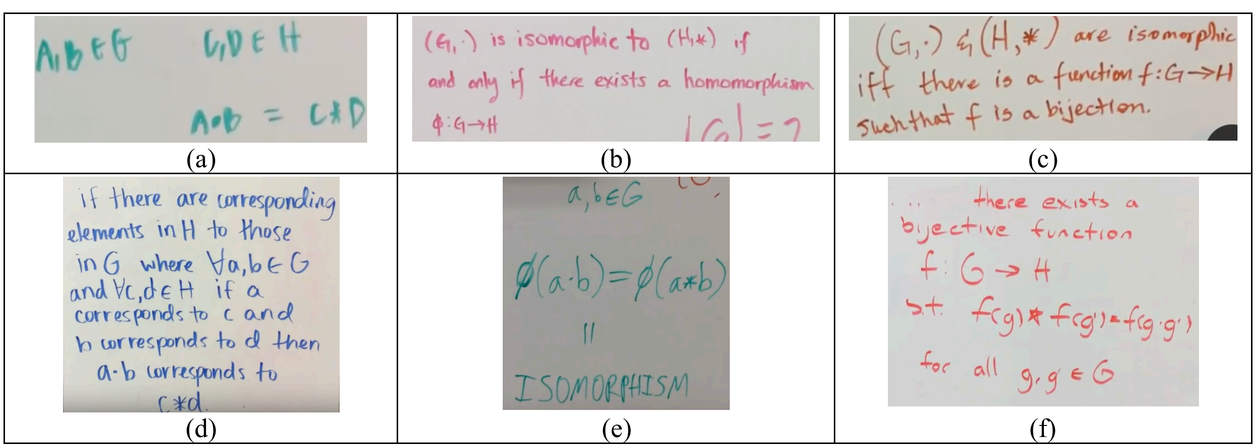

Figure 2. Student written definitions of isomorphic groups from Dr. C's lesson

Comparison of Dr. C's and Dr. K's practices. Here Dr. K and Dr. C exhibited differences in how they elicited student reasoning and contributions. Dr. K elicited characteristics students noticed about isomorphisms and isomorphic groups, but did not seem to elicit their reasoning. Dr. C, however, elicited students' reasoning by asking follow-up questions. Dr. K and Dr. C also exhibited differences in how they responded to student contributions and then used those to inform the lesson. Dr. K primarily elaborated on students' contributions; she accepted one-to-one and onto as characteristics of an isomorphism and then lectured on those definitions, without asking the students why or how they developed those ideas. Dr. C also elaborated on students' responses, but sometimes did not respond, allowing opportunities for other students to respond. Instead of accepting the definition of isomorphism in 2f, Dr. C led students to find counterexamples to see why it is necessary to have a bijective homomorphism between two isomorphic groups. Dr. K and Dr. C also exhibited differences in engaging students in each other's reasoning. Both instructors engaged students in each other's reasoning by assigning follow-up tasks in response to their contributions. However, during class discussion, Dr. K's students primarily talked to her and not each other, whereas Dr. C's students often spoke to each other about their reasoning without being called on. Although Dr. K seemed to do most of the talking in her lesson, she claimed in her entrance interview that she was trying to get better at not dominating class discussions. She said she wanted students to talk to each other instead of talking to her, but this did not seem to be enacted in her class discussion. In Dr. C's entrance interview, he claimed his overarching goal is to get everybody involved somehow. He also explained his class rule that his students are not allowed to judge each other. This might contribute to students' evident ease in participating in class discussion.

Discussion

Provision of equitable participation opportunities for students seems to be related to equitable learning outcomes. Since women in Dr. C's and Dr. K's classes had different achievement outcomes on the GTCA (Melhuish, 2015), we explored the differences in provision of opportunities for men and women to participate in class discussion. Women in Dr. C's class participated proportionally more than men did, while women in Dr. K's class participated less than men did, considering the equity ratios of participation. We found women in both classes were under-represented when Dr. C or Dr. K called on a volunteer. This finding aligns with Leyva's (2017) claim that mathematics is a masculine space, and men are more confident in volunteering to participate. The cause for the disparity in women's participation in Dr. K's class and women's limited volunteerism in Dr. C's class is unknown, yet prior research makes it reasonable to believe that women's lack of mathematical confidence may be a contributing factor (Ellis, Fosdick, & Rasmussen, 2016; Lubienski & Ganley, 2017). Future research can explore how equitable participation in class discussion relates to equitable achievement outcomes.

We also found differences in Dr. C's and Dr. K's instructional approaches during class discussion. We found significant differences in their methods for soliciting student participation and in their responses to student contributions. Our qualitative descriptions of Dr. K's and Dr. C's classes also highlighted some differences in their teaching practices, which we hypothesize might contribute to the differences in their students' achievement. We found Dr. K primarily elaborated on student responses without asking follow-up questions, whereas Dr. C asked follow-up questions to students' responses to inquire into their reasoning. Also, students in Dr. K's class primarily talked to her during class discussion instead of to other students, while Dr. C's students talked to each other, possibly because of his rule of no judgment. We hypothesize women might benefit from instructors inquiring into their reasoning, as this might position them as competent learners of mathematics. Women might also benefit from participating in class discussion in an atmosphere of no judgment, for this might enhance their mathematical confidence. Future research can explore which teaching practices give equitable learning opportunities for women.

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