Hypothetical Learning Trajectory Leveraging Proportional Reasoning

Ishtesa Khan

This poster presents conceptual analysis and hypothetical learning trajectory for learning proportionality which was previously limited only to figure out missing value using cross-multiplication. Based on a series of clinical interview that investigated students' meaning of proportionality in an online format, I found that students tend to use only cross-multiplication strategy to reason proportionally which did not help them to reason proportionally. That emphasizes how learning proportionality along with the constant rate of change among quantities given in any specific word problem helps to reason proportionality conceptually.

Keywords: Proportional Reasoning, Conceptual Analysis, Hypothetical Learning Trajectory.

Introduction and Theoretical Framework

In proportional reasoning, we are interested in comparing quantities in relation to one another instead of finding the 'missing' number of given situations. The fundamental concept we need for proportional reasoning is the idea of 'ratio'. A ratio is a binary relation which involves ordered pairs of quantities. (Lesh, Post, & Behr, 1988). According to Thompson (1994), a ratio is a result of comparing two quantities multiplicatively. When we discuss proportionality we not only consider one ratio, we compare two ratios with likely quantities. And the rate of change of both ratios remains the same constant in this relationship. By Thompson (1994), a rate is a reflectively abstracted constant ratio. Both definitions of ratio and rate followed by Thompson's 1994 paper are fundamental perspectives to look forward to proportional reasoning.

Conceptual Analysis and Hypothetical Learning Trajectory

A conceptual analysis is a way to describe what students might understand about an idea to reason the way it should be understood (Thompson 2008). To conceptualize and reason proportionality I conjecture that the student will need to achieve seven learning goals I tried to identify in this poster. Simon's (1995) development of hypothetical learning trajectory(HLT) is consist of the goal for the student learning, and hypotheses of the students' learning (Simon, M. & Tzur, R., 2004). Generalizing conceptual analysis (Thompson 2008) and HLT (Simon 1995), this poster is going to present an HLT for proportional reasoning-

1. Students will draw a picture which represents the given situation
2. Students will identify quantities and determine whether they are varying or fixed quantities, and they will always verbalize them with corresponding units.
3. Students will be able to represent the situation graphically with scaled measurements.
4. Students will identify the varying quantities in the given situation and will be able to relate these quantities to the constant rate of change.
5. Students will understand that one quantity is as many times bigger or smaller as the second quantity. If there are more than two quantities in one situation they will be able to understand the relationship among them as well.
6. Students will avoid seeing ratio and proportions only as a tool for performing calculations, applying rules and formulae and manipulating numbers and symbols in proportion equations.

References

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