A Student's Meanings for the Derivative at a Point

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Abstract: The purpose of this study is to examine the meanings and interpretations a student has about the derivative at a point. The responses given by the student is representative of many Calculus 1 students and their beliefs about derivative.

Key Words: Derivative at a Point, Calculus, Student Thinking

This poster discusses one particular student's reasoning about the following task:

Task 4 – The Approximation Derivative Problem

Given that *P(t)* represents the weight (in ounces) of a fish when it is *t* months old,

a. Interpret the statement $P'(3) = 6$

b. If $P(3) = 15$ (and $P'(3) = 6$) estimate the value of $P(3.05)$ and say what this value represents.

Figure 1: Task 4. Interpreting Derivative at a Point

The purpose of this study is to build models of students' mathematics, termed the mathematics of students (Steffe & Thompson, 2000). In this study I attempt to build a model of a student's understandings of the derivative at a point and the factors that might have contributed to the student's responses. The study of derivatives is fundamentally about change and thus dynamic situations, yet as evidenced by Zandieh (2006) students tend to recall the finished static product and not the dynamics involved. The research questions this poster endeavors to address is "What images do students have of derivative at a point? Is it dynamic or static?".

Due to students' wide range of beliefs about functions (Szydlik, 2000) and students' tendency to recall a finished product Zandieh (2006), I theorize that students' will not consider the derivative at a point as concerning a small interval, but rather a point. This notion is reminiscent of Harel and Kaput's (1991) discussion of pointwise versus uniform operators and bolsters this idea on student thinking about functions. Students are often introduced to function as a correspondence (Sfard, 1992) and see one input being mapped to one output. It should be natural then that as this notion is rarely challenged, student's conception of function as a mathematical object (Thompson & Sfard, 1994) has the property of only being concerning with a singular input value. Despite dynamic teachings of the derivative that involve secant lines converging towards a tangent line, Zandieh (2006) notes that students forget the dynamic motion and recall the finished product of the tangent line. This informs the possibility that students may interpret a statement such as $P'(3) = 6$ as one that is focused solely on one point.

This poster presents a study of one student's meaning for the derivative at a point in a quantitative context. I take the constructivist approach (Glasersfeld, 1995) espousing that it is impossible to know completely a student's knowledge and hence the goal is to model the student's beliefs. In this study, the student's responses indicate a consistency with Zandieh's (2006) assertion that students recall a finished product. His responses either noted completed change, or anticipation of change to come both of which lacked dynamic imagery for the point involved. This student's responses indicate a need for teaching of the derivative to flesh out meanings of the derivative at a point so that students might construct a productive meaning for it.

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