Learning Through Play: Using Catan in an Inquiry-Oriented Probability Classroom

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Research has documented the power of play to affect learning at all ages. This research shares the kinds of mathematical student thinking elicited by incorporating the board game Catan into an inquiry-based classroom. The class was composed of 25 students, not majoring in STEM fields, who were enrolled in a freshman seminar course intended to provide an opportunity to engage in research-like inquiry. Students had played Catan in class and to engaged inquiry-oriented instruction sessions focusing on the relationships between mathematics and Catan. Student work is provided for arguments related to the value of each resource and selecting locations for initial settlements, with connections between this work and topics traditionally taught in probability classes

Keywords: probability, games, inquiry-oriented instruction

The phrase "learning through play" often evokes images of elementary age students, however play can be an important part of the learning process for learners of all ages (Rieber 1996). In mathematics games have often been used to address relatively simple probability concepts such as rolling dice or choosing cards from a deck (Goering, 2008; Hoffman & Snapp, 2012/2013). The board game, Catan (Mayfair Games, 1995), provides unique opportunities for learners to engage with both these probability basics as well was several other mathematical ideas including more advanced ideas related to probability and expected value. Here I share evidence of how Catan provided a relevant context in which students used concepts of probability and expected value to address two important questions related to strategic game play: (1) determining which resource is most valuable and (2) deciding where to place one's initial settlements.

A Brief Introduction to Catan

Catan is a property building and resource trading based Euro-style board game for 3-4 players. Since its introduction in 1995, Catan has gained world-wide recognition, winning several awards, being translated into over 30 languages, and revolutionizing the board game industry (Law, 2010). At the start of every game the board is constructed from resource tiles, numbers, and ports. This makes Catan a perfect setting for abstract thinking about patterns and generalities rather than strategies that optimize play on one unchanging board. The benefits of this dynamic nature has already been noted by both mathematicians (Austin & Miller, 2015) and computer scientists (Szita, Chaslot & Spronck, 2010).

The board is assembled by arranging 19 hexagon tiles into a larger hexagonal formation as shown in Figure 1. These tiles dictate what resource will be produced at a location: brick/clay (3 tiles), lumber/wood (4 tiles), wool/sheep (4 tiles), grain/wheat (4 tiles), ore/stone (3 tiles), or nothing/desert (1 tile).   Next each tile except the desert receives a number chip, this will dictate when the resource is produced. There is one ship for 2, one for 12, and two for every number from 3 to 11 excluding 7. Whenever the number on a tile is rolled by any player, the players with settlements or cities adjacent to that tile will collect the given resources. Finally, eight ports are distributed around the board, but they do not affect the mathematics discussed in this work. Settlements will be placed at the intersection of the corners of the hexagon tiles. Roads will be built along the edges of the tiles. Only one settlement or road may occupy a given space and there must be at least two edges between any two settlements on the board.

The game begins with each player placing two settlements in reverse draft order. Choosing initial settlements is a very important part of the game because it affects what resources you have immediate access to and which resources you may have the opportunity to build to; all new settlements must be connected to one of the player's initial settlements by at least two roads.

On a player's turn they roll two standard dice and all players who have a settlement on a tile with that number collect the corresponding resources. If a seven is rolled the player whose turn it is has the opportunity to move the robber, which is another detail which does not affect the mathematics in this research. Next the player whose turn it is has the opportunity to build additional road or settlements, upgrade a settlement to a city, or purchase a development card, which gives various bonuses. Each of these things contributes to a player earning victory points; the first player with 10 victory points wins. For complete rules please see, https://www.catan.com/en/download/?SoC_rv_Rules_091907.pdf.

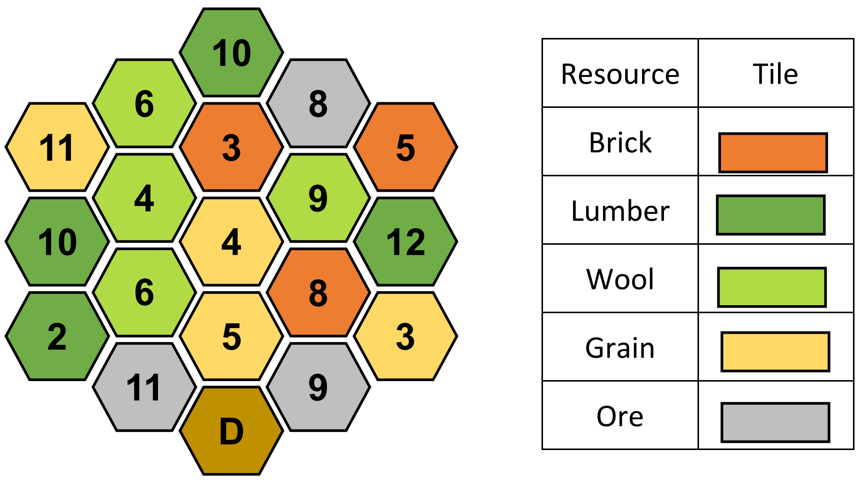


Figure 1: This is one sample Catan board, but there are many more possibilities.

Setting and Methods

Data for this study were collected from a freshman learning community focusing on the relationship between mathematics and Catan; it consisted of the same cohort of students enrolled in both a common pre-calculus course section and a freshman seminar section which provided the majority of the focus on Catan. Students in the learning community were all first semester freshman at a large college in the southeastern US. The class included 25 students from a variety of majors, none of whom had officially declared majors in the university's science and math focused college. 24 students consented to participate in the study.

The freshman seminar course was intended to orient students to college life and to provide them with an opportunity to engage in some accessible research-like activities which do not have an easily found answer or explicit method for solving. The course met three days a week for 50 minutes, with roughly 1/3 of time dedicated to orientation content, 1/3 of time dedicated to playing Catan, and 1/3 of time dedicated to inquiry-oriented instruction (Rasmussen & Kwon, 2007) to explore the mathematics of Catan. The mathematical components of the course were presented to students as practical questions in the context of the game, which they were then asked to answer using mathematical reasoning. Explorations were often proceeded or followed by a game played on a board strategically chosen to highlight the concept in question. Methods of exploration included individual explorations, group work, debates, and whole-class student-led solutions. Previous analyses have shown that the course was effective in engaging students in mathematical reasoning (Molitoris Miller & Hillen, 2018). The goal of this report is to provide more detailed analysis of the kinds of mathematical reasoning the students used. These results answer the research question: What kinds of mathematical thinking are elicited by the board game Catan, in a student-driven inquiry-based classroom?

The data for this analysis came from a final exam item, which is very closely related to the student-centered inquiry-based theme of the course. It stated, "Describe five ways you can use mathematics to improve your chances of winning in Catan. For each mathematical application, describe it in detail and provide an example of how it works." Students were given this prompt one week before the exam to think about it in advance but were not permitted to bring any prepared materials into the exam with them and two sample boards were provided. Although different students took more or less vocal roles in classroom or group discussions, the final exams better measure what each individual student eventually learned. Inductive coding was used to code the 120 responses from the 24 consenting students and group them into categories according to the topic they addressed. The results in this paper focus on the two largest overarching themes present in the student's final exam responses, the value of each resource, and settlement location.

Determining Resource Value

The first main theme that demonstrates the kinds of student thinking elicited by the game focuses on assigning value to the resources. During the game, players may opt to trade resources with one another; thus, the value of each type of resource comes into question to determine if a certain trade is advantageous or at least fair.

First students began contemplating the usefulness of any resource in general as they relate to building in the game. In one approach, students highlighted that brick, lumber, wool, and ore are each used in two out of the four building processes, but grain is the only one used in three of the four building possibilities, thus they claimed that grain is the most valuable resource. Their argument rested on the idea that not having any access to grain would greatly limit ones' ability to progress in the game.

Other students used a more weighted average, where the student determined how many of each card was needed to build one of everything. This strategy found brick, lumber, and wool were equally valuable because you would need only two of each of those resources. Grain and ore were also equally valuable, requiring two of each. Students then looked to the number of tiles of each type to determine that ore was more valuable because there are only three tiles which produce ore opposed to four tiles which produce grain. This strategy suggests that while you need access to both grain and ore, you would have a stronger advantage if you have slightly better access to ore.

Both of these solution methods were completely student-generated in class. This variety of arguably equally valid methods or measuring value provide opportunities to discuss the complexity of measuring more abstract judgement-based attributes such as value. This discussion could be taken further to include strategies like tracking how many of each resource is used in an average game, or a combination of general usefulness and rarity on a particular board.

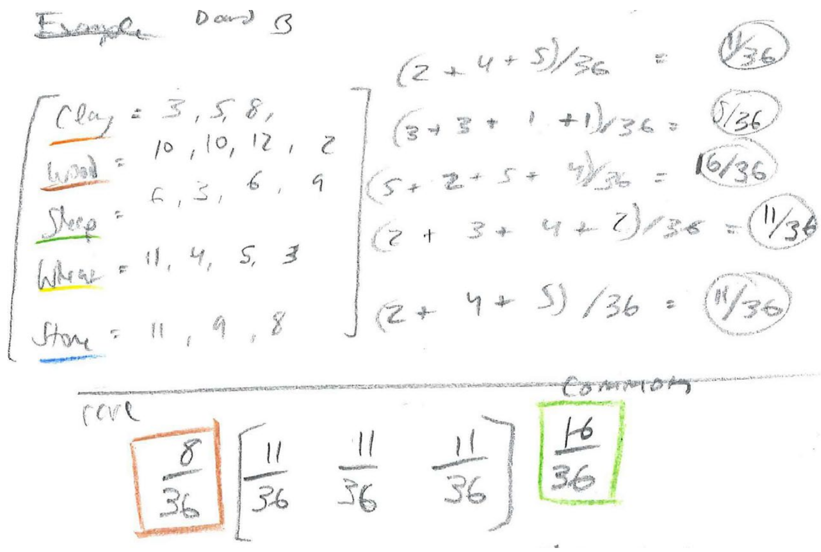
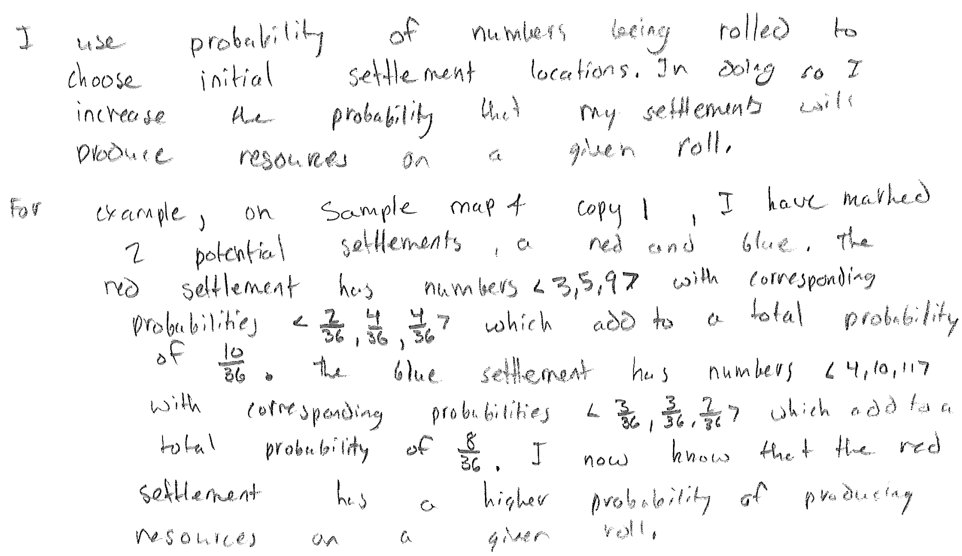


Figure 2: This student used probability to determine which resources will be rare and common

Raising the connection to the game board lead students to consider not only the resources' usefulness in general, but also the usefulness of each resource in terms of supply and demand on a given board. This question was encouraged by asking students to play on a certain board with one particularly rare resource. Students used the expected number of cards of each type produced on any given roll to determine "how many of each resource you would get if you rolled all 36 possible rolls each exactly once." Student work corresponding to this strategy can be found in Figure 2. This work involved considering the probability of rolling each number two through twelve with a pair of standard six-sided dice, as well as when adding probabilities is appropriate or not, and how to handle duplicate numbers, such as two grains tiles with 5's on them.


I use probability of numbers being rolled to choose initial settlement locations. In doing so I increase the probability that my settlements will produce resources on a given roll.

For example, on Sample map 4 copy 1, I have marked 2 potential settlements, a red and blue. The red settlement has numbers $<3,5,9>$ with corresponding probabilities $<\frac{2}{36}, \frac{4}{36}, \frac{4}{36}>$ which add to a total probability of $\frac{10}{36}$. The blue settlement has numbers $<4,10,11>$ with corresponding probabilities $<\frac{3}{36}, \frac{3}{36}, \frac{2}{36}>$ which add to a total probability of $\frac{8}{36}$. I now know that the red settlement has a higher probability of producing resources on a given roll.

Figure 3: This student compared two potential settlement locations based on expected number of cards produced.

Settlement Placement

Knowing which resources are most important in general or on a specific board is only one part of what informed student's mathematical justification of their decision process when choosing where to place initial settlements. The most basic intuition is that being on more resource producing tiles is better than being on fewer; however, when presented with this proposal, students were able to create examples where it could be statistically advantageous to be on a single very productive tile over a location with three low production tiles. After considering these ideas students began to evaluate each location based on the expected number of cards each location would produce, as shown in Figure 3. Students also considered resource rarity to determine which tiles were most important to settle near. This lead to an informal exploration of conditional probability and possible applications of reasoning aligned with Bayes formula.

Other Catan Applications

The student work provided above demonstrates the kinds of mathematical thinking that can be elicited by the use of a board game such as Catan in a course focused on exploring probability and expected value. Other topics explored included, the probability a certain number would be rolled before it is your turn to build again, the largest number of cards you could have in your hand with no more than three-of-a-kind and not be able to build anything, and various combinatorial considerations related to how to acquire the required ten victory points and win the game. These questions are not unlike others seen in probability classes but they are uniquely motivated because of the relevance to the game and game play. Recall that this work was completed with freshman non-math majors who were co-enrolled in a pre-calculus course. Employing similar techniques in a higher level probability or discrete math course intended for mathematics majors would likely lead to the same conclusions more quickly and provide further opportunities for extensions.

References

Austin, J.& Molitoris Miller, S. (2015). The Settlers of Catan: using settlement placement strategies to explore probability and expected value. *College Math Journal. 46(4)*.

Hoffman, T. R., and B. Snapp. (2012/2013)"Gaming the Law of Large Numbers." *The Mathematics Teacher 106, no. 5,* 378-383.

Law, K. (2010). Mental Floss, Settlers of Catan: Monopoly killer? Retrieved from: http://mentalfloss.com/article/26416/settlers-catan-monopoly-killer.

Mayfair Games. (1995). "Catan."

Molitoris-Miller, S., & Hillen, A. F.(2018) Using Catan as a vehicle for engaging students in mathematical sense-making. Proceedings of Special Interest Group of the Mathematical Association of America Conference on Research in Undergraduate Mathematics Education. Mathematical Association of America. San Diego, CA.

Rasmussen, C. and Kwon, O. N. (2007) An inquiry-oriented approach to undergraduate mathematics. *Journal of Mathematical Behavior*, 26, 189-194.

Rieber, L. P. (1996). Seriously considering play: designing interactive learning environments based on the blending of microworlds, simulations, and games. *Educational Technology Research and Development 44, no. 2*, 43-58.

Szita, I., G. Chaslot, and P. Spronck. (2010). Monte-carlo tree search in Settlers of Catan. *Advances in Computer Games*, 21-32. Springer, Berlin.