Calculus TAs' Reflections on Their Teaching of the Derivative Using Video Recall

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This paper addresses the characteristics of first time Calculus I TAs' teaching practice of the derivative and reflection on their teaching using video-stimulated recall. Our analysis using three views of function – correspondence, variation, and covariation – shows that regardless of representations that TAs adopted, their discussion mainly addressed correspondence, and most TAs used correspondence to justify the difference quotient (DQ) in the limit definition as a function, and transition from the derivative at a point and the derivative as a function. TAs also emphasized different uses of letters as an input of the derivative in such transition from students' point of view although they as mathematicians did not see the difference. Some TAs addressed the variational or covariational view in class and/or during reflections but in a limited way by simply acknowledging that quantities "change" without describing how they change.

Keywords: Video recall, Teaching, Reflections on Teaching, Teaching Assistants, Calculus

Graduate teaching assistants (TAs) for Calculus courses play a crucial role in educating Science, Technology, Engineering and Mathematics (STEM) students through significant interactions with students during their classes and office hours (Ellis, 2014). Unfortunately, the ongoing national effort to improve STEM education has found that many students leave STEM after their first year in college and report poor-quality teaching as their reason for leaving (Connolly et al., 2016) and many students still struggle with crucial ideas in Calculus necessary to understand various STEM phenomena (Park, 2013; Thompson & Carlson, 2017). Currently there are ongoing efforts to support TAs as novice teachers of STEM students (e.g., MAA, 2017), but what we as a field know about TAs' teaching practice is still limited. Based on this observation, this paper investigates TAs' teaching of one of the crucial Calculus concepts, the derivative by analyzing their video-recorded lessons and their reflections on their teaching by analyzing the interviews with them, in which they watched the videos and explained their instructional choices in class. The following research questions guided our study:

1. How did TAs discuss the derivative at a point and of a function in class?
2. How did TAs reflect on their class discussions on the derivative?

To answer these questions, we adopted three ways to conceptualize function – correspondence, variation, and covariation (Confrey & Smith, 1994; Thompson & Carlson, 2017) in our analysis of dominant approaches in TAs' classes and consistency between their teaching and reflection.

Theoretical Background

This paper builds upon two bodies of literature: video recall as a tool for teacher learning, and quantitative reasoning addressing mathematical concepts related to functions.

Teacher Learning Through Video Recall

Videos have been widely used as an effective tool in teacher education and professional development to enhance teachers' ability to notice or reflect on the recorded lessons or students' work. Researchers have argued that videos foster ways teachers think about teaching and learning. For example, van Es and Sherin (2006) studied the impact of videos with two groups of teachers who learned to notice different mathematical aspects of students' thinking depending on how the authors designed the use of videos during professional development. Rosaen et al.

(2008) showed that when videos were used for recall, pre-service teachers made more specific observation of their own teaching, focused more on teaching itself than classroom management, and commented more on students than themselves than when they were based on their memory.

Some researchers used videos of teacher's recorded lessons to help them reflect on and improve their teaching practice. Speer and Wagner (2009) used selected videotaped lessons where an instructor had trouble orchestrating class discussions to examine what occurred from the instructors' view point, and with the videos the teacher revisited what occurred at certain moments of teaching and critiqued his methods. Meade and McMeniman (1992) adopted the stimulated recall with videos to make teachers implicit beliefs and assumptions explicit, and Muir (2010) argued that videoed taped lessons are a powerful medium for teacher's deliberate reflection, and eventually led to change in teaching practice that was more effective for students.

This study also uses video footage to stimulate TAs recall and reflection of their teaching practice. Video stimulated recall is often defined as a research method, where "the subject is shown video records of his or her work on a task…immediately after the recoding" (Busse & Ferri, 2003, p. 257). However, due to the research design of this study, we adopted this method without the immediacy. Since we were interested in particular mathematical aspects of the lesson – how the TAs addressed the quantitative reasoning behind the discussion about the derivative – we, researchers, watched videos and selected the clips for crucial moments, before watching them with the TAs instead of immediately showing the whole videotaped lesson to TAs.

Quantitative Reasoning in Calculus

Calculus mainly deals with how quantities vary or covary in terms of rates of change or accumulated rates of change. To understand the teaching and learning of concepts in calculus, researchers have proposed multiple ways to investigate quantitative reasoning for functions which is pervasive in calculus concepts. The first view is *Correspondence* which conceptualizes a function as a process of building "a rule that allows one to determine a unique *y*-value from any given *x*-value," and thus "a correspondence between *x* and *y*" (Confrey & Smith, 1994, p.137), and another way of viewing function is through *Variation* and *Covariation* which focus on varying quantities involved in functions and their relations (Thompson & Carlson, 2017). Researchers who have advocated for variational/covariational reasoning emphasize the importance of understanding the context where functions were used, the quantities involved in functions, and how their coordination changes simultaneously (Confrey & Smith, 1994; Thompson, 1994). Thompson and Carlson (2017) further detailed levels for variational and covariational reasoning starting from no (co)variation towards smooth continuous (co)variation. We adopted and modified their levels by adding new categories to analyze our data specific to the derivative. Due to the limited space, we will only discuss the categories relevant to our data.

Regarding the derivative, one could consider two functions. First, while defining the derivative a point $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$, one can consider the DQ, $\frac{f(a+h)-f(a)}{h}$ as a function of h with:

(a) correspondence where an h value corresponds to a specific value of the DQ,
(b) variation where DQ is changing either smoothly and continuously (smooth continuous variation), or simply approaches (gross variation), or
(c) covariational where DQ and h simultaneously covary smoothly and continuously (smooth continuous covariation), or simply approaching together (gross covariation).

Second, once the derivative at a point is defined, one can apply the definition on an interval where the derivative exists. One can conceptualize this process with:

(a) correspondence by considering each x value corresponding to the value of the derivative at that point,

(b) variation where the derivative changes over an interval either smoothly and continuously (smooth continuous variation), simply increases or decreases (gross variation), and

(c) covariation where the derivative and the independent variable simultaneously covary either smoothly and continuously (smooth continuous covariation) or simply increases or decreases as the independent variable increases (gross covariation).

Using these three approaches, we will investigate how the first year Calculus TAs addressed the quantitative reasoning in their teaching practice and their reflections on their teaching.

Research Design

This study is part of a larger study that includes a semester-long content-specific professional development (PD) for first time TAs for Calculus I. The PD consisted of five 75-min sessions during the semester starting from a week before the TAs started teaching the derivative focusing on varying and covarying quantities involved in the derivative. The current study focuses on TAs' teaching of the derivative and reflections on it. Five TAs, who taught Calculus I recitations in Fall, 2016 and Spring, 2017, participated in this study. Three TAs – Amy, Kay, and Dan – had no previous classroom teaching experience, and two TAs – Lia and Edi – had taught as an instructor before they entered graduate school. At the institution where the study was conducted, Calculus was offered as a large section for 80-180 students and taught by faculty instructors three times a week, and small recitations consisting of 30-35 students were taught by TAs twice a week. The study design consisted of three phases: video-recording of class, the first and second interviews for their reflections on teaching. In Fall 2016, we video-recorded TAs' recitation sections five times when they taught the derivative. The current study focuses on the first two lessons where they defined and used the derivative at a point and the derivative of a function in various contexts. Once the recording was done, we watched the videos and identified video clips for critical moments based on our framework. Then, we invited the TAs to individual one-hour interviews, where we showed each the selected video clips, and asked three questions:

1. What was the main idea that you want to discuss with your students here?
2. What do you think about your wording or representations? Do you have any thing that you want to modify? If so, why?
3. In this episode, do you see anything varying? What is or are varying mathematically?

The first interview occurred in the first week of Spring, 2017 before the PD started. The second interview occurred when they finished teaching the derivative towards the end of Spring, 2017 and the PD ended. The interviews were video recorded and transcribed.

Results

TAs' Approach to the Derivative at a Point with Symbols

While discussing the derivative at a point through the limit process on the DQ with symbols, all TAs addressed only the correspondence view in class except Kay, who additionally mentioned gross covariation (Table 1). During reflections, two TAs consistently addressed correspondence whereas the other two addressed variation or covariation. One TA, Kay addressed correspondence and mentioned gross covariation in class and during reflections.

All TAs started the derivative unit by defining and computing the derivative at a point with symbols and used the correspondence as the main approach in teaching and reflections. They conceptualized the DQ as a function, i.e., the corresponding value of the function DQ of the independent variable (e.g., h approaching 0). One of the TAs, Edi, explicitly used the word "function" for the DQ in class while emphasizing the correspondence by saying, "the limit as x goes to 1 of that function and just plug in 1 if the function exists there and negative 2 is the answer” for $\lim_{x\to 1}\frac{\frac{2}{x}-2}{x-1}=\lim_{x\to 1}\frac{-2}{x}=-2$. Edi and another TA, Lia, whose reflection also addressed the correspondence, used the word “function” again for the DQ and emphasized its existence at the point where the limit is computed. Edi even chose the correspondence as his main view on the limit process over the variational view; to the interviewer’s question “what is or are varying?” in his algebraic computation of the derivative at a point, he responded, “it is a tangent line or a secant becoming a tangent line, but I guess I kind of see it more as evaluating the limit.” It should be noted that Lia also mentioned variational view in her reflection as a response to the same question, but it was a simple mention about the limit symbol (e.g., $\lim_{h\to 0}$) by saying “*h* is varying here” without any connection to the DQ.

Another TA, Kay, used the function composition notation to compute the DQ for a given function, which also emphasized the correspondence view (Figure 2):

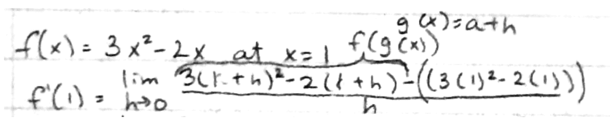

Figure 2. Kay’s use of function composition for the DQ.

> A very difficult step, often when we're doing this limit definition of the derivative, is this plugging in step right here (gestures to $3(a+h)^2-2(a+h)$). It can be very difficult because you have to plug in *a*+*h* anywhere you see an *x*, and that can be confusing. But if you think of it as a composition, it might be a little easier. So I'm gonna let *g(x)* be *a*+*h* (writes *g(x)*=*a*+*h*) and *f(x)* is your function to begin with, then the first part here (puts a curly bracket around $3(a+h)^2-2(a+h)$ and writes *f(g(x))*)…wherever I see an *x* in my function *f*, I'm gonna plug in all of the *g(x)*.

It should be noted that in Figure 2, the input *g(x),* which Kay emphasized, shed light on Kay’s discussion of the quantity that varies in the DQ. With the limit symbol, $\lim_{h\to 0}$ attached to the DQ in computing $f'(1)$, *h* is an independent varying quantity in $(a+h)$. Therefore, a natural way to set up a function for $(a+h)$ would be $g(h)$ rather than $g(x)$. While watching this video, she did not comment on her use of *x*, but emphasized the substitution process to simplify computation.

The reflections of three TAs – Amy, Kay, and Dan – addressed variation or covariation. Both Amy’s and Kay’s reflections included that *h* and the DQ are varying simultaneously, but described this as a simple gross behavior of “changing” or “getting closer to.” Dan, took a different approach from the first reflection to the second. In the first reflection, he addressed the gross variation on the location by simply mentioning “*h”* as “a variable…go[ing] to 0” in $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. In the second reflection, he identified both the DQ and the location as covarying at the gross level using graphical terms and connected it back to the algebraic expression:

> This (pointing to $\frac{f(x)-f(a)}{x-a}$ on the screen) is actually the slope of the secant line. So this one, corresponding, is changing. But, here (pointing to $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$) we are interested in the limit…Because *x* is approaching *a*…I expressed that in terms of *x*. And finally, when we carry out this calculation, *x* goes to *a*. So we can give out an exact number. We know where the limit is. So that's how we get the slope of the tangent line.

TAs' Approach to the Derivative at a Point with Graphs

Two TAs, Amy and Lia addressed the limit process with two types of graphs: non-linear and piece-wise linear. On both graphs, they drew a few secant lines approaching the tangent line at a point. Amy drew secant lines without marking x values and only described their behavior approaching the tangent line with gross variation. In comparison, Lia plotted both the x values and the secant lines and described how they covary with gross covariation.

TAs' reflections were consistent; Lia' reflection addressed the gross covariational view of the slopes of the secant lines and the corresponding location of x without explicitly mentioning how both quantities vary or covary. Amy's first reflection was also consistent with her teaching addressing the gross variation of the slopes of secant lines without the location on the non-linear function graph, but the gross covariation of the piece-wise linear function ("I can get as close as I want [bring hands closer to each other horizontally, and it [the slope of secant line] will always be positive"]). However, a piece-wise linear function only provides a limited context to discuss variational or covariational reasoning since the slopes of the secant lines are constant. During the second reflection, Amy explicitly addressed the missing component – the location, and the covarying relationship between the location and the change at the gross level: "the values of x are changing and the slopes of the secant lines are therefore changing."

TAs' Approach to the Derivative of a Function with Symbols

All TAs' discussions on the derivative as a function in class using symbols addressed correspondence. Their reflections on those discussions also addressed the correspondence. In the discussion of the correspondence between the input x value and the derivative value, TAs focused on the input, and some TAs also emphasized different uses of letters as an input.

In the discussions of the derivative function, all TAs used most of their class period computing the derivative of a function given as an equation applying the limit on the simplified DQ. The derivative process became only explicit when TAs substituted a number in for x to compute the derivative at a number. Three TAs – Dan, Amy, and Lia – introduced the derivative as a function using the correspondence between an input and the output value of the derivative. For example, Dan used a feeding mechanism analogy:

> [DF-1] What we have done f prime a is equal 2a *(writes $f'(a) = 2a$)*. So we can think of, like *the function* as a lazy dog…You feed the dog something and the dog will come up with something, so you feed in an a, and you get a result 2a. But if we changed the variable, here is a fixed number *(points to a)*, x equals to a *(writes x=a)*, but let's just say in general, cause x always equals to a, right? So if we feed in x, you will get f prime x *(writes $f'(x) = 2x$)*, right, so therefore f prime x is the function. So, we have done f prime negative 1 *(writes $f'(-1)=$)* and we'll get a equals -1, *(points to a in 2a)*, so it's negative 2 *(writes -2)*. So any number we're feeding, any real number will get an actual number (points to "$f'(x) = 2x$").

Here his analogy of feeding a lazy dog highlights the corresponding relation between an input and the corresponding output. Specifically, he interpreted the function as the correspondence between an input changed from a as "a fixed number," then "x" in general, and then a number a = -1, and its corresponding derivative 2a, $2x$, and -2, respectively.

Three TAs – Dan, Amy, and Kay – emphasized the role of the input of the derivative function again during their reflections using a correspondence perspective. They specifically explained why "a" or "x_0" are different from "x" from the student point of view while acknowledging that a and x are the same mathematically. For example, in his second reflection about the lesson above [DF-1], Dan said:

> [DF-2] Here, from our point of view, it's [sic] just change x into a or change a to x. You can get from one to another. But um, from the learner's point, they may not see that easy, oh right, you just change the letter x uh into a, so you claim they're the same thing. Like, it's not that easy because again, usually a is a constant. x is a variable of a function that's not to be touched.
> [DF-3] I think here if I actually graph this, we can say okay, we can actually graph this, so this looks like it's a function, right? So, then I should probably say, okay, because a is, you can put in any number here. So why not just make it to be a variable? Instead of some fixed number.

Here, Dan said that mathematically "a" and "x" are not different, but explicitly differentiated "a" as constant or fixed from "x" as "not to be touched," ([DF-2]) and justified the transition from a fixed number "a" to the variable "x" by that any number can be substituted in "a" ([DF-3]). His use of "x" as something "not to be touched" is consistent of his use of "a=-1" instead of "x=-1" in his class ([DF-1]). In reflection, he explained that a direct transition "a" in $f'(a)$ "x" in $f'(x)$ would be hard for students because "they are not taught that we can substitute a constant by a variable."

It should be noted that two TAs identified varying/covarying quantities in the limit process on DQ rather than the change of letters or numbers as an input of the derivative function. Specifically, while reflecting on class discussions on the computation of the derivative at a point, Dan and Kay explicitly chose h in $\lim_{h \to 0}$, and then the corresponding DQ as changing whereas stating that they did not view changing the input for the derivative function from a letter x or a to a number as varying.

TAs' Approach to the Derivative of a Function with Graph

All TAs used graphs to discuss the derivative of a function while graphing it or finding its range in class using a correspondence perspective, and their reflections also addressed the correspondence view. However, but three TAs simply mentioned gross covariation between x and the derivative (e.g., when x changes, the slope changes) in their second reflection when they reflected on their teaching involving graphs. TAs' class discussions were limited from the variation or covariation point of view; graphing and describing the behavior of the derivative of a function was mainly based on the correspondence and the sign of the derivative of a function without addressing how the associated quantities vary or covary. For example, two TAs, Edi and Kay, drew the derivative of a function given as a graph but based on the correspondence rather than on variation or covariation. Specifically, Edi graphed the derivative of a piece-wise linear function, which provided only limited context for variational and covariational aspects of it. He mainly read the slope for each interval to graph the derivative function without discussing any variation. Kay drew the graph the derivative of a non-linear function, but only considered the sign of the slope of the function on the intervals partitioned by the critical values and graphed the derivative as if it were piece-wise linear (Figure 2):

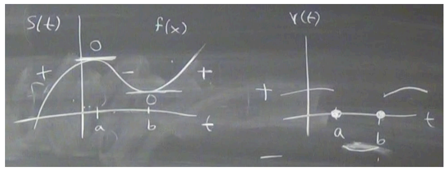

Figure 2. Kay's graph of the derivative of a function

Kay: You're gonna find out…the value of the slope, the exact value, the magnitude, but sometimes it's actually really more important just to know the sign of the slope. So, over here (points to most left), what is the sign of my slope?

Students: Positive

Kay: It's positive, right? So I'm just gonna write a little positive here. *(puts + next to curve)* Where is my slope zero?

Student: At the top

Kay: At this little crest? Right? And right here right? *(gestures to first and second critical points of curve, draws horizontal line, and writes 0 near).*

As Kay pointed out the point of discussion was to know the sign of the slope of the distance function on each interval instead of how the slope varied as a quantity. Moreover, how the original function behaved was not discussed in the class. Instead, the graph of the derivative was drawn completely by reading the sign of the slope of the original function from the graph without associating it with discussing how the original graph behaved.

Discussion and Conclusion

Our analysis of TAs teaching practice of the derivative showed that the correspondence was their main approach to discussing the derivative at a point through the limit process on the DQ, and the derivative as a function. Correspondence view was prominent in class, even when the TAs used graphs, which are often used to visualize the behavior of a function that is often overlooked with algebraic representations. Most TAs either chose limited contexts involving a linear original function whose rate of change is constant, and even when they chose non-linear functions as original functions, they graphed the derivative mainly focused on the sign of the slope on intervals partitioned by critical values without considering how the quantities involved in the function or the derivative vary or covary between those values. During the reflection, TAs often addressed a gross variational or covariational view in addition to correspondence. TAs, especially the ones who had not taught before, showed progress towards covariational reasoning by identifying missing quantities from their class discussions, and describing their relation in terms of how the quantities involved covary.

The results of this study provide valuable information for content-specific PD for TAs. TAs' adoption of correspondence was dominant even when they were using graphical representations which are often used to emphasize the behavior of changing quantities and their relationships. Their approach could be extended to include other approaches by letting them think about how different types of context, problems, and representations could promote discussion of varying and covarying quantities. Also, PD material should challenge TAs' own content knowledge for them to revisit the missing components and relations that could be in their teaching of calculus, based on which TAs could build up the mathematical knowledge for teaching to promote quantitative reasoning in students that is crucial in STEM fields.

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References

Busse, A., & Ferri, R. B. (2003). Methodological reflections on a three-step-design combining observation, stimulated recall and interview. *Zentralblatt für Didaktik der Mathematik*, *35*(6), 257-264.

Confrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. In *Learning Mathematics* (pp. 31-60). Springer Netherlands.

Connolly, M., Savory, J., Ward, A., Dang, A., Rubinyi, L., Lee, Y., Benbow, R., Vivyan, E., Hunter, A., Seymour, E., Weston, T, Thiry, H., Harper, R., Holland, D., Ferrare, J., Miller, J. (2016). *Talking About Leaving, Revisited: Exploring the Contribution of Teaching in Undergraduate Persistence in the Science.* Presented at Envisioning the Future of Undergraduate STEM Education (EnFUSE): Research and Practice symposium.

Ellis, J. (2014) Graduate students Teaching Assistants'(GTAs') beliefs, instructional practices, and student success In (Eds.) T. Fukawa-Connelly, G. Karakok, K. Keene, and M. Zandieh, *Proceedings of the 17th Annual Conference on Research in Undergraduate Mathematics Education*, 2014, Denver, Colorado.

Meade, P., & McMeniman, M. (1992). Stimulated recall—An effective methodology for examining successful teaching in science. *The Australian Educational Researcher*, *19*(3), 1-18.

Muir, T. (2010). Using Video-Stimulated Recall as a Tool for Reflecting on the Teaching of Mathematics. *Mathematics Education Research Group of Australasia*.

Park, J. (2013). Is the derivative a function? If so, how do students talk about it? *International Journal of Mathematical Education in Science and Technology 44*(5), 624–640.

Rosaen, C. L., Lundeberg, M., Cooper, M., Fritzen, A., & Terpstra, M. (2008). Noticing noticing: How does investigation of video records change how teachers reflect on their experiences?. *Journal of teacher education*, *59*(4), 347-360.

Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education*, 530-562.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421-456). Reston, VA: National Council of Teachers of Mathematics.

van Es, E. A., & Sherin, M. G. (2006). How different video club designs support teachers in "learning to notice". *Journal of computing in teacher education*, *22*(4), 125-135.