The Institutionalized Paradox: Our Teachers Are Not Trained To Teach

Rochy Flint
Columbia University

In a culture where STEM preparation is rapidly becoming of utmost importance to the nation's economy and educators are challenged to increase diversity and equity amongst students, quality mathematics instruction at the collegiate level is critical. Yet the majority of undergraduate mathematics teachers are not formally trained in pedagogy. This is a systemic issue, an institutionalized paradox, which originates in the mathematicians' training grounds - mathematics PhD programs. This paper provides background on this issue and focuses on a survey of university mathematicians concerning their formal academic training and their outlooks and prioritization of pedagogical training. Attention is drawn to the disconnect between university mathematicians' beliefs about the important role of pedagogical education in mathematics program and their resistance to promoting its implementation as a basic institutional requirement. A call for action is suggested to remedy these institutionalized systemic paradoxes.

Keywords: Pedagogical Training, Pedagogy for Mathematicians, Undergraduate Instruction

Introduction

It is a curious phenomenon that those whose role it is to prepare the next generation of mathematics learners at the college level most often lack basic training in the fundamentals of teaching. Although this pedagogical vacuum is present in other subject matters, the lack is most critical in mathematics, where future success is based on a mastery of progressively complex predecessor functions and disciplines. Mathematics teachers rarely learn how to teach; they learn how to be mathematicians. Granted that to be a good mathematics teacher strong mathematical knowledge is required – as subject matter expertise is a primary critical component of teaching. However, in most cases subject matter knowledge is not sufficient to reliably result in effective and excellent teaching.

The central thesis of this paper is that a discrepancy exists between the way mathematicians think about the importance of pedagogical training for teachers of mathematics and the priority they place on actually implementing pedagogy into mathematics PhD programs. This discrepancy contributes to a situation in which the overwhelming majority of mathematics PhDs will become college level mathematics teachers (i.e. adjuncts, instructors, lecturers, and professors), while mathematics PhD programs across the United States, and often around the world, have little or no pedagogy development to assure that these programs will produce good mathematics teachers. These programs are designed to prepare mathematics researchers. Our future teachers are rigorously trained to "do" mathematics and are not trained to "teach" mathematics. Research shows that this is having an impact on student retention and attrition in undergraduate mathematics programs.

The aim of this paper is threefold: (a) to review research which discusses this systemic problem, (b) to assess contributing factors to this systemic problem by addressing the perspective of university mathematicians on the importance of pedagogical training, and (c) to suggest fundamental changes in the way we approach integrating course requirements for mathematics PhD programs.

The paper is organized as follows: (a) Section 2 is a literature review and discussion of studies related to this paper's central thesis; (b) Section 3 presents a survey on pedagogy and an analysis of survey results, which were conducted at an international conference of mathematicians and at a research seminar at an American university in spring of 2018; (c) Section 4 discusses the ramifications of the literature review and the survey results; (d) Section 5 draws conclusions and suggests a call to action for fundamental change in the curricula of mathematics PhD programs and proposes a study to assess the value of that change.

Literature Review

There is a rich amount of research showing that strong mathematics knowledge is not necessarily an indicator of strong mathematics teaching skills (Bass, 1997; Kennedy, 1991). Universities, have competing goals when they hire faculty: research and teaching. These two goals often conflict. Many universities have a value hierarchy and regard research as more pivotal and will hire faculty primarily for their research abilities, regardless of their pedagogical training and skills (Brand, 2000). Although universities generally do have a process for assessing teaching capabilities of its subject matter experts, it is most often a limited process consisting of a brief model lesson and an observation lesson each semester for beginning teachers and student evaluations (NRC, 2003). Whereas this system may screen out teachers with "poor skills" it almost never results in formal pedagogical training or deep professional development. The system is missing the fundamental step of providing formal pedagogical training prior to graduates becoming teachers.

Mathematics instructors in college mathematics vary widely, from tenured full time professors and full time lecturers with many years of teaching experience, to adjuncts either with PhDs or enrolled in PhD programs, and varying teaching experience especially in the beginner mathematics college courses (Haycock, Majors, & Steen, 2004). Implementing Shulman's directive that to understand a profession one looks at its nurseries, to understand the profession of college mathematics teachers one should look at the mathematics PhD programs (Shulman, 2005). PhD programs most often do not focus on preparation for college teaching, despite teaching being a fundamental component of an academic life (Adams, 2002). Many of these professors and instructors have had no formal pedagogical training. As Bass states, "academic mathematical scientists, who typically spend at least half of their professional lives teaching, receive virtually no professional preparation or development as educators, apart from the role models of their mentors" (Bass, 1997). Moreover, since teaching is often not the primary focus for many university teaching faculty, this results in minimal time to focus on building teaching skills and tends to rely on "learning on the job" to gain classroom skills despite the availability of resources (Boyer, 1990; Fairweather, 2005).

The systemic issue is that regardless of the intention and perspective of the teacher, lacking pedagogical skills often negatively impacts the students in the classroom (Gibbs & Coffey, 2004). Furthermore, it leads to disinterested students and can discourage students from continuing their pursuit of a mathematics degree (Seymour & Hewitt, 1997). Is it okay to have an entire system dependent on idiosyncratic teacher performance? Acknowledging that there are many mathematicians who profoundly care about teaching and who have developed excellent teaching skills on the job (Oleson & Hora, 2014), should there be a systematic approach to developing excellent teachers? Research shows that educators with teacher training are more successful educators than teachers without professional teacher training (Darling-Hammond, 2000). Moreover, teacher practices and skill are not innate but something that is learned

(Darling-Hammond, 2012). Formal pedagogical training is ubiquitously accepted as fundamental and required in the K-12 level of schooling, yet this consensus is not an established norm at the college level despite the prevalent need and public concern (TAC & NRC, 2001).

The field of mathematics education, which was established to study the fundamental issues of pedagogy in mathematics, was founded over a century ago by the renowned mathematician, Felix Klein (Bass, 2005; Eves 1969). Naturally it would seem that a positive symbiotic relationship between mathematicians and mathematics education would ensue. Yet, there is an unfortunate disconnect between the fields (Dörfler, 2003). Under the umbrella of mathematics education there is a plethora of rich research, knowledge, tools and resources that focus on pedagogy for postsecondary mathematics instruction, e.g. Transforming Postsecondary Education in Mathematics (TPSE Math) (Holm & Saxe, 2016). There exist communities of scholars and programs consisting of mathematicians and mathematics educators that focus on pedagogical related issues for undergraduate teaching in mathematics, e.g. programs such as SIGMAA on RUME, the Mathematical Association of America (MAA) Project NExT, the Preparing Future Faculty (PFF), and the International Commission on Mathematical Instruction (ICMI), to name just a few. Furthermore, there exist teams of scholars addressing mathematicians' knowledge of teaching (Loewenberg Ball, Thames, & Phelps, 2008) and active studies (e.g. see Miller, 2017) finding the best teaching methods at the collegiate level. Nevertheless, the majority of mathematicians are generally unaware of these resources (Nardi et al, 2005). Many PhD programs that are training future mathematics educators fail to acknowledge and integrate this fundamental body of knowledge. This failure can have tremendous impact on the quality of teaching and hence negatively impact the quality of mathematics learning at the collegiate level.

The principal problem is that this body of research does not enter the curriculum of mathematics PhD programs. It is just not part of the system. There are some PhD mathematics programs that have begun to require and offer pedagogy training in the form of mentoring, but even a rigorous mentoring program is not sufficient for ensuring student learning. In addition, most mentoring training programs fail to offer basic courses such as a methods class, or a multicultural mathematics education course which would better equip teachers in increasingly diverse populations of undergraduate classrooms. Many PhD mathematicians are not pedagogically trained at all, as is highlighted in the survey below.

Survey on Pedagogy for Mathematicians

The Survey

During his 90th birthday celebration mathematician Dr. Henry Pollack humorously told the crowd that when he teaches his mathematics education students mathematical modeling he tells them "I'll teach you math, and you'll teach me how to teach." This sentiment resonates with many mathematicians. To highlight this perspective, which resonated deeply with me while training to be a mathematician, I decided to conduct a survey of fellow mathematicians to ascertain what they thought about the importance of pedagogy for mathematicians. To date, I have conducted the survey with two groups: (a) at a recent international mathematics research conference consisting of a group of mathematicians actively engaged in advanced mathematics - faculty, postdocs, and graduate students in PhD mathematics programs who often have teaching requirements at the undergraduate level; and (b) at an American university mathematics seminar in advanced mathematics. For both groups, the survey was intended to elicit participants'

thoughts on their training and their views on the value of pedagogical training for mathematicians.

In total, 64 participants completed the survey. The majority of the survey participants have teaching obligations at the undergraduate level in mathematics. The responders consisted of 32 faculty, 13 postdocs, 16 students, and 3 unidentified. The international conference had a total of 77 participants from 50 different universities worldwide. In total 57 responded. The survey at an American university was given after a research seminar talk to a small group of 7. The questions on the survey were chosen to be direct, short, and easy to answer in order to attract a high volume response rate. There were four questions:

1. How many pedagogical courses have you taken during the course of your mathematics education? (a) none, (b) 1 or 2, (c) 3 or more.
2. How important is it to have pedagogical training for mathematics PhD programs? (a) Not important, (b) Somewhat important, (c) Very important.
3. Should mathematics graduate programs offer courses in pedagogy? (a) No, (b) Yes, (c) Unsure.
4. If you answered yes to #3, should the courses be required? (a) No, (b) Yes.

Following these questions, the survey included a section for comments, and an option to describe the individual's position as faculty, postdoc, or student.

Survey Results

The questions and corresponding responses are indicated in Table 1. The numbers indicate the number of responses for each option per question; adjacent are the corresponding percentages with respect to the total number of responders for that particular question indicated as well.

When administering the survey at the international conference I requested that responders write the name of the country in which they took their pedagogical training (if they had any pedagogical training). The participants in the conference were from a diverse collection of countries. 21 of the 30 responders who had pedagogical training identified the country in which they took pedagogical courses, (see Table 2). It will be interesting for further research to determine whether there is any significant variance amongst geographical locations concerning the perceived importance of pedagogical training.

Table 1. Survey questions along with participants' responses.

Survey on Pedagogy for Mathematicians Results						
1. **How many pedagogical courses have you taken during the course of your mathematics education?**	**None**		**1-2**		**3+**	
	34	53%	25	39%	5	7.8%
2. **How important is it to have pedagogical training for mathematics PhD programs?**	**Not important**		**Somewhat Important**		**Very Important**	
	9	14.3%	33	52.4%	21	33.3%
3. **Should mathematics graduate programs offer courses in pedagogy? With three options to respond:**	**No**		**Yes**		**Unsure**	
	8	12.7%	37	58.7%	18	28.6%
4. **If you answered yes to #3, should the courses be required?**	**No**		**Yes**			
	16	44.4%	20	55.5%	--	--

Table 2. Countries where participants received teacher training.

Country where pedagogical courses where given			
Country	# of responders	1 or 2 courses	3 or more courses
Hungary	1	--	1
Israel	1	1	--
Japan	1	--	1
Korea	1	1	--
US	17	16	1

The results led to insightful findings highlighting the systemic issue, as follows:

1. The majority (53%) of respondents did not have any pedagogical training.
2. The overwhelming majority (86%) replied that pedagogical training for mathematics PhD programs is "somewhat" to "very important".
3. Less than 15% answered that it is "not important".
4. The majority (59%) did agree that graduate programs should offer courses in pedagogy.
5. Less than 13% answered that mathematics graduate programs should not offer pedagogical courses, and approximately 29% were unsure.
6. Of those who responded yes to question 3 (Should mathematics graduate programs offer courses in pedagogy?), approximately 55% replied that it should be required.
7. Very few (less than 8%) have taken 3 or more pedagogical courses.

Many were eager to complete the survey and expressed concern about the lack of focus on pedagogical training and attitudes about pedagogical training in mathematician circles. Of note, the majority of mathematicians in this survey were not trained formally in teaching, yet an overwhelming majority believe pedagogical training for mathematics PhD programs is important. 55% of those who responded yes to question 3 (59% of total participants) about whether mathematics PhD programs should offer pedagogical training courses said it should be required. This means that in total, only 31% of all responders believe that pedagogy training courses should be required. This underscores a common perspective amongst mathematicians, namely that many mathematicians do not think that formal pedagogy training is essential yet they still acknowledge that it is important. Most outstanding is that only 8% of all the responders had formal training of 3 courses or more.

After conducting the survey many responders reached out to discuss the issue of teacher training in mathematics PhD programs. Additional anecdotal insights provided by the survey participants conveyed that there was a sense that pedagogical training is not an issue that mathematicians think deeply about but is something that is vital. Few expressed concern about how ill-prepared they felt to fulfill their teaching obligations. Some were proud to praise the programs their universities had to guide their students in teaching. One faculty responder noted that his program for requiring students to take a 2-semester course on lecturing created the outcome that "our students result in the best presenters, regardless of the nature of their content".

The survey responses conveyed similar results with slight variation when observing data based on cross tabulation of the following subgroups (a) faculty, (b) postdocs, (c) students, (d) research seminar, and (e) international conference.

This survey highlights several key critical findings: (a) the lack of pedagogical training for mathematicians, (b) the overall belief that training is important, and (c) there exists a disconnect and reluctance of mathematicians regarding the fundamental importance of formal pedagogy training in PhD programs as a required part of the curriculum. The fact that only 31% think pedagogical courses should be required highlights the disconnect and lack of awareness of the vast body of knowledge in undergraduate mathematics education supporting the vital role of pedagogical training in the development of mathematics educators.

Discussion on Ramifications of Literature Review and Survey Results

There is substantial support, both in the literature and amongst the sample surveyed, that pedagogical training for college mathematics is important to produce good teachers, and more importantly, to produce good mathematics learners (students). This viewpoint runs counter to the notion that students' innate affinity for mathematics is the major determinant of successful mathematics learning and that students' lack of innate affinity for mathematics is the major determinant of failure to learn well (Rattan, Good, & Dweck, 2012).

There are two inherent paradoxes that emerge from the survey and literature review. (a) The majority of mathematics educators on the collegiate level are not trained in pedagogy; simply speaking, our teachers are not trained to teach. (b) The majority of mathematicians in the survey think pedagogy training is important, yet only a minority believes it should be required.

The failure to have pedagogically trained teachers contributes to poor outcomes of collegiate teaching in the STEM fields and blocks the emergence of mathematical talent across many demographics. This is of utmost concern given the global economic paradigm shift from agricultural-and industrial-based jobs to STEM-based careers, creating a need to prepare generations of students who are STEM-career ready. Yet we are not producing an adequate number of STEM degreed graduates to meet our national need (Hall et al, 2011). Moreover, those who are graduating are predominantly non-diverse—this is in part due to lack of interest in the field projected by ineffective teachers (Nardi, 2007). A significant cause of attrition is not students' ability but rather poor pedagogical practices by faculty (Seymour & Hewitt, 1997).

Understanding the needs of the student body and being equipped to teach collegiate students in mathematics are crucial for student success. The undergraduate curriculum for STEM-oriented majors requires proficiency in the "gateway" courses of calculus and linear algebra. Moreover, many students coming into college are missing basic mathematics skills and are placed in remedial mathematics courses such as college algebra or pre-college algebra (Bryk, & Treisman, 2010). These students require well-trained educators to succeed. Those students who don't pass entry-level courses either are blocked from furthering any STEM-based education or they drop out because their failure has caused them to believe that they cannot succeed (Bellafante, 2014).

To address this gap, critical care must be given to train the teachers who will be responsible to teach all students including (a) students that are insufficiently prepared and (b) an increasingly diverse student body. It is telling to note that students who are taking "beginner courses" are often taught by adjuncts, who are mostly PhD graduate students who are learning and researching upper level mathematics and have little to no pedagogical training (Harris et al, 2009). (As noted, if the teacher is a full-time professor, often they also don't have pedagogical training.) This systemic failure is having a damaging effect on our students and can be rectified.

Conclusions, Limitations, Future Directions, and A Call to Action

Conclusions

The results of this survey highlight that although most mathematicians have limited training in formal pedagogy, the majority believes that pedagogical training for mathematicians is important. Paradoxically, whilst they believe it is important, only a minority of mathematicians endorse that pedagogy is vital enough to be a basic requirement. The disconnect between the well-researched importance of education in mathematics instruction and the level of education training among mathematicians is represented in this survey and speaks to a surprising gap in current mathematics educational practice.

Lack of pedagogical instruction for university teachers is a systemic and detrimental problem. Absent any pedagogical requirements, and coupled with the dominant viewpoint that teaching is the second fiddle to the virtuoso performance of research, university teaching cannot be expected to be efficient – let alone excellent. The literature on the subject of teacher training overwhelmingly demonstrates that trained teachers produce better mathematics learners than untrained teachers, however brilliant these teachers may be in their field. Yet university systems impose no formal pedagogical requirements on their teachers.

Limitations and Future Directions

The survey was conducted on a small convenience sample of mathematicians in a particular area of pure mathematics. Thus, a larger sample size is needed to more broadly assess the attitudes and beliefs of mathematicians across a broader range of university settings and across a more random sample of mathematics specialties.

This study's survey provides preliminary evidence of a gap in mathematicians' knowledge about the importance of formal pedagogical training and readiness to promote educational change on an institutional level. Further research is needed to explore the nature of this discrepancy, its driving/contributing factors, and the impact of raising mathematicians' awareness and increasing knowledge for teaching. A pilot study in which beliefs and attitudes of a randomized group of mathematicians are assessed before and after exposure to pivotal articles in mathematics education research would shed light on whether a lack of crosstalk between the fields of pure mathematics and mathematics education is an important contributing factor to the gap highlighted in this paper. The study would also assess whether individual attitudes and beliefs about the importance of formal pedagogy in mathematics generally, and as a basic requirement in teaching undergraduate mathematics courses specifically, are changed pre- and post- exposure to the selected articles.

Call to Action

To solve this systemic problem requires a systemic solution. Both institutional requirements and public policy require change. The following possible solutions are suggested. Institutionally, mathematics PhD programs should offer and require pedagogical training for all their students, and universities should require all members of their teaching staff to be trained in pedagogy. In order to implement such change, public policy must be affected at a national level. Notwithstanding that change can happen systematically, it may require a grass roots effort (one school at a time) to adopt this policy. Further research is needed to better understand attitudes and beliefs among influential mathematicians toward and in resistance to imposing these educational requirements are crucial to guide future effective and targeted public policy change.

Acknowledgments

I am deeply indebted to Avraham Kamman for stimulating conversations regarding pedagogy for mathematicians and for consistently providing poignant feedback. I wish to thank Nick Wasserman for introducing me to SIGMAA on RUME. I also thank Erica Walker for encouraging me to pursue this line of inquiry. I am most appreciative to the international conference organizers for allowing me to conduct the survey.

References

Adams, K. A. (2002). *What Colleges and Universities Want in New Faculty. Preparing Future Faculty Occasional Paper Series*. Association of American Colleges & Universities, 1818 R Street, NW, Washington, DC 20009.

Bass, H. (1997). Mathematicians as educators. *Notices of the American Mathematical Society*, 44(1), 18–23.

Bass, H. (2005). Mathematics, mathematicians, and mathematics education. *Bulletin of the American Mathematical Society*, *42*(4), 417-430.

Bellafante, G. (2014). Community college students face a very long road to graduation. *New York Times*.

Boyer, E. L. (1990). *Scholarship reconsidered: Priorities of the professoriate*. Princeton University Press, 3175 Princeton Pike, Lawrenceville, NJ 08648.

Brand, M. (2000). Changing faculty roles in research universities: Using the pathways strategy. *Change: The Magazine of Higher Learning*, *32*(6), 42-45.

Bryk, A. S., & Treisman, U. (2010). Make math a gateway, not a gatekeeper. *Chronicle of Higher Education*, *56*(32), B19-B20.

Council, T. A., & National Research Council. (2001). *Educating teachers of science, mathematics, and technology: New practices for the new millennium*. National Academies Press.

Darling-Hammond, L. (2012). *Powerful teacher education: Lessons from exemplary programs*. John Wiley & Sons.

Darling-Hammond, L. (2000). How Teacher Education Matters. *Journal of Teacher Education*, 51(3). 166-173.

Dörfler, W. (2003). Mathematics and mathematics education: Content and people, relation and difference. *Educational Studies in Mathematics*, *54*(2-3), 147-170.

Eves, H. (1969). *An Introduction to the History of Mathematics*. New York: Holt, Rinehart and Winston.

Fairweather, J. (2005). Beyond the Rhetoric: Trends in the Relative Value of Teaching and Research in Faculty Salaries. *Journal of Higher Education,* 76: 401-422.

Gibbs, G., & Coffey, M. (2004). The impact of training of university teachers on their teaching skills, their approach to teaching and the approach to learning of their students. *Active learning in higher education*, *5*(1), 87-100.

Hall, C., Dickerson, J., Batts, D., Kauffmann, P., & Bosse, M. (2011). Are We Missing Opportunities to Encourage Interest in STEM Fields? *Journal of Technology Education*, *23*(1), 32-46

Harris, G., Froman, J., & Surles, J. (2009). The professional development of graduate mathematics teaching assistants. *International Journal of Mathematical Education in Science and Technology*, 40(1), 157-172.

Haycock, K., Majors, G. M., & Steen, L. A. (2004). Thinking K-16. *Education Trust*, *8*(1), 1-36.

Holm, T., & Saxe, K. (2016). A Common Vision for undergraduate mathematics. *Notices of the*

American. Mathematics. Society.
Kennedy, M. M. (1991). Some surprising findings on how teachers learn to teach. *Educational leadership*, *49*(3), 14-17.
Links. *TPSE Math*, www.tpsemath.org/links.
Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of teacher education*, *59*(5), 389-407.
Miller, E. R. (2017). A new methodological approach for examining mathematical knowledge for teaching at the undergraduate level: Utilizing task unfolding and cognitive demand. *Proceedings of the 20th Annual Conference on Research in Undergraduate Mathematics Education*, 39-52
Nardi, E. (2007). *Amongst mathematicians: Teaching and learning mathematics at university level* (Vol. 3). Springer Science & Business Media.
Nardi, E., Jaworski, B., & Hegedus, S. (2005). A spectrum of pedagogical awareness for undergraduate mathematics: From" tricks" to" techniques". *Journal for research in mathematics education*, 284-316.
Oleson, A., & Hora, M. T. (2014). Teaching the way they were taught? Revisiting the sources of teaching knowledge and the role of prior experience in shaping faculty teaching practices. *Higher Education*, *68*(1), 29-45.
Rattan, A., Good, C., & Dweck, C. S. (2012). "It's ok—Not everyone can be good at math": Instructors with an entity theory comfort (and demotivate) students. *Journal of Experimental Social Psychology*, *48*(3), 731-737.
Seymour, E., Hewitt, N. M., & Friend, C. M. (1997). *Talking about leaving: Why undergraduates leave the sciences* (Vol. 12). Boulder, CO: Westview press.
Shulman, L. S. (2005). Signature pedagogies in the professions. *Daedalus*, *134*(3), 52-59.