## Students' Proving as a Collaborative Work-in-Progress: The Case of a Graduate Course in Topology

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We observed recordings of instances from a graduate course in topology where students engaged in proving theorems on the whiteboard in a collaborative environment. We considered the written component on the whiteboard as "the proof", which was aided, in 17 out of 20 instances by some form of verbal explanation. The peculiarity of the class structure allowed each lesson to be followed by an open discussion regarding "the proof". As a result of the discussions, the written component of each proof would undergo improvements. When analyzing the developments of the proofs in this course, we employed the thematic of proof introduced by Mariotti. Stemmed by these proof-presentations, we introduce the idea of proving as a "work-inprogress" activity.

## Keywords: Collaborative learning, Proof and proving

As part of a larger project on teaching and learning of topology in a collaborative and discursive classroom setting, we analyzed video recordings of an activity that has been atypical to traditional university courses (Pinto & Karsenty, 2018) – students proving theorems on the whiteboard and discussing their proofs with their peers and the course teacher. Our study was aimed at characterizing such an activity with a special focus on what is said, what is written, what is gestured, and the coordination between the three. In 17 out of 20 instances of the activity that took place during a semester, the written component was treated as "the proof", when the verbal counterpart played an auxiliary role of explaining "the proof". The remaining three instances converged to the written components only as the provers did not accompany their work by verbal speech.

After provers have concluded the described phase, the classroom floor was opened for a discussion, in which the rest of the students raised clarification questions and offered suggestions on how the written component could be improved. This part often resulted in the prover revisiting their writing to account for the received feedback. These developments instantiate what Mariotti describes how all proofs have to undergo through a negotiation that leads to social acceptance. An acceptance, in our context, occurred in a classroom setting. Accordingly, this phase can be associated with Mariotti's theoretical frame positioning the activity of proving "as work-in-progress". Indeed, even after the described improvements, it is plausible to think that students' proofs could be further enhanced.

Several conclusions could be drawn from the presented conceptualization. First, proving as a work in-progress is a social and situational activity that mimics to some extent the activity of professional mathematicians. Second, every proving instance of such a kind is unique since neither the coordination of its written, spoken, and gestured components nor the following discussion may be replicated. This uniqueness challenges a common view of students and teachers, in which a resulting text that emerges from a proving activity is treated as an object with proving powers that are indifferent to time, place, and people who engage with it.

## References

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