Understanding Calculus Students' Thinking about Volume

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We present the methodology and preliminary findings from a pilot study undertaken at three institutions during Spring 2018. Our purpose is to uncover student reasoning around volumes of solids of revolution. Initial findings suggest issues arise in the Product layer of the Riemann Integral Framework (Sealey, 2014).

Keywords: Geometric and Spatial Thinking, Calculus, Post-Secondary Education

Motivation and Research Question

The purpose of this study is to explore student thinking of volume in the context of a second semester calculus course. We are interested in sharing our methodology and in exploring its applicability to a follow-up research project.

The central research question that guides this study is, *how do students think about the Riemann Sum and Integral when approximating and computing volumes of solids of revolution?* We consider the Riemann Integral Framework (Sealey, 2014), and the role of visualization (Giaquinto, 2007; Tall, 1991) to make sense of student thinking evident in our data.

Methods

Student participants in this study were enrolled in Calculus II. We videotaped interviews of three pairs of students from three institutions as they collaborated on mathematical tasks. Students were paired so that we might capture student thinking through discourse. The tasks we created focused on volumes of spheres. Our first problem prompted students to approximate the volume of a rotated semi-circle by slicing into a fixed number of pieces. Follow-up prompts asked students to approximate using an arbitrary number of slices and to compute the exact volume of the resulting sphere. This task also raised questions around the role of dx and Δx in students' computations. In addition to using routine tasks (Brestock and Sealey, 2018), we included Kepler's volume approximation of a sphere (using infinitesimal cones) and asked students to explain this method. Non-routine problems had the potential to reveal student thinking as they moved from *n* subdivisions to the actual volume of a solid.

Results and Discussion

Preliminary results reveal that students had a strong conceptual grasp of the role of dx and Δx in their computations and they understood the exact volume as the limit of a finite sum. However, students had difficulty visualizing appropriate slices of the solid of revolution as cylinders. We partially attribute this difficulty to teaching, where approximating area is given more instructional time than approximating volume. Students also had difficulty distinguishing between varying and constant quantities in setting up a Riemann Sum, e.g. height and radius of a disk. We position this difficulty within the Product layer of the Riemann Integral Framework (Sealey, 2014). As we frame our study based on this pilot project, we hope to revise tasks so that we may gain additional insight into such difficulties. We are also considering how visual thinking arises in the Riemann Integral Framework (Sealey, 2014), as we found this to be essential in constructing volume approximations and deciding on quantities that vary in such problems.

References

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