Developing Pedagogical Content Knowledge: Can Tutoring Experiences be Used to Train Future Teachers?

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This preliminary report explores data from a larger study investigating the nature of preservice elementary teachers' content knowledge and pedagogical content knowledge (PCK) in the area of number theory. A prominent theme emergent from the data – a contributing factor in participants' PCK – was the theme of tutoring experiences. Participants explicitly and regularly referenced their tutoring experiences when responding to hypothetical students in PCK tasks. The influential nature of preservice elementary teachers' tutoring experiences on their PCK holds implications for teacher-training, but further investigation is necessary. Questions concerning the design of a future study are proposed for discussion.

Keywords: Pedagogical content knowledge, preservice elementary teachers, tutoring

Research efforts to improve on mathematics education in the United States focus on a variety of contributing factors, key among them is the professional development and education of teachers. The literature has consistently linked student success in mathematics with teacher pedagogical content knowledge or PCK (e.g., Hill, Rowan, & Ball, 2005; Speer & Wagner, 2009), which is an understanding of content that is specific to teaching. A recent study also indicated a link between teachers' mathematical content knowledge and student achievement (Campbell et al., 2014). However, the research suggests that many preservice elementary teachers (undergraduates enrolled in elementary teacher education programs) may lack the mathematical content knowledge and the mathematical PCK necessary to teach mathematics for understanding (e.g., Conference Board of Mathematical Sciences, 2012). This suggests a need for preservice elementary teachers to have additional opportunities to develop their mathematical content and pedagogical content knowledge.

Some researchers have argued that mathematical PCK can only be developed through authentic interactions with students (e.g., Van Driel & Berry, 2010). However, preservice elementary teachers have few, if any, opportunities to engage elementary school students with mathematics prior to their student teaching internships. At many schools, mathematics methods courses for future elementary school teachers emphasize planning and preparation rather than practicum. Practicum can include classroom observations and, occasionally, the teaching of a mini-lesson. Neither of these activities allow for sufficient interactions with students for developing robust mathematical PCK. And during preservice elementary teachers' student teaching experiences, mathematics is never the primary focus; it is only one of many subjects that elementary education majors are required to teach every day.

In this preliminary report, I use data from a larger study exploring preservice elementary teachers' number theory PCK to suggest that tutoring experiences might be used to develop preservice elementary teachers' mathematical PCK. While suggestive, the evidence I present is hardly definitive. I conclude this report with a list of discussion questions concerning the design of a future study with which to further investigate the effects of structured tutoring experiences on preservice elementary teachers' mathematical PCK.

Theoretical Framework

The most prevalent model for mathematical PCK in the U.S. mathematics education literature is Ball and colleagues' (e.g., Hill, Ball, & Schilling, 2008) Mathematical Knowledge for Teaching (MKT). This model distinguishes between types of subject matter knowledge and types of PCK, and it identifies three constructs of mathematical PCK. According to Hill, Schilling, and Ball (2004), one of the three constructs of PCK is knowledge of content and students (KCS), which pertains to "knowledge of students and their ways of thinking about mathematics – typical errors, reasons for those errors, developmental sequences, strategies for solving problems" (p. 17). Another construct, knowledge of content and teaching (KCT), combines knowing about teaching with knowing about mathematics and pertains to instructional decisions as they relate to mathematics. A third construct, knowledge of curriculum, is a knowledge of programs developed for the teaching of a particular subject, concepts covered at a given level, and instructional materials available.

The emergent perspective (Cobb & Yackel, 1996) served as the lens for collecting and analyzing data. I primarily used the psychological lens because the bulk of the data represent individual conceptions. On the other hand, via the social lens I explored the classroom norms, expectations, and experiences that framed participants' perspectives on mathematics teaching and learning. I also drew from Ball and colleagues' MKT (e.g., Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008) in designing my interview tasks to elicit PCK and again to analyze responses.

Methodology

Data for this report came from an interpretive case study (Merriam, 1998) centered on preservice elementary teachers who were seeking a mathematics concentration and enrolled in a number theory course. Data included classroom observational notes, student coursework for 13 volunteers, as well as responses from two sets of one-on-one task-based interviews with six purposively chosen participants (a subset of the 13 volunteers), which served as the focus of the data analysis. During the interviews, all six interview participants admitted to having had mathematics tutoring experiences, either with grade school students, their peers, or both (see Table 1).

Table 1. *Interview participants' tutoring experiences*

	Brit	Cara	Eden	Gwen	Isla	Lucy
Grade School Tutoring	X		X	X	X	X
Peer Tutoring	X	X		X		X

Many of the interview tasks posed hypothetical student scenarios, designed to elicit number theory PCK. To elicit KCS specifically, I asked participants to identify the hypothetical students' mathematical conceptions and misconceptions. I also asked participants to describe how they might respond to the students in the scenarios in order to elicit KCT. Many of the students scenario task also included a meta-cognitive piece; I asked participants to reflect on why they responded to the hypothetical student in that way.

During the initial stages of my data analysis, I coded data according to the primary constructs detailed in my theoretical framework: KCS, KCT, classroom norms, etc. Within those general, umbrella codes, I conducted open, thematic coding. Finally, I conducted constant-comparative coding (Corbin & Strauss, 2008) until I achieved saturation. Among my efforts to ensure trustworthiness, I used member checking during the interviews and data triangulation afterwards.

Results: The Influence of Tutoring on Participants' Responses

One theme that emerged from the data was the theme of tutoring experiences. Participants frequently referred to past tutoring experiences, sometimes spontaneously, sometimes in response to the meta-cognitive interview questions, in order to justify their proposed responses to the students in the hypothetical scenarios. It was clear from their responses that participants' tutoring experiences contributed to their demonstrated PCK in general, and their KCT specifically. To depict this influence, I detail the results of one such interview task.

During the first round of interviews, I posed the scenario, "Mark suggested that the least common multiple (LCM) of two numbers is equivalent to their product." I asked participants to validate Mark's conjecture, identify his conceptions and misconceptions (KCS), respond to Mark in a way that helped him improve his understanding (KCT), and explain their reasoning for how they responded to Mark. All participants determined that Mark's conjecture was incorrect, and found appropriate counterexamples, but only Brit, Cara, and Lucy correctly determined that Mark's conjecture works for pairs of relatively prime numbers.

When I asked participants why they thought Mark might believe his conjecture to be true, they responded with a variety of insights, which I coded as "KCS" if the statement pertained to "students and their ways of thinking about mathematics – typical errors, reasons for those errors, developmental sequences, strategies for solving problems" (Hill, Schilling, & Ball, 2004). I also coded the KCS statements as "student reasoning" if the participant referred to *why* a student might believe a statement, claim, or conjecture about number theory is true or false. All six participants had acknowledged at some point during the interview that Mark's conjecture works for some pairs of numbers. Cara, Eden, Gwen, and Isla explicitly cited this as a reason for why Mark may have formed his conjecture. I coded this as "KCS", and "student reasoning", more specifically, because it was a reasonable explanation for why Mark might have believed his conjecture to be true.

After I asked participants why Mark might believe his conjecture to be true, I asked them how they would respond to Mark to help him correct his misconceptions, hoping to elicit KCT. Eden suggested that she would explicitly tell Mark which types of numbers worked. However, Eden's limited understanding of the concepts behind Mark's conjecture led to an inaccurate response. "You could go in and say, 'Yes, this method does work but only for certain types of numbers. And these certain types of numbers would be the prime numbers.'" Here, Eden's content knowledge weakened her KCT.

Gwen suggested that she would discuss a confirmatory example (four and five) with Mark so that he would better understand why it worked, but her explanation was insufficient. The data suggested that Gwen's understanding of the content may have limited any explanation with regards to the role factors play in finding the LCM of two numbers.

In their responses to Mark, Brit and Cara also suggested they would point out that while the product of two whole numbers is a multiple, it is not always the *least* common multiple. This instructional decision did draw attention to the inaccuracy in Mark's conjecture, so I coded it as "KCT." At some point in their responses to Mark, all participants claimed they would present him with a counterexample to explore. I coded these statements as "KCT" as well, because not only were they hypothetical instructional responses, but by strategically picking a specific counterexample the statements pertained to the specific mathematics related to the misconception. Isla suggested the counterexample of two and six, and she went so far as to explain to Mark *why* it was a counterexample.

> *Isla:* If we had the numbers two and six, and if you multiply them together, you get 12. But in the sense of the least common multiple of two and six, it can be six, because six times one is six and two times three is six.

Cara claimed that she would also provide Mark with a counterexample. However, her tack was very different than Isla's. Isla said that she would fully explain the counterexample to Mark, while Cara insisted that Mark explore the counterexample on his own. The other four participants, Brit, Eden, Gwen, and Lucy, suggested that they would give Mark a counterexample to explore using Cuisenaire rods. They all claimed that this would help Mark see, in a tactile and visual way, that his conjecture was not always true. This decision seemed to draw from participants' SCK and experience with Cuisenaire rods from their number theory course. Brit also used this opportunity to draw attention to common factors. "With six and eight, they have that two in common, so they have that stuff to match up before they actually multiply together." Brit went on to say that "we have to look at what they have in common and whether we can match up [the trains] before [the product]." Not only did Brit create an opportunity for Mark to realize that his conjecture was invalid, but she demonstrated KCT by also creating an opportunity for Mark to understand *why* his conjecture does not always work.

Participants offered a multitude of reasons for why they responded to Mark in the ways that they did. I coded all of these responses as "insight to KCT." Brit, Eden, and Isla all cited their tutoring experiences with elementary and middle school students. Brit said that her response to Mark was "just a natural thing" for her because of her years of experience tutoring students.

Eden's tutoring experiences led her to believe that students can be quite adamant that their answers are correct and that it can take a bit of work to convince them of an invalid answer or procedure.

> *Eden:* I tutor some kids in math, and they always think that their method is right, but you kinda show them that, 'if you do it this way I get this answer and it's not the same as yours. How come?' And you kinda slowly take what they're saying and slowly show them why it's wrong. And hopefully they'll connect to it saying, oh yeah, that is wrong.

It is evident from Eden's response that her tutoring experiences contributed to her general strategy for responding to Mark. Eden was one of three students (including Brit and Isla) that had not yet taken a mathematics education course. Later in the interview, she claimed that her tutoring experiences were the only experiences that contributed to her responses to the hypothetical students in the student scenarios. She also suggested that her strategy when working with students mostly consisted of trial and error. She said she would "see what works and what doesn't work."

Isla claimed that her tutoring experiences helped her to recognize the conflicts that arise when teachers tell their students that a "rule" always works when, in fact, it may not. She frequently tutored her younger cousin, a 5th grade student, and she witnessed her cousin attempt to make generalizations about her mathematical understandings from earlier grade levels in order to better understand the current material. Isla claimed that this was problematic. She said, "You're told this rule applies for all, but it really doesn't."

Rarely did participants have experience tutoring the specific content of the interview tasks, but when they did, they would bring it up during the task to justify their responses. In some cases, as we see with Eden's response to Mark, insufficient content knowledge can negatively affect PCK, thus making content knowledge another important contributing factor to preservice teacher PCK.

Discussion

While some might argue that teachers may only demonstrate true PCK (KCT, in particular) in the classroom, others suggest that demonstrations of PCK in a clinical interview may be a sort of pre-knowledge or a subset of the knowledge they could demonstrate in the classroom (Hauk, Jackson, & Noblet, 2010). Even Hill (2010), a contributor of MKT, developed and implemented PCK test items that proposed to elicit KCT. If clinical settings can elicit a subset of a future teacher's mathematical PCK, then more authentic teaching experiences like tutoring sessions are very likely to do so. The data from this preliminary study suggests that not only would tutoring experiences elicit PCK, but they appear to contribute to PCK development. However, to better understand the potential for using tutoring experiences to develop preservice elementary teachers', further inquiry is necessary. Such inquiry should also take into account the effects of content knowledge on PCK. In designing a future study, we might consider the following questions.

Questions for Audience Consideration:

1. How might we structure or supplement mathematics tutoring sessions with primary students in order for the tutors (preservice elementary teachers) to best learn from the experience?
    a. Which design elements will encourage the development of the tutors' KCS? KCT?
    b. How might we ensure that participants' content knowledge is sufficient and being used appropriately?
2. It can be argued that the format of a tutoring session greatly distinguishes tutoring from an authentic field experience. What are some of the limitations (and benefits) of using tutoring sessions in the research design?

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