Critical Features and Representations of Vectors in Student-Generated Mindmaps

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The purpose of this study is to investigate multivariable calculus students' communication of vectors by examining how their responses on a mindmap assignment change over time. A mindmap is a visual network of connected and related concepts often with one image or topic centrally located. Through this open-ended instrument, we conduct a qualitative analysis to explore the connections students make between different aspects and multiple representations of vectors.

Keywords: mindmaps, vectors, variation theory, multiple external representations

While basic vector concepts, representations, and operations are presented in both high school and college mathematics, students continue to have significant conceptual difficulties with them. Much research on student understanding of vectors explores students' misconceptions of physical concepts such as force and motion, but students' misconceptions regarding vector concepts, properties, and fluency in vector operations are not explored directly (Aguirre & Rankin, 1989; Barniol, Zavala, & Hinojosa, 2013; Flores, Kanim, & Kautz, 2003; Govender & Gashe, 2016; Hestenes & Wells 1992; Hestenes, Wells, & Swackhamer, 1992; Miller-Young, 2013). While some researchers provide more explicit consideration of students' understanding of vector concepts, representations, and operations outside of a kinematic context, the focus has not been on how students make connections between vector operations and between different representations of vectors (Barniol & Zavala, 2014; Knight, 1995; Kustusch, 2016; Nguyen & Metzler, 2003; Van Deventer & Wittmann, 2007; Wang & Sayre, 2010; Zavala & Barniol, 2010). The overarching research goal for this paper is to investigate multivariable calculus students' communication of vectors by examining how their responses on a mindmap assignment change over time. More specifically, the three research questions we consider are: what changes are noted with respect to the

1. critical features students address in the mindmaps?
2. connections that are made between these features in the mindmaps?
3. representations (e.g., graphical, verbal, symbolic, or numeric) used in the mindmaps?

Mindmaps and Concept Maps

In recent years, educators have begun using software mapping tools for a variety of pedagogical and research purposes (Govender & Gashe, 2016; Ayal, Kusuma, Sabandar, & Dahlan, 2016; Davies, 2011; Edmondson, 2005). These diagrammatic representations of ideas and their relationships "may not be the panacea … , but they do represent an approach that more effectively taps the dimensions of student thinking that many traditional assessment formats miss" (Edmondson, 2005, p. 36). Here, we identify what critical features and representations of vectors students present in a series of mindmap assignments.

The terms concept map and mindmap are used interchangeably by software developers and educators, but in the research literature there are distinctions. *Mindmaps* are networks of connected, related concepts often with one topic centrally located; they typically use line thickness, colors, and pictures to communicate ideas and connections (Davies, 2011). Mindmaps are used to study student understanding of a concept by providing a deliberately ambiguous central topic without suggesting relationships (Bandera, Eminet, Passerini, & Pon, 2018). A concept map can be thought of as a specific type of mindmap. *Concept maps* are more tightly structured, hierarchical networks with descriptive phrases such as “leads to,” “results from,” “is a part of,” etc. characterizing the connections linking two ideas (Davies, 2011; Edmondson, 2005).

Because of the more formal structure of concept maps, automated and quantitative scoring rubrics are typically used to count the number and complexity of linkages, placing less emphasis on the content. Few guidelines and protocols exist for qualitative assessment, and most focus on the structure of the concept map independent of content (Keppens & Hay, 2008; Kinchin, 2000). While time consuming, a qualitative content analysis of concept maps can document change over time among a group of participants with varied backgrounds (Hough, O’Rode, Terman, & Weissglass, 2007).

Multiple Representations

Multiple External Representations (MERs) of mathematical and scientific concepts are commonly used to support learning by integrating and/or coordinating more than one source of information. However, this integration requires the ability to translate between different representations, which students often find difficult to do (Ainsworth, 2006; Kozma, 2003). How well an individual is able to move between different representations depends on several individual characteristics including domain knowledge and representational fluency (Ainsworth, 2006). In this study we not only consider the vector knowledge that students communicate in their mindmaps and how it is organized, but also which MERs they chose to include in their mindmaps. For the purposes of this study we use a modification of Shield and Galbraith’s (1998) taxonomy of modes of representations of written mathematics: symbolic (i.e., algebraic), numeric, verbal, and graphical (Neira & Amit, 2004).

Theoretical Framework

Our work combines Simon’s (2017) theoretical construct of “mathematical concept” with Marton and Booth’s variation theory (Rundgren & Tibell, 2009). We begin with the assumption that effective mathematics instruction and assessment of student understanding requires clear articulation about the mathematical learning goal which is often too broadly described as “understanding a topic” (Simon, 2017, p. 128). Simon’s construct of “mathematical concept” and the notion of “critical feature” from variation theory taken together have the potential to provide a way to more precisely define what it means to “understand vectors.”

Simon defines a *mathematical concept* to be “a researcher’s articulation of intended or inferred student knowledge of the logical necessity involved in a particular mathematical relationship” (Simon, 2017, p. 123). Like Simon’s definition of mathematical concept, variation theory also focuses on intended and inferred student knowledge. In variation theory, the term *critical feature* refers to an aspect of or condition of a topic that is necessary for learning. According to variation theory, learning takes place when students perceive critical features, and students can only discern a critical feature if they experience variation of it (Runesson, 2006).

To specify a mathematical concept, Simon (2017) recommends observing contrasts in individuals’ mathematical functioning, whether it be between: a student and an expert, two students, or observations at different times of a single student. The mathematical concept then arises as a specific explanation of differences observed. Simon (2017) cautions that further research or pedagogical activity will reveal modifications to the mathematical concept. As a first step towards developing a mathematical concept of vectors in multivariable calculus, critical features of the vector cross product have been identified: magnitude, direction, angle between two vectors, location of the vectors, and orientation of the cross product to the two vectors that form it (VanDieren, Moore-Russo, Wilsey, & Seeburger, 2017). Our study tests the validity of these critical features with different data and on a broader range of vector concepts. We begin the process of developing mathematical concepts for the nebulous goal of "understanding vectors" by contrasting work of students in a multivariable calculus class over time attending to differences in their communication of critical features.

Methodology

Context of the Study

The participants were 30 students in a multivariable calculus course at a private, regional university. On the first day of the semester, the first author introduced the mindmap activity to the students and explained its purposes to: (a) provide students the opportunity to organize their thoughts on vectors, (b) identify connections between different features, applications, and operations of vectors, and (c) serve as resource during the first exam. Students were allowed to include any items including images from textbooks, links to online tutorials, or photos of handwritten notes in their work. The first author suggested to students to use Inspiration and Lucid Charts software to create the mindmaps, but ultimately students could choose their preferred software. A sample mindmap on geometry content and a tutorial for creating a mindmap in Inspiration were offered to the students. Students were assigned to create a mindmap of what they knew about vectors at three points of time during the first three weeks of the semester during which the topics of vectors, vector operations, lines, and planes were covered in class. After each submission students were given feedback on their work including suggestions for adding graphical depictions, applications, or missing concepts in future submissions.

Data Analysis

Of the 30 students in the study, 24 students submitted at least one mindmap. One student was removed from the sample because his work was not in the form of a mindmap. Of the 23 students who submitted first and final drafts of the mindmaps, only 15 submitted an intermediate draft during the second week. Therefore, we report results from only the first and final mindmaps of the 23 students. An iterated coding analysis was conducted on the mindmaps.

Development of coding. Two days of discussions between the authors led to a first round of analysis, which was based on eight *a priori* content or topic categories (*vectors, scalar multiplication, addition, subtraction, dot product, cross product, projection,* and *other*). These categories were used to sort the content in each mindmap. These content categories were further refined according to critical features of vectors (*direction, magnitude, angle between vectors,* and *location*). Each category that was marked as present was then coded according to its representation on the mindmap (*graphical*, *verbal*, *symbolic* and/or *numerica*l). These were coded for presence and not for accuracy. For each of these categories, whether the mindmap included an application (e.g., force, work, etc.) was also coded. Finally, the researchers coded whether each concept was presented with two- and/or three-dimensional representations. A second set of codes was used to characterize the relationship between pairs of concepts and how they were depicted (by lines or words) and whether these connections represented declarative, procedural, and/or conceptual knowledge (Sarwar & Trumpower, 2015). We will not discuss the declarative, procedural, and conceptual knowledge coding further in this paper.

A sample of four student mindmaps was coded by the first author. Issues with this coding scheme were then discussed with the second author. A new coding scheme was proposed that added generic content categories. Based on emergent themes from this sample, three new codes were added to the scheme: “unit vectors” and “basis vectors” were added as subcategories of *vectors* and “orthogonal component of projection” was also added as a subcategory of *projection*. The *words* category for connections between content areas was split into three categories: “words,” “multimedia static,” and “multimedia dynamic” to distinguish verbal descriptions from graphs or images and video links. The category of 2D or 3D was only assessed over the entire mindmap and not on individual concepts because the previous level of refinement was deemed unnecessary. Similarly the *applications* code was evaluated at the level of topic and not critical feature. The original sample of four students plus two additional mindmaps were coded by the second author according to this new scheme. The authors then discussed some discrepancies in coding from these rounds. Clarifications were made in the codebook and the first author then coded the initial four and the additional two mindmaps with the new scheme. The codings of the two authors on the sample of six mindmaps were compared, discrepancies discussed, and clarifications to the codebook added. The dot product subcategories were eliminated from the codebook because these critical features did not apply to a scalar value.

Interrater reliability measures. Since the codes were not mutually exclusive categories, the measure Mezzich’s kappa of interrater agreement for multivariate nominal data was used (Mezzich, Kraemer, Worthington, & Coffman, 1980; Eccleston, Weneke, Armon, Stephenson, & MacFaul, 2001). Mezzich’s kappa statistic for this sample indicated 63% agreement. Most of the disagreement stemmed from the interpretation of the categories “verbal” and “declarative” in the fifth mindmap in the sample. The authors discussed these disagreements and came to a consensus that was then addressed in the codebook. Making adjustments to these codings based on the new consensus, brought Mezzich’s kappa to 73%. Because this sample of six mindmaps did not exhibit every code in the codebook, two additional mindmaps were selected and coded by both authors independently. Results were compared resulting in Mezzich’s kappa equal to 75%. At this stage, the codebook was finalized and the first author coded the remaining mindmaps.

The codebook. The codebook can be separated into two parts: <u>content</u> and <u>connections</u>. The <u>content</u> coding included the topic categories: *vector* (V)*, scalar multiplication* (S)*, addition* (A)*, subtraction* (B)*, cross product* (X)*, dot product* (D)*,* and *projection* (P)*.* There was also an *other* (O) category to capture ideas (e.g., lines and planes) not directly fitting into these topics. Each topic category was marked for presence and whether the mindmap included an example of an application of each topic (V-app, S-app, A-app, B-app, X-app, D-app, P-app, and O-app respectively). In addition, any relevant critical features of these categories present on the mindmap were also coded. Table 1 below describes some of the subtopic codes that were observed along with examples. In addition to the subtopic codes listed in Table 1 and the application codes, the full list of subtopic codes included: Ag (general addition), Bg (general subtraction), Xg (general cross product), Xd (cross product direction or orientation in relation to the two vectors that form it), Xm (cross product magnitude), Xa (cross product as orthogonal to the two vectors that form it), Dg (general dot product), Pg (general projection), Pd (projection direction), Pm (magnitude of the projection), Po (orthogonal component of projection), and O (other). Finally, for all codes, except the application codes, it was noted whether or not the topic and/or critical feature was described *verbally, numerically, graphically,* or *symbolically.*

We used <u>connection</u> codes between topics and how those connections were represented. For example, the codes VS-W, VA-W, VB-W indicated the topic of vector was connected, respectively, to scalar multiplication, addition, and subtraction through words. An example was a bubble with the words “Vector operations” that connects to three bubbles with “scalar multiplication,” “addition,” and “subtraction.” If these three are also connected through lines on the mindmap, then VS-L, VA-L, and VB-L were also coded. Other connections could be in the form of a static image (MS) or a dynamic multimedia clip (MD). For example, a video clip of an instructor working through a problem demonstrating **u** + (-**v**) = **u** - **v** was coded as AB-MD. The complete list of connecting codes included all pairs of topics (V, S, A, B, X, D, P) and all four types of connections (L, W, MS, MD).

Table 1. A sample of the commonly used codes, their descriptions, and representative examples.

Subtopic Codes	Description	Examples
Vg	General Vectors - any mention of vectors at all.	Different notations of vectors; graph of a vector; conversion between different representations of vectors; any of the examples under the “V” codes below; if there is very little information on the mindmap, this may be the only category coded.
Vd	Vector Direction	Picture of a vector with the direction marked; mention of change in x and change in y; image that marks the angle the vector makes with the x-axis; explanation of the process of how to find the angle that the vector makes with the x-axis
Vm	Vector Magnitude	Image of a vector with the length marked; formula or computation of the length of the vector; use of the Pythagorean Theorem for computing length
Va	Angle between Two Vectors	Image with angle between two vectors marked in a graph; image with angle between two vectors defined in a formula for dot or cross product (the presence of “theta” in a formula without a geometric or verbal definition would not be coded)
Vl	Vector Location	Statement that vectors can be moved or that location doesn’t matter
Vu	Unit Vectors	Definition, formula or explanation for finding a unit vector in a given direction
Vb	Basis Vectors	Definition or graph of the **i**, **j**, **k** vectors
Sg	General Statement about Scalar Multiplication	Mention that scalar multiplication combines a scalar with a vector, Including the notation cv, any of the examples in the “S” codes below
Sd	Scalar Multiplication Direction	Indication that c**v** is parallel to **v**; mentioning the impact of -1 on the direction
Sm	Scalar Multiplication Magnitude	Demonstration the effect of the magnitude of c on the length of c**v**

Results

The vast majority of students included both two- and three-dimensional representations of vectors in their initial (87%) and final mindmaps (96%). On the other hand, very few students included applications of vectors in their initial mindmaps. Among the initial mindmaps, only seven students (30%) provided any application, but twenty students (87%) included an application of vectors in their final mindmaps. The distribution of the kinds of applications the students mentioned in their mindmaps appears in Table 2. Almost no students provided applications of scalar multiplication, addition, subtraction, and projection. Table 3 and Table 4 report the frequency counts of the other codes.

Table 2. Frequency comparison (counts) of application codes in the initial and final mindmaps (n=23).

Mindmap	V-app	S-app	A-app	B-app	X-app	D-app	P-app	O-app
Initial	7	0	1	0	0	0	0	0
Final	10	1	1	0	16	14	1	1

Table 3. Frequency comparison (counts) of topics and representations in the initial and final mindmaps (n=23).

Types of Representations	Vg	Vd	Vm	Va	Vl	Vu	Vb	Sg	Sd	Sm	Ag	Bg	Xg	Xd	Xm	Xa	Dg	Pg	Pd	Pm	Po	O
Initial Mindmap																						
Geometric	7	1	4	2	1	2	2	2	2	2	4	3	1	1	1	1	3	0	0	0	0	0
Numerical	17	5	5	3	0	4	12	9	0	0	8	10	0	0	0	0	3	0	0	0	0	0
Symbolic	20	4	12	4	0	6	13	12	1	2	12	12	1	0	0	0	6	0	0	0	0	0
Verbal	22	17	19	6	5	15	6	16	4	6	13	11	2	1	1	1	5	0	0	0	0	0
Final Mindmap																						
Geometric	13	6	7	5	1	4	4	5	4	4	11	9	7	9	4	6	8	6	1	2	3	2
Numerical	17	5	6	3	0	4	13	10	0	0	8	10	6	0	1	0	9	1	0	0	0	3
Symbolic	21	4	13	7	0	9	14	16	1	2	14	15	19	2	8	1	23	7	2	1	1	8
Verbal	21	17	20	12	7	16	6	17	4	6	12	10	17	8	4	6	19	6	1	1	4	8

Table 4. Frequency count of connections in the initial and final mindmaps (n=23).

Types of Connections	VS	VA	VB	VX	VD	VP	SA	SB	SX	SD	SP	AB	AP	BP	XD	DP
Initial Mindmap																
Lines	20	19	19	1	6	0	0	0	0	0	0	1	0	0	0	0
Words	9	8	7	0	1	0	1	0	0	0	0	6	0	0	0	0
Multi-media Static	3	1	3	1	0	0	1	1	0	0	0	0	0	0	0	0
Multi-media Dynamic	0	1	1	1	2	0	0	1	1	0	0	0	0	0	1	0
Final Mindmap																
Lines	22	20	21	21	22	7	0	0	0	0	0	1	0	1	0	2
Words	11	9	8	6	6	2	1	1	0	0	0	7	1	0	7	4
Multi-media Static	6	10	10	14	11	4	1	5	1	0	0	3	1	1	4	0
Multi-media Dynamic	0	1	1	1	2	0	0	1	1	0	0	0	0	0	1	1

Discussion

We first consider the topics and critical features in the mindmaps. Since the initial and final mindmaps were created three weeks apart and more material on vectors was presented in class during this time frame, it is not surprising to see more topics on the final mindmaps. Once students addressed a concept in the initial mindmap, they rarely made any changes or additions to that concept in their final mindmap. Therefore, the statistics reported below represent whether a category was coded on at least one of the initial or final mindmaps for each student.

Almost all students included the vector operations (scalar multiplication, addition, subtraction, cross product, dot product), but only nine (39%) of the students mentioned projection. Nearly all students described the direction and magnitude of the vector, but only 14 (61%) of the students mentioned the angle between two vectors. This echoes a study of pre-service teachers' concept maps of vector kinematics in which the most common code was "vectors have magnitude and direction," and only one concept map mentioned angle (Govender & Gashe, 2016, p. 331). Furthermore, in our study when discussing the cross product, less than half mentioned the direction or magnitude. Therefore, while students identified relevant vector operations, they did not communicate all critical features related to the operation.

Considering the connections between the topics, it is notable that students tended to treat the vector operations in isolation, especially scalar multiplication. Additionally, only eight students (35%) made a connection between addition and subtraction. Even straightforward connections between topics were not reported. For example, only five of the nine students who mentioned projection included the formula and/or connected it with the dot product. When students did demonstrate a connection between two of the operations, they did not explicitly draw a connecting line, but implicitly connected the ideas through a formula or a static image.

Students' representation of vectors changed over time. More students provided geometric representations of vectors and applications in their final mindmap than in their initial mindmap. This could be attributed to re-reading the assignment instructions and receiving instructor feedback after submitting the initial mindmap. However, geometric and numeric representations were sparsely used for every topic on both the initial and final mindmaps. Despite being given encouragement to graphically display information, students tended to rely on verbal and symbolic representations of vectors, and few initially reported applications of vector concepts. These observations are consistent with a think-aloud study of engineering students working through three-dimensional force problems (Miller-Young, 2013). Furthermore, the nature of the connections that students made and their use of representations supports research that shows that novices organize their groupings by surface-level features and use only one or two representations, while experts tend to cluster apparently different situations together into meaningful groups using a greater variety of representations (Kozma, 2003).

Limitations

Because the students were allowed to access class notes, the textbook, and online resources, the mindmaps created may not reflect the students' understanding of vectors. However, since students were allowed to use these mindmaps on their in-class exam, the mindmaps may reflect what students viewed as important or critical information about vectors. Also, since students were allowed to copy material from other sources, the representations that they added to their mindmaps may indicate what they found readily available versus the representation that they would have chosen to create on their own. Because the assignment was carried out on the computer, technological constraints may have influenced the representations that the students chose to include in their mindmaps. For instance, a student may have found it more convenient to type a verbal description rather than a symbolic description involving subscripts. Furthermore, the assignment instructions and instructor feedback to include multiple representations may have influenced students to include representations of vectors beyond what they would have chosen on their own. Finally, this study was limited to a sample of students from one section of multivariable calculus. A broader sample of students from different schools may provide varied results based on instructor emphasis and student background.

Conclusion

By examining how students communicate vector topics on mindmaps over time, this study contributes to the body of research on student understanding of vectors. Knowledge about which connections and which representations the students communicate can inform pedagogical practices and the development of technological environments to help students coordinate the ideas and representations (Kozma, 2003). Our study triangulates research on critical features of vectors (VanDieren et al., 2017). Additionally, our research serves as a testing ground for Simon's theoretical construct (2017) at the undergraduate level, since it was originally developed for K-12 content. Finally, critical features, and as Simon (2017) suggests, identifying student differences between one another and over time can both be used to articulate mathematical concepts that may later be used for assessment of student understanding.

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